Neural Networks and Learning Theory

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Exercise Sheet 5
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Exercise 1.

Let $X := \mathbb{R}$ and the concept space be the set of all positive "half-lines", i.e. $C := \{[a, \infty)|a \in \mathbb{R}\}$. Consider the algorithm that first, after seeing a training set S which contains m labeled examples of the form $(x_i, c(x_i))$ where $x_i \in \mathbb{R}$, define $(x_r := max_{i,c(xi)=0}xi)$ and $(x_l := max_{i,c(xi)=1}xi)$. The algorithm outputs an hypothesis $h = [a, \infty)$, where a is a real number arbitarily selected from the open interval (x_r, x_l). Prove that this algorithem satisfies the requirements of PAC learning and therefore proves that C is learnable by $H := C$.

Exercise 2.

PAC learning of hyper-rectangles: An axis-parallel hyper-rectangle in $\mathbb{R}^n$ is a set of the form $[a_1, b_1] \times ... \times [a_n, b_n]$. Show that axis-parallel hyper-rectangles are PAC-learnable by extending the proof given in the lecture for the case $n = 2$.

Exercise 3.

Concentric circles: Let $X := \mathbb{R}^2$ and consider the set of concepts of the form $c = \{(x, y)|x^2 + y^2 \leq r2\}$ for some real number r. Show that this class can be PAC-learned from training data of size $m \geq (1/\epsilon)log(1/\delta)$.

Exercise 4.

Given any set T of $n + 2$ points in $\mathbb{R}^n$. Show that there is a partition T_1, T_2 of T such that $convhull(T_1) \cap convhull(T_2) \neq \emptyset$.

Exercise 5.

Consider the sample spac $X := \mathbb{R}$. What is the VC-dimension of

(a) $H_1 := \{(a, \infty)|a \in \mathbb{R}\}$ and

(b) $H_2 := \{(-\infty, a) \cup (a, \infty)|a \in \mathbb{R}\}$.