Neuronale Netze, exercise sheet 3

April 27, 2015

Exercise 1

Prove or disprove the following statements.

a) A symmetric matrix $A \in \mathbb{R}^{n \times n}$ is positive-definite if and only if A has a positive eigenvalue.

b) Let $f : \mathbb{R}^n \to \mathbb{R}$ be continuously differentiable on $\mathbb{R}^n$. The function f is convex on $\mathbb{R}^n$ if and only if $\forall x, y \in \mathbb{R}^n (f(y) - f(x) \geq Df(x) \cdot (y - x))$.

c) $x \mapsto x^T Ax$ is a convex function if and only if A is positive-semidefinite.

d) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be any convex function on $\mathbb{R}^2$. If f has a local minimum on $M := \{x \in \mathbb{R}^2 | \|x\| \geq 1\}$, then this is a global minimum on M.

Exercise 2

Find the point of the ellipse $E := \{(x, y) \in \mathbb{R}^2 | x^2 + xy + y^2 - 3 = 0\}$ that is closest to the origin $(0, 0)$. Also find the the point that has the largest distance to $(0, 0)$.