Neural Networks and Learning Theory

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Exercise Sheet 3
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Exercise 1.
Suppose $f : \mathbb{R}^n \to \mathbb{R}$ is differentiable over an open domain. Then, the following are equivalent:

(a) f is convex.

(b) $f(y) - f(x) \geq Df(x)^T(y - x)$, $\forall x, y \in dom(f)$.

Exercise 2.
Let $M \subseteq \mathbb{R}^n$ be convex and $f : M \to \mathbb{R}$ be differentiable and convex. Show that:

(a) Any point x that satisfies $Df(x) = 0$ is a global minimum.

(b) If f is strictly convex, then the global minimum (assuming it exists) must be unique.

Exercise 3.
Find the QR-factorization of the matrix M:

$$M = \begin{pmatrix} -2 & -2 & -2 & 1 \\ -2 & -1 & -1 & 2 \\ 1 & 0 & -1 & 0 \end{pmatrix} = Q \cdot R.$$

where $Q \in \mathbb{R}^{3\times4}$ has orthogonal columns, i.e. $Q^T \cdot Q$ is a diagonal matrix and $R \in \mathbb{R}^{4\times4}$ is an upper triangular matrix in which the entries of the main diagonal are all 1.

Exercise 4.
Solve the XOR problem using an **RBF network**, that has the points $c^{(1)} := (0, 0)$ und $c^{(2)} := (1, 1)$ as the RBF centers and the Gaussian component functions $\phi_i(x) := e^{-\|x-c^{(i)}\|_2^2}$, $i = 1, 2$. Can you solve the problem with this network but using the functions $\phi_i(x) := \|x - c^{(i)}\|_2^2, i = 1, 2$?

Exercise 5.
Design an RBF network to approximate the following functions:

(a) $f(x) = (1 - x + 2x^2) \cdot e^{-\frac{x^2}{2}}$, $x \in [-10, 10]$.

(b) $h(x_1, x_2) = 2sin(\frac{x_1}{4})cos(\frac{x_2}{2})$, $x_1, x_2 \in [0, 10]$.