Algebraische Rechenmodelle, exercise sheet 5

November 27, 2014

Exercise 1

For any pair of univariate polynomials f, g over $\mathbb{R}$ there exists a polynomial R with integer coefficients in the coefficients of f and g which vanishes iff f and g have a common root in $\mathbb{C}$. This is called the resultant of f and g. If $f(x) = f_0 + f_1x + \ldots + f_mx^m$ and $g(x) = g_0 + g_1x + \ldots g_nx^n$, then R is a polynomial expression with integer coefficients of degree nm in $f_0, \ldots, f_m, g_0, \ldots, g_n$. Can you use this to prove that the algebraic numbers form a field?

Exercise 2

Let $A \subset \mathbb{R}^{n+1}$ be semi-algebraic and define

$$\Pi A := \{y \in \mathbb{R}^n | \exists x \in \mathbb{R} \text{ such that } (y, x) \in A\}.$$

Consider the following two statements and give a proof or a counter example.

- $(\Pi A) \cup (\Pi B) = \Pi(A \cup B)$
- $(\Pi A) \cap (\Pi B) = \Pi(A \cap B)$

Exercise 3

Let $\mathcal{P} = \{p_1, \ldots, p_s\}$ be a set of univariate polynomials from $\mathbb{R}$ to $\mathbb{R}$ each of degree at most d. How many valid sign vectors can this family of polynomials have at most?

Exercise 4

Let $\mathcal{P} = \{p_1, \ldots, p_s\}$ be a set of multivariate polynomials from $\mathbb{R}^n$ to $\mathbb{R}$ each of degree at most d. Can you compute (with a BSS algorithm in polynomial time) a bound b in terms of d and the coefficients of $p_1, \ldots, p_s$ such that if $\mathcal{P}$ has a common zero, then it has a common zero $(x_1^*, \ldots, x_n^*)$ with $x_i \leq b$ for all $i = 1, \ldots, n$. Or can you prove that such a bound cannot be computed?