Algebraische Rechenmodelle, exercise sheet 9

January 15, 2015

Exercise 1

Given polynomials $f_1, \dots, f_s$ and $u_1, \dots, u_s$ in $\mathbb{C}[x_1, \dots, x_n]$, show that $LM(f_1 u_1 + \dots + f_s u_s) \leq \max_{1 \leq i \leq s}(LM(f_i)LM(u_i))$. Does equality necessarily hold? (Prove or disprove.)

Exercise 2

Let $m < n$. Prove that any monomial ordering on $\mathbb{N}_0^m$ is the restriction of a monomial ordering on $\mathbb{N}_0^n$.

Exercise 3

Show that the polynomials $f_1 = x - y^2 w$, $f_2 = y - zw$, $f_3 = z - w^3$, $f_4 = w^3 - w$ in $\mathbb{C}[x, y, z, w]$ form a Gröbner basis for the ideal they generate, with respect to the lexicographic order with $x > y > z > w$. Show that they do not form a Gröbner basis with respect to the lexicographic order with $w > x > y > z$.

Exercise 4

Let $I = (f_1, f_2)$ with $f_1 = x^3 - 2xy$ and $f_2 = x^2 y - 2y^2 + x$. Construct a Gröbner basis for I with respect to the monomial ordering defined by $x^{\alpha_1} y^{\alpha_2} < x^{\beta_1} x^{\beta_2}$ if $\alpha_1 + \alpha_2 < \beta_1 + \beta_2$. If $\alpha_1 + \alpha_2 = \beta_1 + \beta_2$, then compare α_1 and β_1 to decide which monomial is smaller.

Exercise 5

Let $I \subseteq \mathbb{C}[x_1, \dots, x_n]$ be an ideal generated by a set G of non-zero terms (e.g. $G = \{4x_2^4 x_5, -x_1 x_2 x_3, x_3^3\}$). Show that G is a Gröbner basis for I.