Algebraische Rechenmodelle, exercise sheet 2

October 23, 2014

Exercise 1

Let $p(x) := \sum_{i=0}^{n} a_i x_i$ be any degree n polynomial from $\mathbb{R}$ to $\mathbb{R}$. Show that $Z_+(p) \equiv \mathrm{VW}(a) \mod 2$ and show that $Z_+(p) = \mathrm{VW}(a)$ in case that p has only real zeros. Can you find a general lower bound on the number of real zeros of p (counting multiplicities) in terms of $\mathrm{VW}(a)$?

Exercise 2

Fourier-Motzkin elimination is an algorithm which can be used to project a convex polyhedron $P(A, b) := \{x|Ax \leq b\}$ on a hyperplane of the form $H := \{x|x_1 = 0\}$. Here A is a matrix in $\mathbb{R}^{m\times n}$ and b is a vector in $\mathbb{R}^m$. The algorithm produces a matrix $D \in \mathbb{R}^{r\times n-1}$ and a vector $d \in \mathbb{R}^r$ such that

$$D(x_2, \dots, x_n)^T \leq d \Leftrightarrow \exists x_1 A(x_1, \dots, x_n)^T \leq b.$$

The rows in A that have a zero as first entry are copied into D. For every pair of rows in A in which one has a positive number as first entry and the other has a negative number as first entry a row is added to D which is a certain linear combination of these two rows.

a) Can this be used to produce an algorithm which decides whether a given system of linear inequalities has a solution?

b) What is the algebraic complexity of this algorithm?

c) How does this algorithm work in detail?

d) If you answered question c), can you prove its correctness?