Approximation Algorithms, exercises week 3

October 29, 2013

MAX-3SAT

Consider the following decision problem. Given a 3SAT instance and an integer k, decide whether there exists an assignment that satisfies at least k clauses in the instance.

Is this problem in NP? Is it complete? If you would be given an efficient algorithm that solves this problem, could you use it to construct an algorithm that receives as input an instance of MAX-3SAT and outputs an assignment that satisfies a maximum number of clauses?

Kruskal's algorithm to compute a mimimal spanning tree

The algorithm receives as input a connected graph $G = (V, E)$ with some weight function w on the edges. First it orders the edges in E according to their weights. Let $E = \{e_1, \ldots, e_n\}$ with $w(e_1) \leq w(e_2) \leq \cdots \leq w(e_n)$. It sets $T := \emptyset$ and then for $i = 1, 2, \ldots, n$, if adding e_i to T does not create a cycle in T then e_i is added to T.

Prove that the set T that this algorithm produces is a minimal spanning tree of G.

Odd degree

Prove that the number of vertices with odd degree in a graph is even.