Lehrstuhl Theoretische Informatik
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Neural Networks and Learning Theory - Summer Term 2024

Sheet 0
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Algorithms for training of neural networks heavily depend on derivatives, we will therefore start by repeating the basic concepts from calculus.

Exercise 1. Give the derivatives f' of the following functions f and draw both f and f':

a) $f(x) = 2x^2 + x + 3$

b) $f(x) = cos(x)$

c) $f(x) = 2^x$

d) $f(x) = \sqrt{x}$

Exercise 2. Repeat/research the following properties of derivatives, give one example each and prove them if possible.

a) Linearity/Sum rule

b) Product rule

c) Chain rule

d) Inverse function rule

e) Quotient rule/Reciprocal rule

f) Power rule

Exercise 3. A function is called sigmoidal if it admits the following properties:

- Differentiable
- Strictly increasing
- Exactly one inflection point
- $\lim\limits_{x \to -\infty} f(x) = a$, $\lim\limits_{x \to \infty} f(x) = b$ exist

Give a function with all these properties and draw it. What can be said about the derivative of such a function?

Exercise 4. Give the derivatives f' of the following functions f and draw both f and f':

a) $f(x) = log(exp(x) + 1)$

b) $f(x) = x^x$

c) $f(x) = x^a - a^x$

d) $f(x) = sin(x)^{ln(x)}$

e) $f(x) = \sqrt[3]{cos(x)}$

Exercise 5. If a function f depends on more than one variable, we write $\frac{\partial}{\partial x}(f(x, y, z, ...))$ for the derivative with respect to the variable x. If you are not familiar with the concept, research "partial derivative". Compute the following:

a) $\frac{\partial}{\partial x}\frac{\partial}{\partial y}(x^y - y^x)$

b) $\frac{\partial}{\partial x}\frac{\partial}{\partial x}(x \cdot sin(x))$

c) $\frac{\partial}{\partial x}\frac{\partial}{\partial y}(ln(2x) + sin(xy^2))$

d) $\frac{\partial}{\partial y}\frac{\partial}{\partial x}(ln(2x) + sin(xy^2))$

e) $\frac{\partial}{\partial x}\frac{\partial}{\partial y}f(0,0)$, $f(x, y) = \begin{cases} 0 & x = y = 0 \\ \frac{x^3y - y^3x}{x^2+y^2} & \text{else} \end{cases}$

f) $\frac{\partial}{\partial y}\frac{\partial}{\partial x}f(0,0)$, $f(x, y) = \begin{cases} 0 & x = y = 0 \\ \frac{x^3y - y^3x}{x^2+y^2} & \text{else} \end{cases}$