# Approximation Algorithms, exercise sheet 9

December 16, 2013

## 1.

Show that $\mathrm{PCP}(O(\log(n)), O(1)) \subseteq \mathrm{NP}$.

## 2.

Can you find functions $r(n)$ and $q(n)$ such that the language GNI of pairs of nonisomorphic graphs is in $\mathrm{PCP}(r(n), q(n))$? So the input is a pair $\langle G_0, G_1 \rangle$ of graphs that both have $n$ nodes and the verifier wants to accept if these graphs are not isomorphic and reject if they are isomorphic.

## 3.

In the lecture a PCP verifier was defined to work in three steps. In the third step it computes in polynomial time whether it accepts or rejects. For this computation is may use the input, the string of random bits that was generated and the answers it got from the queries it made to the certificate.

If we would modify the definition, so that in the third step the verifier is only allowed to use the input and the result of its queries but not the sequence of random bits that was generated, does the PCP theorem (NP = PCP(O(log($n$),O(1))) still hold then?

For example, if the verifier decides in the second step to query $y_4$, $y_2$, $y_3$, $y_4$, $y_3$ and $y_5$ in that order and $y = 1101100$ then in the third step the verifier has to make its decision using only the input $x$ and the vector 110101. Note that the verifier may query a component multiple times. Note also that it does not know that in the vector 110101 the first bit comes from $y_4$, the second from $y_2$ etc., it just knows that the first query resulted in a 1, the second query resulted in a 1, the third query resulted in a 0 etc.