Lösungen der Übungsaufgaben TM III

1 Methoden der Analytischen Mechanik

Aufgabe 1

a) $\delta z = \frac{l}{2}\cos\alpha\,\delta\alpha \qquad \delta x = l\sin\alpha\sqrt{\frac{2}{1+\cos\alpha}}\,\delta\alpha$

b) $\delta W^e = \left(-\frac{Gl}{2}\cos\alpha + Sl\,\sin\alpha\sqrt{\frac{2}{1+\cos\alpha}}\right)\delta\alpha$

c) $S = \frac{\sqrt{2}}{4}G\,\cot\alpha\,\sqrt{1+\cos\alpha}$

Aufgabe 2

a) $\delta W^e = (mg - 4cx)\,\delta x$ b) $x_0 = \frac{mg}{4c}$

Aufgabe 3

a) $\delta x_1 = \delta x_2$ b) $\delta W^e = (Mg - mg\sin\alpha)\,\delta x_1 \qquad \frac{m}{M} = \frac{1}{\sin\alpha}$

Aufgabe 4

b) $\delta W^e = \left(mg\sin\alpha - \frac{1}{4}cx\right)\delta x \qquad x_0 = \frac{4mg}{c}\sin\alpha$

c) $\left(m\ddot{x} - mg\sin\alpha + \frac{1}{4}cx\right)\delta x = 0 \qquad \ddot{x} + \frac{c}{4m}x = g\sin\alpha$

Aufgabe 5

a) $\delta W^e = (mg - 4cx)\,\delta x \qquad x_0 = \frac{mg}{4c}$

b) $(7m\ddot{x} + 4cx - mg)\,\delta x = 0 \qquad \ddot{x} + \frac{4c}{7m}x = \frac{1}{7}g$

2 Diskrete Schwingungssysteme

Aufgabe 1

a) $T = \frac{3}{4}\,m\,\dot{x}^2 \qquad U = 2cx^2 - mgx$ b) $\frac{3}{2}m\ddot{x} + 4cx - mg = 0$

Aufgabe 2

a) $T = \frac{7}{4}mr^2\dot{\varphi}^2 \qquad U = mgr\,(\sin\alpha - 2)\,\varphi$ b) $\frac{7}{2}mr^2\,\ddot{\varphi} + mgr\,(\sin\alpha - 2) = 0$

Aufgabe 3

a) $T = \frac{m}{2}\dot{x}^2\,, \quad U = \frac{c}{8}\,x^2 - mg\sin\alpha\,x$ b) $m\,\ddot{x} + \frac{c}{4}\,x - mg\sin\alpha = 0$

c) $x_0 = \frac{4mg}{c}\sin\alpha$

Aufgabe 4

a) $T = \frac{7}{2}m\,\dot{x}^2 \quad U = 2c\,x^2 - mg\,x$ b) $7m\ddot{x} + 4cx - mg = 0$ c) $x_0 = \frac{mg}{4c}$

Aufgabe 5

a) $T = \frac{1}{2}m\dot{s}^2 + \frac{1}{6}ml^2\dot{\varphi}^2 - \frac{1}{2}ml\sin\varphi\,\dot{s}\,\dot{\varphi} \qquad U = -\,mg\left(s + \frac{l}{2}\cos\varphi\right) + \frac{1}{2}cs^2$

b) $m\ddot{s} - \frac{1}{2}ml\,\sin\varphi\,\ddot{\varphi} - \frac{1}{2}ml\,\cos\varphi\,\dot{\varphi}^2 - mg + cs = 0$

$-\frac{1}{2}ml\,\sin\varphi\,\ddot{s} + \frac{1}{3}ml^2\ddot{\varphi} + \frac{1}{2}mgl\,\sin\varphi = 0$

c) $s = \frac{mg}{c} \qquad \varphi \in \{0,\,\pi\}$ d) $m\ddot{\tilde{s}} + c\tilde{s} = 0 \qquad \frac{1}{3}ml^2\ddot{\tilde{\varphi}} + \frac{1}{2}mgl\,\tilde{\varphi} = 0$

Aufgabe 6

a) $T = \frac{m}{2}\left(\dot{l}^2 + l^2\dot{\varphi}^2\right) \qquad U = -\,mgl\cos\varphi + \frac{c}{2}(l - l_0)^2$

b) $ml^2\,\ddot{\varphi} + 2ml\,\dot{l}\,\dot{\varphi} + mgl\,\sin\varphi = 0$

$m\ddot{l} - ml\,\dot{\varphi}^2 - mg\,\cos\varphi + c\,(l - l_0) = 0$

c) $\varphi = 0,\ l = l_0 + \frac{mg}{c}$ bzw. $\varphi = \pi,\ l = l_0 - \frac{mg}{c}$

d) $m\left(l_0 + \frac{mg}{c}\right)\ddot{\tilde{\varphi}} + mg\,\tilde{\varphi} = 0 \qquad m\ddot{\tilde{l}} + c\tilde{l} = 0$

3 Freie Koppelschwingungen konservativer Systeme

Aufgabe 1

a) $T = \frac{1}{2}\left(I_1 + m_1 L_1^2 + m_2 r^2\right)\dot{\varphi}_1^2 + \frac{1}{2}\left(I_2 + m_2 L_2^2\right)\dot{\varphi}_2^2 + m_2 r L_2 \dot{\varphi}_1 \dot{\varphi}_2 \cos(\varphi_1 - \varphi_2)$

$U = -\left(m_1 g L_1 + m_2 g r\right)\cos\varphi_1 - m_2 g L_2 \cos\varphi_2$

b) $J_{11}\ddot{\varphi}_1 + J_{12}\cos(\varphi_1 - \varphi_2)\ddot{\varphi}_2 + J_{12}\sin(\varphi_1 - \varphi_2)\dot{\varphi}_2^2 + C_1 \sin\varphi_1 = 0$

$J_{12}\cos(\varphi_1 - \varphi_2)\ddot{\varphi}_1 + J_{22}\ddot{\varphi}_2 - J_{12}\sin(\varphi_1 - \varphi_2)\dot{\varphi}_1^2 + C_2 \sin\varphi_2 = 0$

mit: $J_{11} = I_1 + m_1 L_1^2 + m_2 r^2 \qquad J_{22} = I_2 + m_2 L_2^2 \qquad J_{12} = m_2 r L_2$

$C_1 = m_1 g L_1 + m_2 g r \qquad C_2 = m_2 g L_2$

c) $\dfrac{C_1}{J_{11} + J_{12}} = \dfrac{C_2}{J_{12} + J_{22}}$

d) $\begin{bmatrix} J_{11} & J_{12} \\ J_{12} & J_{22} \end{bmatrix}\begin{bmatrix} \ddot{\tilde{\varphi}}_1 \\ \ddot{\tilde{\varphi}}_2 \end{bmatrix} + \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}\begin{bmatrix} \tilde{\varphi}_1 \\ \tilde{\varphi}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

e) $f_1 = 0.54\ \text{Hz} \qquad f_2 = 0.59\ \text{Hz}$

Aufgabe 2

a) $\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1 + c_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

b) $\omega^4 - \left(\dfrac{c_1}{m_1} + \dfrac{c_1 + c_2}{m_2}\right)\omega^2 + \dfrac{c_1 c_2}{m_1 m_2} = 0$

c) $f_1 = 0.76\ \text{Hz} \qquad f_2 = 10.5\ \text{Hz}$

d) $\dfrac{a_2}{a_1} = 1 - \dfrac{m_1 \omega_i^2}{c_1}$

e) für ω_1: gleichphasig; für ω_2: gegenphasig

Aufgabe 3

a) $\begin{bmatrix} m & & \\ & m & \\ & & m \end{bmatrix}\ddot{y} + \begin{bmatrix} c & -c & \\ -c & 2c & -c \\ & -c & c \end{bmatrix} y = \mathbf{0}$

b) $m\omega^2\left(c - m\omega^2\right)\left(3c - m\omega^2\right) = 0$ Eigenfrequenzen: $f_1 = 0,\ f_2 = 1.59\ \text{Hz},\ f_3 = 2.76\ \text{Hz}$

c) Eigenvektoren: $\tilde{y}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},\ \tilde{y}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix},\ \tilde{y}_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

d) $y(t) = -\frac{1}{3}t\,\tilde{y}_1 - 0.05\cos\left(10t - \frac{\pi}{2}\right)\tilde{y}_2 - 0.0096\cos\left(17.32t - \frac{\pi}{2}\right)\tilde{y}_3$

<u>Aufgabe 4</u>

a) $\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 2c & -2c \\ -2c & 4c \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

b) $\omega^4 - 6\,\frac{c}{m}\,\omega^2 + 4\,\frac{c^2}{m^2} = 0$

c) $f_1 = 8.2\ \mathrm{Hz} \quad f_2 = 21.5\ \mathrm{Hz}$ Eigenvektoren: $\frac{a_2}{a_1} = 1 - \frac{m\,\omega_i^2}{2c}$

d) für ω_1: gleichphasig; für ω_2: gegenphasig

4 Erzwungene Schwingungen konservativer Systeme

Aufgabe 1

a) $\boldsymbol{Y} = \begin{bmatrix} 0.6573 & 3.1341 \\ 2.9775 & -8.9950 \end{bmatrix}$

b) $\hat{y}(0) = \begin{bmatrix} 0.0591 \\ 0.02 \end{bmatrix}; \quad \dot{\hat{y}}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

c) $\hat{y}(t) = \begin{bmatrix} 0.06 \cos 4t \\ 0.02 \cos 15t \end{bmatrix}$

d) $\boldsymbol{y}(t) = \begin{bmatrix} 0.0394 \cos 4t + 0,0627 \cos 15t \\ 0.1786 \cos 4t - 0.1799 \cos 15t \end{bmatrix}$

e) $\hat{\boldsymbol{h}}_0 = \begin{bmatrix} 0.0947 \\ 0.4513 \end{bmatrix}$

f) $\boldsymbol{y}_p(t) = \left(\frac{1}{16 - \Omega^2} \begin{bmatrix} 0.062 \\ 0.282 \end{bmatrix} + \frac{1}{225 - \Omega^2} \begin{bmatrix} 1.414 \\ -4.06 \end{bmatrix} \right) \cos \Omega t$

Aufgabe 2

a) $\boldsymbol{Y} = \begin{bmatrix} 0.0580 & 0.0255 \\ 0.1449 & -0.3822 \end{bmatrix}$

b) $\hat{\boldsymbol{h}}_0 = \begin{bmatrix} 5.8 \\ 2.5 \end{bmatrix}$

f) $\boldsymbol{y}_p(t) = \left(\frac{1}{3.43^2 - \Omega^2} \begin{bmatrix} 0.34 \\ 0.84 \end{bmatrix} + \frac{1}{3.69^2 - \Omega^2} \begin{bmatrix} 0.06 \\ -0.96 \end{bmatrix} \right) \cos \Omega t$

Aufgabe 3

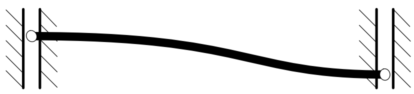
a) $\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1 + c_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ c_2 u_0 \end{bmatrix} \cos \Omega t$

b) $\boldsymbol{y}_p(t) = \boldsymbol{r}_0 \cos \Omega t, \qquad \begin{bmatrix} c_1 - m_1 \Omega^2 & -c_1 \\ -c_1 & c_1 + c_2 - m_2 \Omega^2 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} 0 \\ c_2 u_0 \end{bmatrix}$

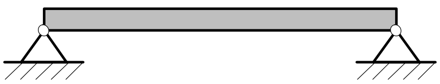
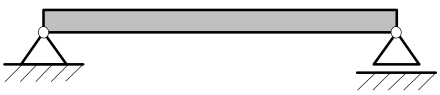
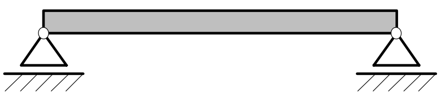
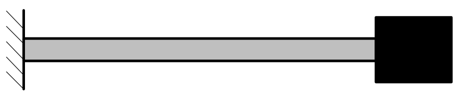
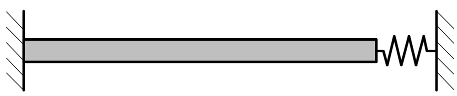
c) $x_1(\Omega)$ – dicke Linie $\qquad x_2(\Omega)$ – dünne Linie

5 Kontinuierliche Schwingungssysteme

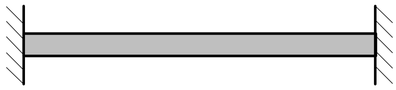
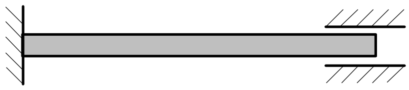
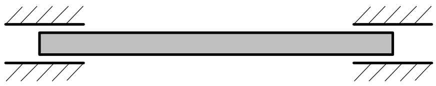
Aufgabe **1**

Einspannung		Randbedingungen
fest – fest		$w(0,t) = 0$ $w(L,t) = 0$
fest – frei		$w(0,t) = 0$ $w'(L,t) = 0$
frei – frei		$w'(0,t) = 0$ $w'(L,t) = 0$

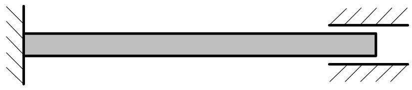
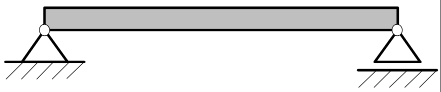
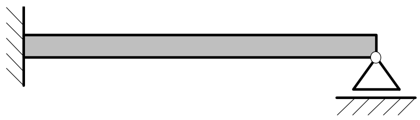
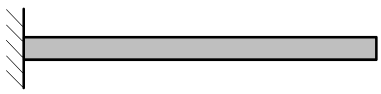
Aufgabe **2**

Lagerung		Randbedingungen
fest – fest		$u(0,t) = 0$ $u(L,t) = 0$
fest – frei		$u(0,t) = 0$ $u'(L,t) = 0$
frei – frei		$u'(0,t) = 0$ $u'(L,t) = 0$
fest – Endmasse m		$u(0,t) = 0$ $u'(L,t) = -\frac{m}{EA}\,\ddot{u}(L,t)$
fest – Endsteifigkeit k		$u(0,t) = 0$ $u'(L,t) = -\frac{k}{EA}\,u(L,t)$

Aufgabe **3**

Lagerung		Randbedingungen
fest – fest		$\varphi(0,t) = 0$ $\varphi(L,t) = 0$
fest – frei		$\varphi(0,t) = 0$ $\varphi'(L,t) = 0$
frei – frei		$\varphi'(0,t) = 0$ $\varphi'(L,t) = 0$

Aufgabe **4**

Lagerung		Randbedingungen
fest – fest		$w(0,t) = 0$ $w(L,t) = 0$ $w'(0,t) = 0$ $w'(L,t) = 0$
gelenkig – gelenkig		$w(0,t) = 0$ $w(L,t) = 0$ $w''(0,t) = 0$ $w''(L,t) = 0$
frei – frei		$w''(0,t) = 0$ $w''(L,t) = 0$ $w'''(0,t) = 0$ $w'''(L,t) = 0$
fest – gelenkig		$w(0,t) = 0$ $w(L,t) = 0$ $w'(0,t) = 0$ $w''(L,t) = 0$
fest – frei		$w(0,t) = 0$ $w''(L,t) = 0$ $w'(0,t) = 0$ $w'''(L,t) = 0$

6 Erzwungene Schwingungen konservativer Systeme

Aufgabe 1

b) $D_k = \sqrt{\frac{2}{L}}$

Aufgabe 2

a) $\ddot{\varphi} = c^2 \varphi''$ mit $c = \sqrt{\frac{G}{\rho}}$ b) $\varphi'(0,t) = 0 \quad \varphi(L,t) = 0$

c) $\cos\frac{\omega}{c}L = 0$ d) $\omega_k = \frac{2k-1}{2}\frac{\pi c}{L}$ für $k = 1, 2, ...$

e) $f_1 = 775 \text{ Hz}, \quad f_2 = 2325 \text{ Hz}$ f) $W_1 = C_1 \cos\frac{\pi}{2}\frac{x}{L} \qquad W_2 = C_2 \cos\frac{3\pi}{2}\frac{x}{L}$

Aufgabe 3

a) $\ddot{u} = c^2 u''$ mit $c = \sqrt{\frac{E}{\varrho}}$ b) $u(0,t) = 0 \qquad u'(L,t) + \mu\frac{L}{c^2}\ddot{u}(L,t) = 0$

c) $\cot\frac{\omega L}{c} = \mu\frac{\omega L}{c}$ d) $f_1 = 1298 \text{ Hz}$

e) $f_1 = 710 \text{ Hz}$

f) $\mu = 0{:}\ W_1 = D_1 \sin\frac{\pi}{2}\frac{x}{L} \qquad \mu = 1{:}\ W_1 = D_1 \sin 0.86\frac{x}{L}$

Aufgabe 4

a) $\ddot{u} = c^2 u''$ mit $c = \sqrt{\frac{E}{\varrho}}$ b) $u(0,t) = 0 \quad u'(L,t) + \frac{k}{EA}u(L,t) = 0$

c) $\tan\frac{\omega L}{c} = -\frac{1}{\mu}\frac{\omega L}{c}$ d) $f_1 = 1671 \text{ Hz}$

e) $\mu \to \infty \qquad \sin\frac{\omega}{c}L = 0$ d.h. Lagerung: fest – fest

$\mu \to 0 \qquad \cos\frac{\omega}{c}L = 0$ d.h. Lagerung: fest – frei

7 Eigenschwingungen des Balkens

Aufgabe 1

a) $\ddot{w}(x,t) + \frac{EI}{\rho A} w^{IV}(x,t) = 0 \qquad w''(0,t) = w'''(0,t) = w''(L,t) = w'''(L,t) = 0$

b) $1 - \cos\gamma L \ \cosh\gamma L = 0$

c) $W_i(x) = C_i\left(\cos\gamma_i x + \cosh\gamma_i x + \frac{-\cos\gamma_i L + \cosh\gamma_i L}{\sin\gamma_i L - \sinh\gamma_i L}(\sin\gamma_i x + \sinh\gamma_i x)\right)$

Aufgabe 2

Torsionsschwingungen, $f_1 = 775$ Hz

Längsschwingungen, $f_1 = 1298$ Hz

Biegeschwingungen, $f_1 = 7.3$ Hz

Aufgabe 3

a) $\frac{\omega_{V_k}}{\omega_{H_k}} = \frac{h}{b}$

Aufgabe 4

a) $f_1 = 0.81$ Hz b) $f_1 = 0.43$ Hz c) $f_1 = 0.42$ Hz

Aufgabe 5

$L^2 = \frac{(\gamma_1 L)^2 \, c \, s}{\omega_1 \, 2\sqrt{3}} \qquad L \approx 0.031 \text{ m}$

8 Freie Schwingungen kontinuierlicher Systeme

Aufgabe 1

a) $\omega_k = \frac{2k-1}{2}\frac{\pi c}{L}$ $\qquad W_k(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{2k-1}{2}\frac{\pi x}{L}\right)$

b) $w(x,t) = \sqrt{\frac{2}{L}}\sum_{k=1}^{\infty}\sin\left(\frac{2k-1}{2}\frac{\pi x}{L}\right)y_{0k}\cos\left(\frac{2k-1}{2}\frac{\pi c}{L}t-\varphi_k\right)$

c) $w_0(x) = \frac{a}{L}x$ $\qquad \dot{w}_0(x) = 0$

d) $y_{0k} = (-1)^{k+1}\frac{4\sqrt{2L}}{(2k-1)^2\pi^2}a$ $\qquad \varphi_k = 0$

e) $w(x,0) = w_0(x)$ $\qquad w\left(x,\frac{L}{c}\right) = 0$ $\qquad w\left(x,\frac{2L}{c}\right) = -w_0(x)$

Aufgabe 2

a) $\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ $\qquad \boldsymbol{\Omega}^2 = \begin{bmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{bmatrix}$ mit $\omega_1^2 = \left(\frac{\pi c}{2L}\right)^2$ $\quad \omega_2^2 = \left(\frac{3\pi c}{2L}\right)^2$

b) $\boldsymbol{y}(0) = \begin{bmatrix} y_{0_1} \\ y_{0_2} \end{bmatrix}$ mit $y_{0_1} = \frac{4a\sqrt{2L}}{\pi^2}$ $\quad y_{0_2} = -\frac{4a\sqrt{2L}}{9\pi^2}$, $\qquad \dot{\boldsymbol{y}}(0) = \boldsymbol{0}$

c) $y_k(t) = y_{0_k}\cos\omega_k t$

d) $w(x,t) = \frac{8a}{\pi^2}\left(\sin\frac{\pi x}{2L}\cos\frac{\pi ct}{2L} - \frac{1}{9}\sin\frac{3\pi x}{2L}\cos\frac{3\pi ct}{2L}\right)$

Aufgabe 3

a) $y_k(0) = \frac{9a\sqrt{L}}{\sqrt{2}k^2\pi^2}\sin\frac{2k\pi}{3}$ $\qquad \dot{\boldsymbol{y}}(0) = \boldsymbol{0}$

b) $y_k(t) = y_{0_k}\cos\omega_k t$ mit $y_{0_k} = y_k(0)$ $\qquad \omega_k = \frac{k\pi c}{L}$

c) $w(x,t) = \frac{9a}{\pi^2}\sum_{k=1}^{\infty}\left(\frac{1}{k^2}\sin\frac{2k\pi}{3}\sin\frac{k\pi x}{L}\cos\frac{k\pi ct}{L}\right)$

d) $\frac{1}{a}w(x,0) \approx 0.79\sin\frac{\pi x}{L} - 0.20\sin\frac{2\pi x}{L}$

Aufgabe 4

$w(x,t) = a\sin\frac{\pi x}{L}\cos\omega_1 t$

9 Erzwungene Schwingungen durch verteilte Kräfte

Aufgabe 1

a) $c^2 = \frac{S}{\varrho A} \qquad q(x,t) = \frac{p(x,t)}{\varrho A}$ b) $c^2 = \frac{G}{\varrho} \qquad q(x,t) = \frac{m(x,t)}{\varrho I_p}$

Aufgabe 2

$$\ddot{w} + \frac{EI}{\varrho A} w^{IV} = \frac{p(x,t)}{\varrho A} \equiv q(x,t)$$

Aufgabe 3

a) $p(x,t) = g\varrho A \qquad \ddot{w} = \frac{S}{\varrho A} w'' + g \qquad w(0,t) = w(L,t) = 0$

b) $\omega_k = k\frac{\pi c}{L}\,, \quad k = 1,2,... \qquad W_k(x) = \sqrt{\frac{2}{L}} \sin\frac{k\pi x}{L}\,, \quad k = 1,2,...$

c) $y_k = \begin{cases} \frac{2g\sqrt{2L}\,L^2\varrho A}{k^3\pi^3 S}(1 - \cos\omega_k t) & k - \text{ungerade} \\ 0 & \text{sonst} \end{cases}$

d) $w(x,t) = \frac{4gL^2\varrho A}{\pi^3 S} \sum_{k=1,3,...} \frac{1}{k^3} \sin\frac{k\pi x}{L}(1 - \cos\omega_k t)$

Aufgabe 4

a) $\ddot{w} + \frac{EI}{\varrho A} w^{IV} = \frac{p_0}{\varrho A} \qquad w(0,t) = w(L,t) = w''(0,t) = w''(L,t) = 0$

b) $\omega_k = k^2\pi^2 \sqrt{\frac{EI}{\varrho A L^4}}\,, \qquad W_k(x) = \sqrt{\frac{2}{L}} \sin\frac{k\pi x}{L}\,, \qquad k = 1,2,...$

c) $h_k(t) = \begin{cases} \frac{2\sqrt{2L}\,p_0}{k\pi\varrho A} & k - \text{ungerade} \\ 0 & \text{sonst} \end{cases}$ d) $w_p(x,0) = \frac{4}{\pi^5}\frac{p_0 L^4}{EI} \sum_{k=1,3,...} \frac{1}{k^5} \sin\frac{k\pi x}{L}$

e) Biegelinie: $w_B(x) = \frac{p_0 L^4}{24EI}\left[\frac{x}{L} - 2\left(\frac{x}{L}\right)^3 + \left(\frac{x}{L}\right)^4\right]$

$w_B(L/2) \approx 0.0130\,\frac{p_0 L^4}{EI} \qquad w_p(L/2,0) \approx 0.0131\frac{p_0 L^4}{EI}$

Aufgabe 5

a) $h_k(t) = \frac{F_0\sqrt{2}}{\varrho A\sqrt{L}} \sin\frac{k\pi a}{L} \cos\Omega t$ b) $y_{pk}(t) = \frac{F_0\sqrt{2}}{\varrho A\sqrt{L}} \cdot \frac{1}{\omega_k^2 - \Omega^2} \cdot \sin\frac{k\pi a}{L} \cdot \cos\Omega t$

c) $w_p(x,t) = \frac{2F_0}{L\varrho A} \sum_{k=1}^{\infty} \left[\frac{1}{\omega_k^2 - \Omega^2} \sin\frac{k\pi a}{L} \sin\frac{k\pi x}{L}\right] \cos\Omega t$

10 Erzwungene Schwingungen durch inhomogene Randbedingungen

Aufgabe 1

a) $\ddot{w} = c^2 w''$ $\quad w(0,t) = 0 \quad w'(L,t) = \frac{\hat{F}}{S}\sin\Omega t$

b) $w_R(x,t) = \frac{\hat{F}}{S} x \sin\Omega t$

c) $\ddot{w}_H = c^2 w_H'' + \frac{\hat{F}}{S} x\, \Omega^2 \sin\Omega t$

d) $$w_H(x,t) = \sqrt{\frac{2}{L}} \sum_{k=1}^{\infty}\left[\hat{y}_k \cos(\omega_k t - \varphi_k)\, \sin\frac{(2k-1)\pi x}{2L}\right] - $$
$$- \frac{8\hat{F}L}{\pi^2 S} \sin\Omega t \sum_{k=1}^{\infty}\left[\frac{(-1)^k}{(2k-1)^2}\, \frac{\Omega^2}{\omega_k^2 - \Omega^2}\, \sin\frac{(2k-1)\pi x}{2L}\right]$$

e) $$w(x,t) = \sqrt{\frac{2}{L}} \sum_{k=1}^{\infty}\left[\hat{y}_k \cos(\omega_k t - \varphi_k)\, \sin\frac{(2k-1)\pi x}{2L}\right] + $$
$$+ \frac{\hat{F}L}{S} \sin\Omega t \left\{\frac{x}{L} - \frac{8}{\pi^2} \sum_{k=1}^{\infty}\left[\frac{(-1)^k}{(2k-1)^2}\, \frac{\Omega^2}{\omega_k^2 - \Omega^2}\, \sin\frac{(2k-1)\pi x}{2L}\right]\right\}$$

Aufgabe 2

a) $\ddot{w} = c^2 w''$ $\quad w(0,t) = w(L,t) = a\cos\Omega t$

b) $w_R(x,t) = a\cos\Omega t$

c) $\ddot{w}_H = c^2 w_H'' + a\Omega^2 \cos\Omega t$

d) $$w_{H_p} = \frac{4a}{\pi} \cos\Omega t \sum_{k=1,3,\ldots}\left[\frac{1}{k}\, \frac{\Omega^2}{\omega_k^2 - \Omega^2}\, \sin\frac{k\pi x}{L}\right]$$

e) $$w_p(x,t) = a\cos\Omega t\left\{1 + \frac{4}{\pi} \sum_{k=1,3,\ldots}\left[\frac{1}{k}\, \frac{\Omega^2}{\omega_k^2 - \Omega^2}\, \sin\frac{k\pi x}{L}\right]\right\}$$

Aufgabe 3

a) $\ddot{w} = c^2 w''$ $\quad w(0,t) = -w(L,t) = \alpha\frac{L}{2}\cos\Omega t$

b) $w_R(x,t) = \alpha\left(\frac{L}{2} - x\right)\cos\Omega t$ $\quad$ c) $\ddot{w}_H = c^2 w_H'' + \alpha\Omega^2\left(\frac{L}{2} - x\right)\cos\Omega t$

d) $$w_{H_p} = \frac{2L\alpha}{\pi} \cos\Omega t \sum_{k=2,4,\ldots}\left[\frac{1}{k}\, \frac{\Omega^2}{\omega_k^2 - \Omega^2}\, \sin\frac{k\pi x}{L}\right]$$

e) $$w_p(x,t) = L\alpha\cos\Omega t\left\{\frac{1}{2} - \frac{x}{L} + \frac{2}{\pi} \sum_{k=2,4,\ldots}\left[\frac{1}{k}\, \frac{\Omega^2}{\omega_k^2 - \Omega^2}\, \sin\frac{k\pi x}{L}\right]\right\}$$

11 Wellenausbreitung in eindimensionalen Kontinua

Aufgabe 1

$\frac{c_L}{c_T} \approx 1.6$

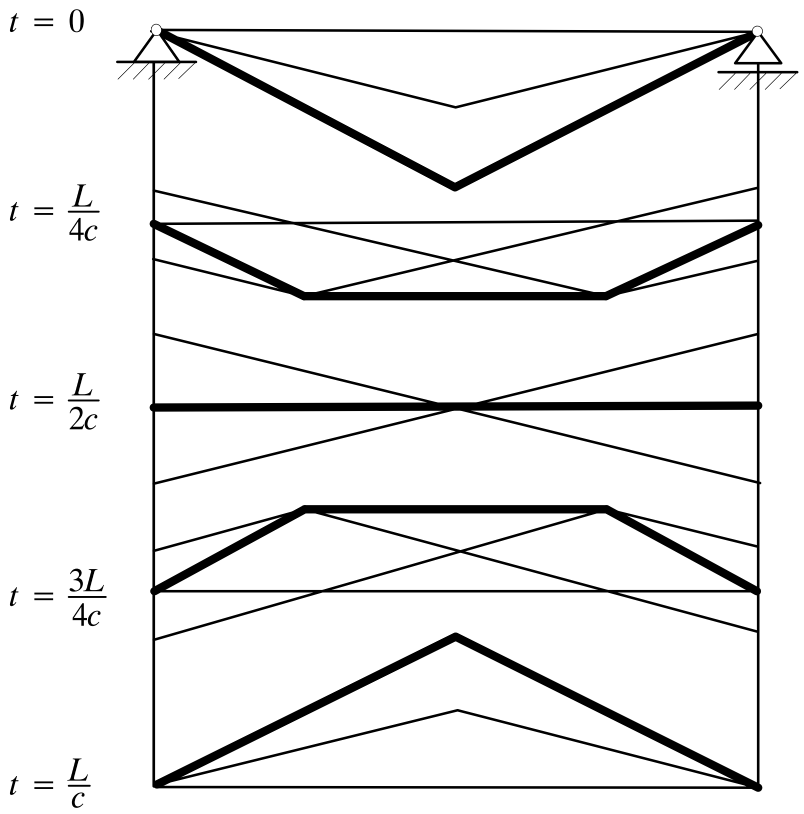
Aufgabe 2

a) $w(x,0) = a \sin \frac{\pi x}{L} \qquad \dot{w}(x,0) = 0 \qquad\qquad f_{1,2}(x) = \frac{a}{2} \sin \frac{\pi x}{L}$

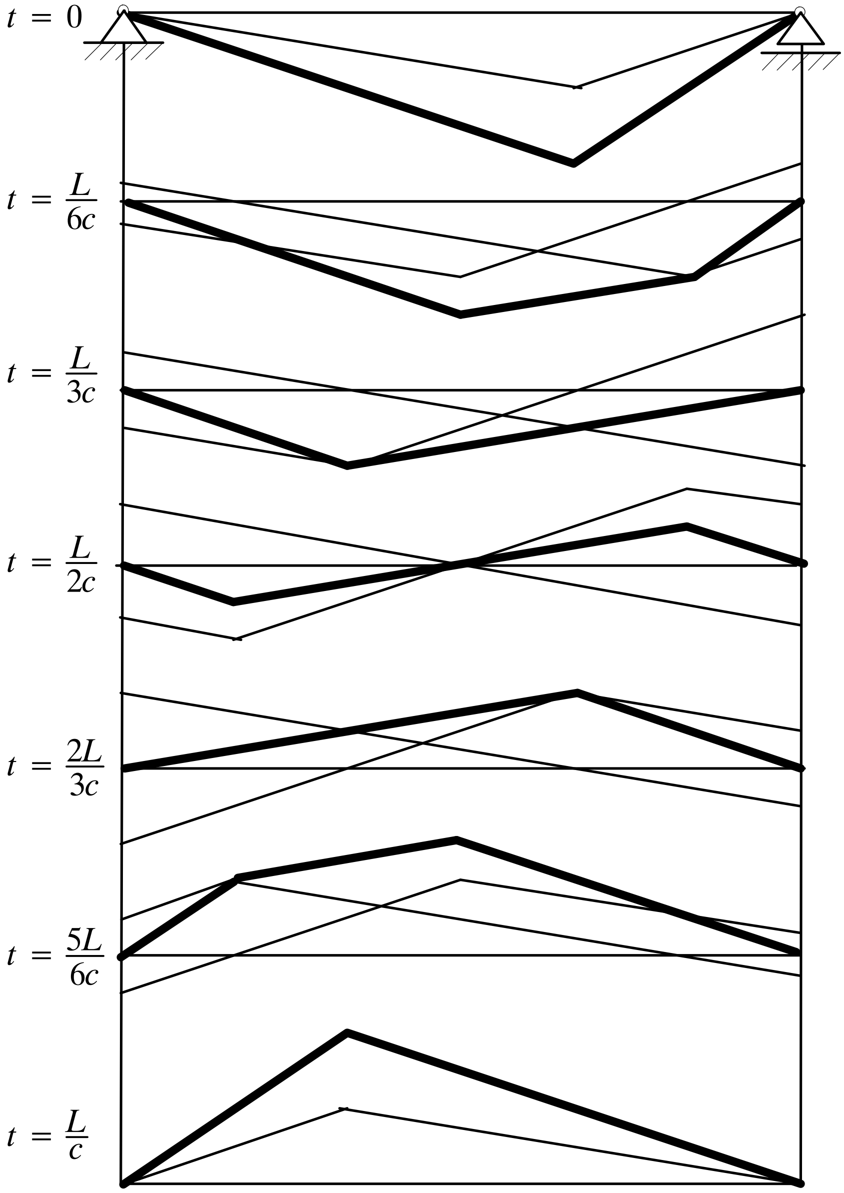
b) $w(x,t) = a \sin \frac{\pi x}{L} \cos \frac{\pi c t}{L}$

c) $w(x,t) = a \sin \frac{\pi x}{L} \cos \frac{\pi c t}{L}$

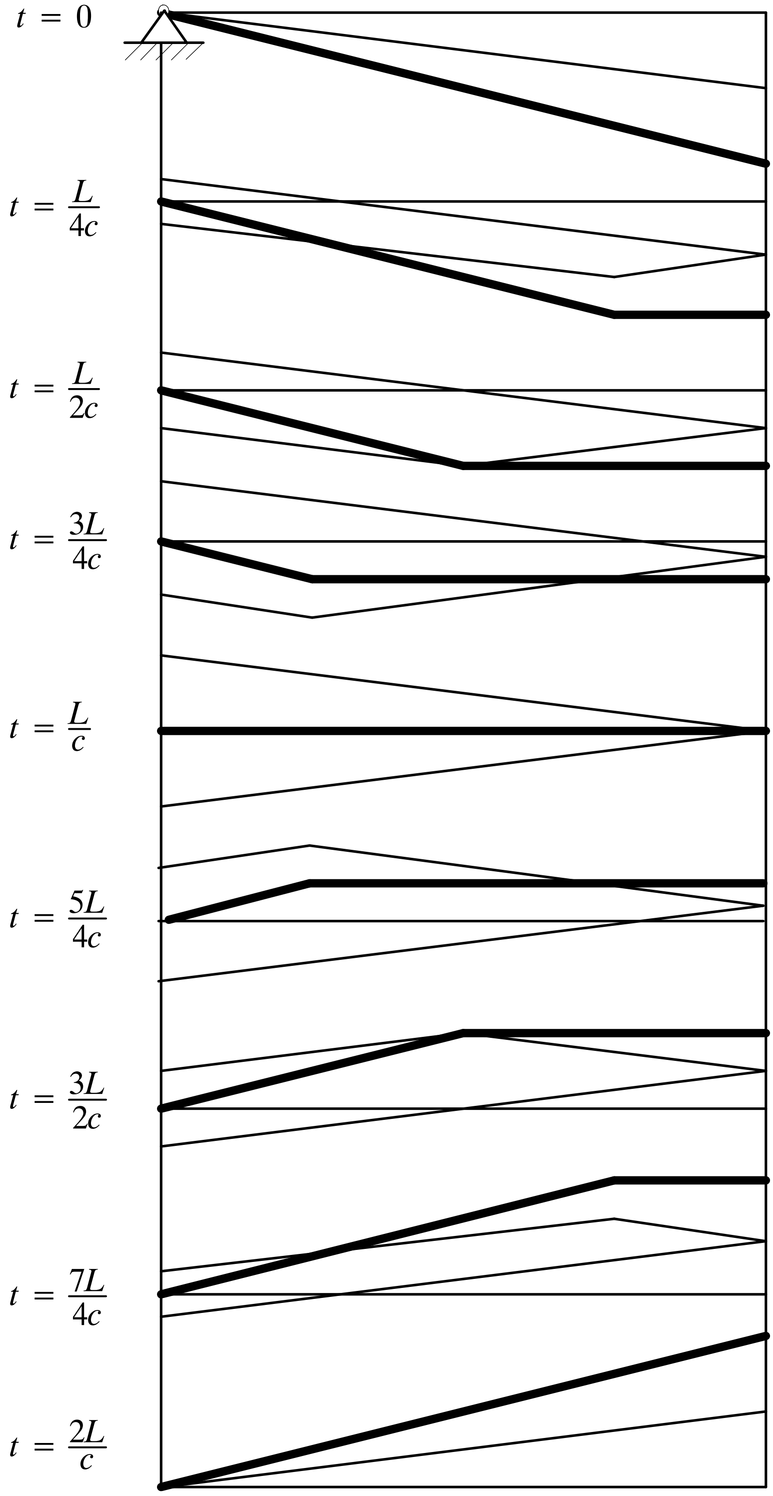
Aufgabe 3



Aufgabe 4


$t = 0$
$t = \frac{L}{6c}$
$t = \frac{L}{3c}$
$t = \frac{L}{2c}$
$t = \frac{2L}{3c}$
$t = \frac{5L}{6c}$
$t = \frac{L}{c}$

Aufgabe 5


$t = 0$
$t = \frac{L}{4c}$
$t = \frac{L}{2c}$
$t = \frac{3L}{4c}$
$t = \frac{L}{c}$
$t = \frac{5L}{4c}$
$t = \frac{3L}{2c}$
$t = \frac{7L}{4c}$
$t = \frac{2L}{c}$

12 Fluidstatik

Aufgabe 1

a) $\dot{\gamma} = 167.8 \, \frac{1}{\mathrm{s}}$ b) $h = 1.19 \cdot 10^{-4} \, \mathrm{m}$

Aufgabe 2

a) $\tau(r) = \frac{M}{2\pi h} \frac{1}{r^2}$ b) $\dot{\gamma} = \frac{dv}{dr} = \frac{\tau}{\eta}$

c) $dv = \frac{M}{2\pi h \eta} \frac{1}{r^2} dr$ RB: $v(r_1) = 0 \quad v(r_2) = \omega \, r_2$

d) $\eta = \frac{M}{2\pi h \omega r_2} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$ e) $\eta = 0.18 \, \mathrm{Ns/m^2} \quad \mu = 2 \cdot 10^{-4} \, \mathrm{m^2/s}$

Aufgabe 3

a) $p_0 = 1013,2 \, \mathrm{hPa}$ b) $h_w = 10.33 \, \mathrm{m}$

Aufgabe 4

a) $\Delta h_O = H \left(1 - \frac{\varrho_2}{\varrho_1} \right)$ b) $\Delta h_M = H \left(\frac{1}{2} - \frac{\varrho_2}{\varrho_1} \right)$ c) $\varrho_1 = 2\varrho_2$

Aufgabe 5

a) $p_2 \approx 2.067 \cdot 10^5 \, \mathrm{Pa}$ b) $p_1 \approx 2.095 \cdot 10^5 \, \mathrm{Pa}$

Aufgabe 6

a) $p = p_0 + \varrho g \, (t + h)$

b) $h_{1,2} = -\frac{1}{2} \left(\frac{p_0}{\varrho g} + t \right) \pm \sqrt{\frac{\left(\frac{p_0}{\varrho g} + t \right)^2}{4} + \frac{p_0}{\varrho g} H}$

c) $t = \frac{p_0}{\varrho g} - \frac{H}{2}$

13 Kräfte auf Behälterwände

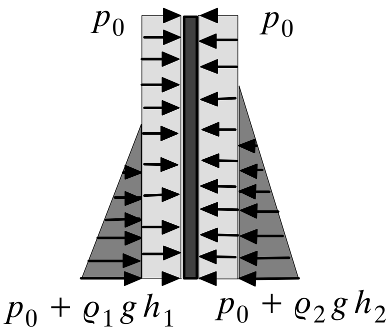
Aufgabe 1

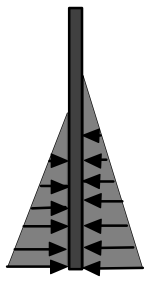
a) $F = \frac{1}{2}\varrho_W g\, h^2 b \qquad z_D = \frac{2}{3}h$ b) $G = 3\varrho_S g\, a^2 b$

c) $h \leq 3a\sqrt[3]{\frac{\varrho_S}{3\varrho_W}}$ d) $\varrho_S = 3\varrho_W$

Aufgabe 2

a)





b) $F_1 = \frac{1}{2}\varrho_1 g\, h_1^2\, b \qquad z_{D1} = \frac{2}{3}h_1 \qquad F_2 = \frac{1}{2}\varrho_2 g\, h_2^2\, b \qquad z_{D2} = \frac{2}{3}h_2$

c) $\frac{h_1}{h_2} = \sqrt[3]{\frac{\varrho_2}{\varrho_1}}$

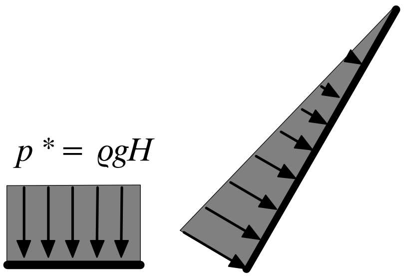
Aufgabe 3

a) $F_{D1} = \varrho\, g\, \pi R^3\left(\frac{H}{R} - \cos\alpha - \frac{2}{3}\right)$ b) $F_{D2} = \varrho\, g\, \pi R^3\left(\frac{H}{R}\cos^2\alpha - \frac{1}{3}\cos^3\alpha\right)$

c) $F_D = \varrho\, g\, \pi R^3\left[\left(\frac{H}{R} - \frac{1}{3}\cos\alpha\right)\sin^2\alpha - \frac{2}{3}(1 + \cos\alpha)\right]$ d) $F_V = 2.6\ \mathrm{N}$

Aufgabe 4

a)



b) waagerechter Schenkel: $F_1 = \varrho g H a b$ Druckpunkt: $\frac{a}{2}$ vom Punkt P

schräger Schenkel: $F_2 = \frac{\varrho g b H^2}{2\sin\varphi}$ Druckpunkt: $\frac{H}{3\sin\varphi}$ von P

c) $H \geq \sqrt{3}\, a \sin\varphi$

Aufgabe 5

a) $\boldsymbol{F} = \begin{bmatrix} -2\varrho gHbr \\ 0 \\ -\frac{\pi}{2}\varrho gbr^2 \end{bmatrix} \qquad M_0 = 0$

b) $\boldsymbol{F} = \begin{bmatrix} -2\varrho gHbr \\ 0 \\ -\frac{\pi}{4}\varrho gbr^2 \end{bmatrix} \qquad M_0 = -\frac{1}{3}\varrho gbr^3$

c) $\boldsymbol{F} = \begin{bmatrix} -2\varrho gHbr \\ 0 \\ -\frac{\pi}{4}\varrho gbr^2 \end{bmatrix} \qquad M_0 = -\frac{1}{3}\varrho gbr^3$

Aufgabe 6

a) $F_x = \frac{1}{2}\varrho\, g\, h^2 b \qquad F_z = \frac{1}{3}\varrho\, g\, h^2 b \qquad F = \frac{\sqrt{13}}{6}\varrho\, g\, h^2 b$

b) $z_D = \frac{2}{3}h \qquad x_D = \frac{3}{16}h$

14 Auftrieb und Schwimmstabilität

Aufgabe 1

$$\omega_0 = \sqrt{\frac{Rmg}{2I_y}}$$

Aufgabe 2

a) $x = L\left(1 - \sqrt{1 - \frac{\varrho_S}{\varrho_F}}\right)$

b) $\sin\alpha = \frac{H}{L\sqrt{1 - \frac{\varrho_S}{\varrho_F}}}$, wenn $H < L\sqrt{1 - \frac{\varrho_S}{\varrho_F}}$; sonst $\alpha = \frac{\pi}{2}$

Aufgabe 3

a) Wasserspiegel sinkt

b) Wasserspiegel bleibt unverändert

Aufgabe 4

a) $t_1 = \frac{G}{\varrho g A}\left(6 - \frac{x}{L}\right) \quad t_2 = \frac{G}{\varrho g A}\left(5 + \frac{x}{L}\right)$

b) $\varphi = \frac{G}{\varrho g A L}\left(1 - 2\frac{x}{L}\right)$

Aufgabe 5

a) $V_K = \pi r^3 \qquad V_F = \pi r^2 t \qquad t = \frac{\varrho_K}{\varrho_F}\, r$

b) $h_M = r\left(\frac{1}{4\lambda} + \frac{\lambda}{2} - \frac{1}{2}\right)$

Aufgabe 6

a) Kippstabilität um y−Achse ist geringer

b) $h_M = \frac{4R}{3\pi}$

15 Eindimensionale Strömungen

Aufgabe 1

a) instationär b) $\boldsymbol{v}(\boldsymbol{x},0) = \begin{bmatrix} x \\ 2y \\ 0 \end{bmatrix}$ c) $\boldsymbol{a}(\boldsymbol{x},t) = \begin{bmatrix} 0 \\ \dfrac{2y}{(1+t)^2} \\ 0 \end{bmatrix}$ d) $\boldsymbol{r}(t) = \begin{bmatrix} 1+t \\ (1+t)^2 \\ 0 \end{bmatrix}$

Aufgabe 2

a) $v = 13.3\ \frac{\mathrm{m}}{\mathrm{s}}$ b) $h = 0.75\ \mathrm{m}$

Aufgabe 3

a) $h = H$ b) $Q = \left(2 + \sqrt{2}\right)\mu A_0 \sqrt{gH}$ c) $T = \frac{1}{\mu}\sqrt{\frac{2H}{g}}$

Aufgabe 5

a) $v_1 = v_2 = v_0$ $A_1 = \mu\, A_0$ $A_2 = (1-\mu)\, A_0$

b) $\sin\varphi = \dfrac{\mu}{1-\mu}$ $F = (1 - \sqrt{1-2\mu})\, \varrho\, A_0\, v_0^2$