

Algebraische Rechenmodelle, exercise sheet 1

October 16, 2014

Exercise 1

Let $a, b, c \in \mathbb{Q}$ and let $\psi(a, b, c) := \exists x \in \mathbb{Q} : ax^2 + bx + c = 0$. Does there exist a quantor free formula that is equivalent to ψ ?

Exercise 2

Given the univariate polynomial $p(x) := a_n x^n + \dots + a_1 x + a_0$ with $a_0, \dots, a_n \in \mathbb{C}$. Can you compute an upper bound for the absolute value of any root of p in the coefficients a_0, \dots, a_n ?

Exercise 3

Let F be a field, let $f, g \in F[x]$ be nonzero polynomials and consider the following two statements.

1. The greatest common divider of f and g is not equal to 1.
2. There exist $s, t \in F[x]$ such that $sf + tg = 0$, the degree of s is strictly less than the degree of g and the degree of t is strictly less than the degree of f .

Does 1. imply 2.? Does 2. imply 1.?