Algebraische Rechenmodelle, exercise sheet 6

December 4, 2014

Exercise 1

Let $f : \mathbb{R}^n \to \mathbb{R}$ be a polynomial. Define $C := \{x \in \mathbb{R}^n | \frac{\partial f}{\partial x_1} = \ldots = \frac{\partial f}{\partial x_n} = 0\}$ and define $S := \{t \in \mathbb{R} | \exists x \in C \quad t = f(x)\}$. What can you say about the cardinality of S?

Exercise 2

Consider the polynomial $p(x_1, x_2) = x_1^2 x_2^4 + x_1 x_2^2 + x_1^2 + a$ where a is a constant. Suppose we want to compute whether p has a zero by first eliminating the variable x_1 using Tarski's algorithm. Compute $q = F_4(p, F_1(p))$. What can you say about the roots of q?

Exercise 3

Let k be a field. We need the following definitions.

(1) Let $f = (f_n)$ be a family of polynomials and let $v(n)$ be the number of variables on which f_n depends. If there exists a polynomial in n which bounds both v and the degree of f_n from above, then f is called a *p-family*.

(2) A function $f \in k[X_1, \ldots, X_n]$ is a *projection* of a function $g \in k[X_1, \ldots, X_m]$ if there are $a_1, \ldots, a_m \in k \cup \{X_1, \ldots, X_n\}$ such that $f(X_1, \ldots, X_n) = g(a_1, \ldots, a_m)$.

(3) If $f = (f_n)$ and $g = (g_n)$ are p-families of polynomials, then f is called a *p-projection* of g, iff there exists a polynomially bounded function t such that for every n, f_n is a projection of $g_{t(n)}$.

Show that $SUM := (SUM_n)$ with $SUM_n := X_1 + \ldots + X_n$ and $PROD := (PROD_n)$ with $PROD_n := X_1 \cdots X_n$ are p-families and prove that they are not p-projections of each other.