Approximation Algorithms, exercise sheet 7

December 4, 2013

1. The complexity class RP

A language L belongs to RP if and only if there is a probabilistic Turing Machine M running in polynomial time such that for any input $x \in L$ the computation of machine M on input x leads to an accepting state with probability at least $\frac{1}{2}$. For any input x which is not in L the machine will reject with probability 1. Would a different class be obtained if it would be required that if $x \in L$ then M accepts with probability at least 0.99? Where does the class RP fit in the complexity hierarchy?

2.

Prove that for every nonnegative random variable V, $\Pr[V > 0] \geq \frac{E[V]^2}{E[V^2]}$.

3.

In the proof that the 2-opt strategy for TSP with triangle inequality has a perfomance ratio that is not worse than $4\sqrt{n}$, it was used that $\sum_{q=1}^{n} \frac{1}{\sqrt{q}} \leq 2\sqrt{n}$. Can you prove this?