Neuronale Netze, exercise sheet 1

April 13, 2015

Exercise 1

The set of points $\{x \in \mathbb{R}^n | w^T x = b\}$ is a hyperplane in $\mathbb{R}^n$ for every choice of $w \in \mathbb{R}^n \setminus \{0\}$ and $b \in \mathbb{R}$. The set of points $\{x \in \mathbb{R}^n | \exists t_1, \ldots, t_{n-1} \in \mathbb{R}(x = x_0 + \sum_{i=1}^{n-1} t_i v_i)\}$ is a hyperplane in $\mathbb{R}^n$ for every $x_0 \in \mathbb{R}^n$ and every set of linear independent vectors $\{v_1, \ldots, v_{n-1}\}$, $v_i \in \mathbb{R}^n$ for $i = 1, \ldots, n-1$.

a) Given the description of a hyperplane in the first form. Is it possible to compute a description of the same hyperplane in the second form? And the other way around?

b) Does $\{x \in \mathbb{R}^n | w^T x = b\} = \{x \in \mathbb{R}^n | w'^T x = b'\}$ imply that $w = w'$ and $b = b'$, or in other words, is this description of a hyperplane unique? Does $\{x \in \mathbb{R}^n | \exists t_1, \ldots, t_{n-1} \in \mathbb{R}(x = x_0 + \sum_{i=1}^{n-1} t_i v_i)\} = \{x \in \mathbb{R}^n | \exists t_1, \ldots, t_{n-1} \in \mathbb{R}(x = x'_0 + \sum_{i=1}^{n-1} t_i v'_i)\}$ imply that $x_0 = x'_0$ and $\{v_1, \ldots, v_{n-1}\} = \{v'_1, \ldots, v'_{n-1}\}$?

c) Given a hyperplane H in $\mathbb{R}^n$ and a point $p \in \mathbb{R}^n$. How can one compute the distance between H and p?

Exercise 2

Design a perceptron which correctly distinguishes the following sets of points (if possible).

a) $\{1, 3, 4, 5\}$ and $\{7, 8, 9\}$ in $\mathbb{R}$

b) $\{1, 3, 4, 5\}$ and $\{2\}$ in $\mathbb{R}$

c) $\{(2, 5), (3, 4), (2, 7)\}$ and $\{(8, 6), (2, 1)\}$ in $\mathbb{R}^2$

d) $\{(1, 2, -1), (3, 5, -4), (6, 2, -2)\}$ and $\{(2, 2, 3), (3, 1, 4), (2, 5, 7)\}$ in $\mathbb{R}^3$

Exercise 3

Given a McCulloch-Pitts neuron with weights w_i and bias b and a McCulloch-Pitts neuron with weights w'_i and bias b'. If for every input both neurons give the same output, does this imply that $w_i = w'_i$ and $b = b'$?

Exercise 4

a) Is it possible to approximate a McCulloch-Pitts neuron with a neuron with activation function $t \mapsto \frac{1}{1+e^{-t}}$?

b) Is it possible, for small values of x_i and w_i, to approximate a neuron which has an activation function $t \mapsto \frac{1}{1+e^{-t}}$ with a neuron which has a linear activation function?

Exercise 5

Find out what the following possible activation functions look like.

$$\phi_1(t) := \frac{1}{1+e^{-ct}}, \quad \phi_2(t) := \tanh(\frac{ct}{2}) := \frac{1-e^{-ct}}{1+e^{-ct}}, \quad \phi_3(t) := \frac{t}{\sqrt{1+t^2}}.$$

Can you express the derivative of the function in terms of the function itself?