Algebraische Rechenmodelle, exercise sheet 4

November 20, 2014

Let $SA(d)$ be the set of all semi-algebraic sets defined by polynomials of degree at most d. For each of the following problems prove that the problem is $\mathrm{NP}_{\mathbb{R}}$-complete or *co*-$\mathrm{NP}_{\mathbb{R}}$-complete.

a) Let $d \geq 2$ and consider the problem
$EMPTY(d) := \{\rho \in SA(d)|$ the semi-algebraic set defined by ρ is empty$\}$.

b) Let $QS := \{f = (f_1, \dots, f_s)|s \in \mathbb{N}, f_i$ has degree at most $2\}$ and consider the problem $QS_{yes} := \{f \in QS|$ there exists a common zero of all $f_i\}$.

c) Let $F^4_{zero,+}$ be the problem whether a degree 4 polynomial has a zero $x^* = (x^*_1, \dots x^*_n)$ with $x^*_1 \geq 0, \dots, x^*_n \geq 0$.

d) Let $d \geq 2$. $CONVEX(d) := \{\rho \in SA(d)|$ the semi-algebrai set defined by ρ is convex$\}$.

e) Let $d \geq 2$. $k{-}FINITE(d) := \{\rho \in SA(d)|$ the semi-algebraic set defined by ρ has at most k elements$\}$

f) Let $d \geq 4$. $POSITIVE(d) := \{f$ a polynomial of degree at most $d|$if $f \in \mathbb{R}[x_1, \dots, x_n]$ then $f(x) > 0\, \forall x \in \mathbb{R}^n\}$.