Neuronale Netze, exercise sheet 2

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Exercise 1

For a differentiable function $f : \mathbb{R}^n \to \mathbb{R}$ we define

$$\frac{\partial f}{\partial x} := \left(\frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n} \right).$$

Compute the following derivatives.

- $\frac{\partial (h^T x)}{\partial x}$, where h is a constant vector in $\mathbb{R}^n$.
- $\frac{\partial (x^T A x)}{\partial x}$, where A is a symmetric matrix in $\mathbb{R}^{n \times n}$. What would be the derivative for a non-symmetric matrix A?
- $\frac{\partial \|x\|}{\partial x}$, where $\|x\| := \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$.

Exercise 2

A reasonable goal for a learning algorithm could be to minimize the difference (error) between what it should ideally compute and what it actually computes. Construct a learning algorithm which does this by expressing the difference (error) in terms of the weights and the bias of the neuron, computing the partial derivatives and using them to determine in which direction the weights and the bias should be changed.

Exercise 3

For each of the following pairs of sets, construct a neural network which separates them (if possible).

a) $\{(0,0)\}$ and $\{(0,1),(1,0),(1,1)\}$ (the logical OR function)
b) $\{(0,1),(1,0),(0,0)\}$ and $\{(1,1)\}$ (the logical AND function)
c) $\{(0,0),(1,1)\}$ and $\{(0,1),(1,0)\}$ (the logical XOR function)

Exercise 4

Construct a neural network which recognizes whether one or both of the diagonals in a black-white picture consisting of 5×5 pixels are colored black (and all the other pixels should be colored white of course). Can you construct a neural network which still recognises this if there is a little bit of noise (i.e. a few pixels have the "wrong" color)?

Exercise 5

Can every feedforward network be transformed into an equivalent multilayer feedforward network?