

# Algebraische Rechenmodelle, exercise sheet 3

November 10, 2014

## Exercise 1

Consider the following questions both over  $\mathbb{R}$  and over  $\mathbb{C}$ . Is every subset of  $\mathbb{N}$  decidable by a BSS machine? If so, what is the complexity?

## Exercise 2

Can  $\lfloor x \rfloor$  be computed by a BSS machine over  $\mathbb{R}$ ? If so, can you find a lower bound for the number of computation steps needed?

## Exercise 3

Show that every problem that is decidable by a BSS machine can be decided by a BSS algorithm which uses only a linear amount of space.

## Exercise 4

An algebraic number is a number that is a root of a non-zero polynomial in one variable with rational coefficients. Numbers that are not algebraic are said to be transcendental.

- Would it make a difference to require that the coefficients are integers?
- How many algebraic numbers are there? How many transcendental numbers are there?
- Consider  $\mathbb{Q}[\sqrt{2}]$  and  $\mathbb{Q}[\pi]$  ( $\pi$  is a transcendental number). What do these rings look like? What are the differences?