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AN INVERSE APPROACH TO IDENTIFY TUNED AERODYNAMIC DAMPING, SYSTEM FREQUENCIES, AND MISTUNING. PART 2: APPLICATION TO BLISKS AT REST.

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ABSTRACT

This work aims at identifying the mechanical system of blade-integrated disks (blisks) from experimental vibration data gathered at standstill employing a least squares approach introduced in the first part of this paper. Based on a linear formulation of the forced response problem the identification of the mistuning pattern in terms of sector-to-sector stiffness variations, modal damping ratios, natural frequencies of the tuned reference and forcing is addressed in this regard. The work has to be understood as both demonstrating the applicability of the method to blisks at standstill and revealing an alternative to existing methods of system identification.

INTRODUCTION

Small variations of mechanical properties from blade to blade denoted as mistuning are known to affect the forced response of blisks and bladed disks. Commonly, it is matter of severe response magnification, which can considerably decrease the service life compared to the perfectly tuned reference. For that reason, the research community has been occupied with the phenomenon since more than 50 years [1]. In particular, the combination of mistuning and small damping facilitates a large forced response. Consequently, huge efforts were made to calculate the forced response magnification due to mistuning. Foremost to mention is the well known work published by Whitehead [2] already in 1966 who formulated a conservative limit with regard to the forced response magnification, which only depends on the number of blades. Many years later Martel and Corral [3] introduced an enhancement of the Whitehead formula by replacing the number of blades with the number of active modes. This less conservative limit takes into account differing natural frequencies in one blade mode family and hence, considers the modal coupling between disk and blade motion. Most recently, Figaschewsky and Kühhorn [4] developed a further improvement for computing the limit by incorporating the standard deviation of mistuning and consequently the magnitude of mistuning. On the contrary, particular patterns of intentional mistuning associated with favourable conditions with respect to aeroelastic interaction can even effect a mitigation of forced response [5], [6].

Independently from the problem, there is a demand to identify the real mistuning of a blisk and update structural models based on experimental data. Hence, advances have been made to provide methods addressing this question in recent two decades. A promising method worth to be mentioned is the inverse approach denoted as FMM-ID (Fundamental Mistuning Model Identification) introduced by Feiner and Griffin [7]. However, in case of strong modal couplings as occurring within a blade mode family of a blisk with a big number of blades e.g. as in case of rear high pressure compressors, the necessary derivation of mistuned mode shapes from experimental data is a challenging task. By contrast the application of the direct method introduced in [8] is more suitable for those cases, however, if closely adjacent blade mode families are apparent, the method is prone to failure.

Following, a new method is applied which is not prone to the problems mentioned above. This least squares approach introduced in the first part of this paper [9] is adjusted to blisks at rest and exemplarily employed for both a radial turbine blisk and two axial turbine blisks in order to identify the mistuning pattern, the tuned natural frequencies, the forcing, and the modal damping ratios as well.

ADJUSTMENT OF INVERSE APPROACH

The approach presented in the first part has been developed for parameter identifications from data of rotating wheels subjected to a particular travelling wave mode (TWM) excitation. Aiming at demonstrating the suitability for data gathered within an experimental modal analysis carried out for a resting wheel, an adjustment of the method is required since the broadband excitation employed affects the presence of all TWMs.

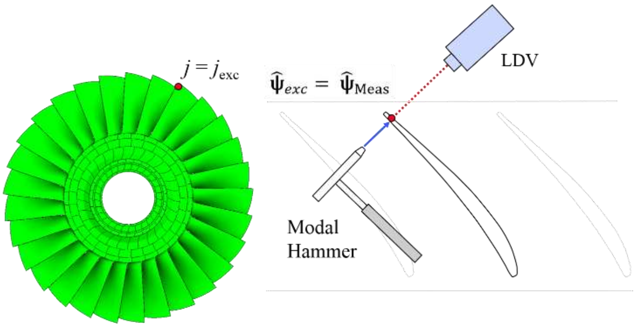

Fig. 1 Schematic of experimental setup (blisk at rest)

Figure 1 shows the principle setup of the experimental modal analysis considered subsequently. The excitation acts upon a single point of the blade with index $j = j_{\text{exc}}$. The strength of excitation force is assumed to be measured during the impact, such that the forced response functions can be derived easily from the measured responses.

The blade individual force on the excited blade equals to the projection of the modeshape vector at the point of excitation onto the normal direction $\boldsymbol{n}_{exc}$ of the blade's surface. Taking into account the simplified representation of the system modes and the fact that forced response functions are measured ($\underline{\hat{f}} \equiv 1$) the modal force of the m-th mode is given by:

$$\underline{\hat{f}}_m^{\text{modal}} = \boldsymbol{n}_{exc}^T \underline{\boldsymbol{\Phi}}_{m,j_{exc}} = \rho_m \boldsymbol{n}_{exc}^T \overline{\boldsymbol{\Psi}}_{exc}^B e^{i2\pi j_{exc} k(m)/N} \quad (1)$$

$$= \rho_m \widehat{\boldsymbol{\Psi}}_{exc} e^{i2\pi j_{exc} k(m)/N} \quad (2)$$

As emphasized already in [9], the modal amplitudes cannot be measured directly. This leads to the appearance of the generalized modal force $\underline{\hat{f}}_m'$ in the governing equations (Eq. 42, Part 1 [9]). The generalized modal force is given by:

$$\underline{\hat{f}}_m' = \rho_m \widehat{\boldsymbol{\Psi}}_{\text{Meas}} \underline{\hat{f}}_m^{\text{modal}} = \rho_m^2 \widehat{\boldsymbol{\Psi}}_{\text{Meas}} \widehat{\boldsymbol{\Psi}}_{exc} e^{\frac{i2\pi j_{exc} k(m)}{N}} \quad (3)$$

The equation above reveals that the amplitude of the m-th modal force is a measure for the mode shape scaling factor ρ_{m}. In the special measurement situation depicted in Fig. 1 with approximately identical points of excitation and response measurement, i.e. $\widehat{\Psi}_{\text{Meas}} \approx \widehat{\Psi}_{\text{exc}}$, the square root of the identified generalized modal force amplitudes equals $\rho_m \widehat{\Psi}_{\text{Meas}}$. Furthermore, it is obvious that each travelling wave mode (TWM) is excited with a particular phase lag as a function of the excited blade index. If the blade indices were renumbered such that $j_{exc} = N$, all generalized forces would become real numbers. Since $\rho_m = \rho_n$ for $k(m) = -k(n)$, the forces of travelling wave modes with opposite direction of rotation are also identical (in the renumbered case) or at least conjugate complex (in the general case). This leads to a reduction of the modal force amplitudes that need to be identified from N to k_{max}. Due to the absence of Coriolis or aerodynamic forces, the tuned natural frequencies as well as the damping should be also symmetric with respect to the TWM index. This allows a further reduction of the unknowns that need to be identified. In order to stabilize the damping identification, the damping of the m-th mode can be approximated by a harmonic expansion with a lower number of harmonics $l = 1, N_H < N$:

$$\delta_m = \sum_{l=0}^{N_H} a_l \cos(2\pi l k(m)/N) \quad (4)$$

Due to the special situation that the values of ρ_{m} can be estimated from the identified modal forces, it is possible to derive an indication for the validity of the assumptions with respect to the simplified approximation of the system mode shapes. Therefore, the ratios of the natural frequencies and the scaling factors are compared to a reference, e.g. the values corresponding to the maximum nodal diameter k_{max}. This yields the following indicator function:

$$\tilde{r}_m = \left| \frac{\rho_m^2}{\rho_{\text{ref}}^2} - \frac{\omega_{0,\text{ref}}^2}{\omega_{0,m}^2} \right| = \left| \frac{\underline{\hat{f}}_m'}{\underline{\hat{f}}_{\text{ref}}'} - \frac{\omega_{0,\text{ref}}^2}{\omega_{0,m}^2} \right| \quad (5)$$

As long as the assumptions of the blade dominated mode family are hold, the indicator function takes a value of zero. Any deviation from the model assumption, e.g. a change of the mode shape or disk participation, will inevitably lead to values greater than zero. In terms of isolated blade mode families with only minor disk participation, the indicator function corresponds to a measure of disk to whole-wheel strain energy ratio of the same mode.

NUMERICAL EXPERIMENT

In order to demonstrate the ability of the adjusted approach for parameter identification, an SNM-based [10] numerical experiment has been created first. The axial turbine blisk chosen for this purpose approximately corresponds to that considered later on in one of the experimental modal analyses. Both a particular stiffness based blade mistuning distribution and TWM-dependent modal damping ratios have been arbitrarily chosen for the 41-bladed wheel. In order to create a basis of comparison, impact excitation acting upon blade 41 has been simulated, which allows to compute 41 frequency response functions (FRF) by processing the unwrapped response calculated at the identical position on each blade as assumed in the previous section.

The simulated response is used as input data for the least squares identification problem. Figures 2, 4 and 5 are revealing that a nearly perfect match of input and identified data is achieved with respect to the frequency mistuning pattern of blade mode 1, the modal damping ratios of BF 1 and the forcing of BF 1. The congruousness of identified and FE-computed natural frequencies of the tuned reference blisk is accurate as well (Fig. 3). Regarding the disk to whole-wheel strain energy ratio (Fig. 6) the overall TWM-dependent shape of the curves is perceptible. However, clear deviations from FE-results appear for low-indexed TWMs, in particular TWM -2, -1, 0, +1 and +2. Indeed this is not surprising since due to a more distinct disk participation a basic assumption of applying the criterion given in Eq. (5) is violated for these TWMs at least in tendency. The disk participation of same TWMs is also indicated in Figure 5.

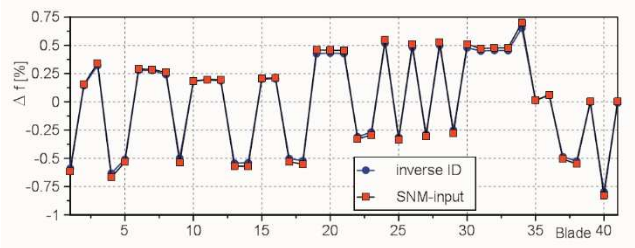

Fig. 2 Frequency mistuning (Mode 1, axial turbine)

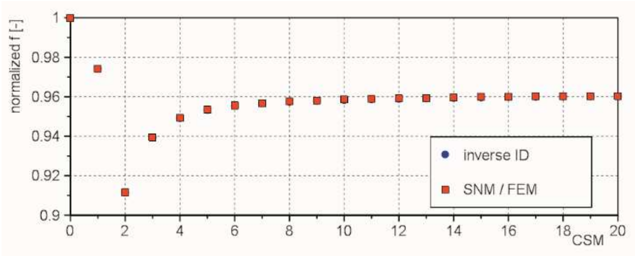

Fig. 3 Natural frequencies of tuned reference (BF 1)

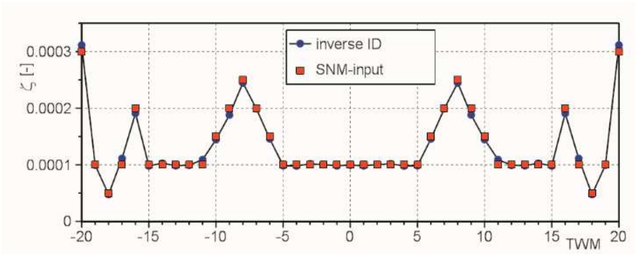

Fig. 4 Modal damping ratios (BF 1)

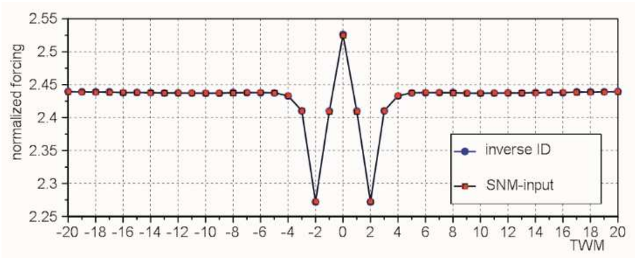

Fig. 5 Forcing (BF 1)

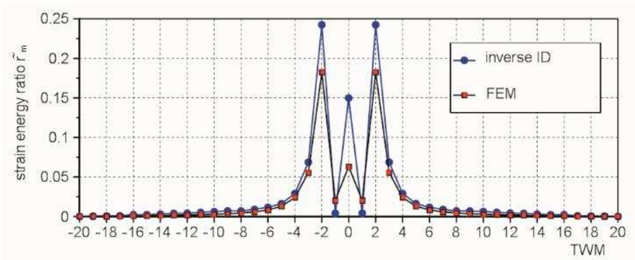

Fig. 6 Disk to overall strain energy ratio (BF 1)

The last comparison taken up is addressing the representative FRF of TWM -6 (Fig. 7). For both, real and imaginary part a high quality match of simulated and identified curves becomes apparent again. More FRFs are given in Appendix A, which confirm both the high accuracy of the method and its suitability for practical blisk applications.

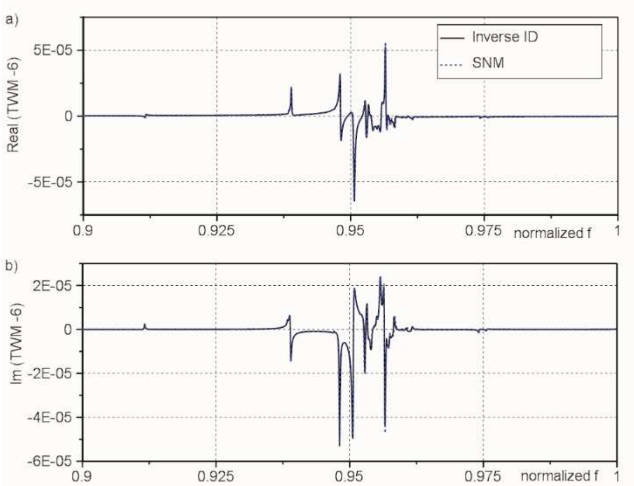

Fig. 7 FRF (BF 1) in TWM coordinates: a) Real part of and, b) imaginary part of TWM –6

EXPERIMENTAL ANALYSES

Turbine Impeller

As first practical example to demonstrate the procedure of system identification a turbine impeller for ship Diesel engine applications is chosen. The 13-bladed wheel has been manufactured in inconel. Two types of experiments have been carried out. Firstly, experimental modal analyses in order to gather data for the purpose of producing the relevant input data for system identification. Secondly, blade by blade mistuning tests as introduced in [8] with the objective to provide a data basis with a view to validation.

The experimental modal testing draws upon miniature hammer excitation and non-intrusive laser Doppler vibrometry. The hammer itself is held by a retaining fixture and released by means of electromagnetic control, which enables impacts at exactly the same position and with identical intensity (Fig. 8). Approximately free support conditions are chosen for each test. The tests are carried out at normal conditions (20°C, 1013 mbar ambient pressure). Since only one FRF is determined per blade, unwrapped mode shapes are identified. However, the 13 FRFs (Fig. 9) themselves are of primary interest with respect to further processing for identifying the mechanical system of the impeller.

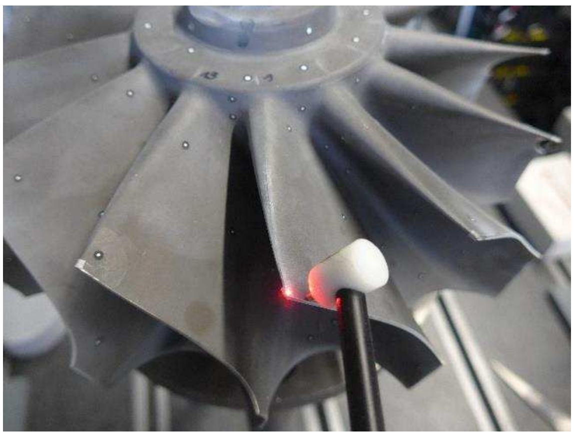
Fig. 8 Impeller experimental modal analysis

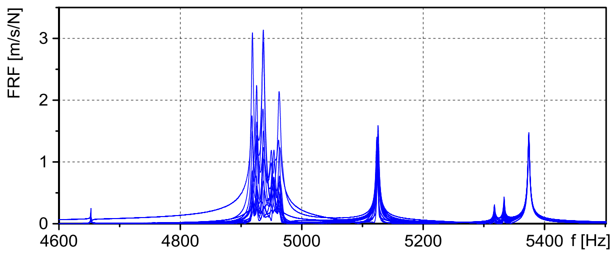

Fig. 9 Magnitudes of FRF (BF 1, turbine impeller, 13 FRF)


Fig. 10 Additional mass detuning while mistuning validation test

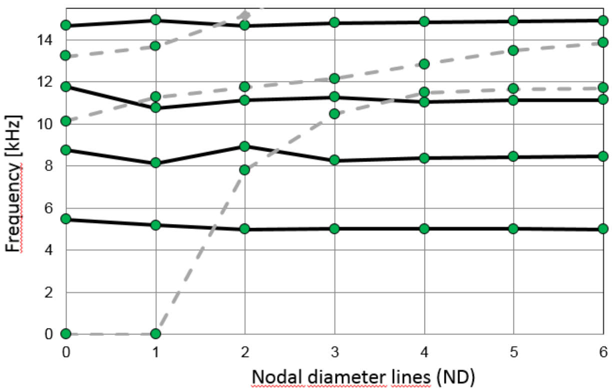

Fig. 11 Nodal diameter plot turbine impeller [13]

The experimental setup is varied with the objective to identify mistuning for the purpose of validation. In order to isolate blade-dominated frequencies additional mass detuning (Fig. 10) is applied on every blade, which is not excited at the same moment. In this manner, the already distorted cyclic symmetry due to mistuning is removed completely. Consequently, the FRF complies with that of a single degree of freedom system since ideally a complete modal decoupling is achieved. Forced response functions are determined only for the excited blade and finally allow for a simple identification of blade-dominated frequency peaks. The procedure has to be repeated for each blade and each blade mode family of interest so that mistuning distributions in terms of blade-to-blade frequency variations could be identified (Fig. 15). However, the success of the method stands and falls with the modal decoupling of different blade mode families. For well-separated blade mode families such as BF 1, 2 or 4 (Fig. 11) the method is working remarkably dependable and robust, whereas in case of strongly participating disk motions, crossing or veering regions [11], [12] the method is prone to failure.

Axial Turbine Blisks

Two prototypes of axial turbine test blisks are analysed as second example type with 41 blades. Hence, regarding the wheel type and the number of blades strongly differing preconditions are present in principle. The first prototype is manufactured with an optimized intentional mistuning pattern in order to limit the impact of low engine order excitation (LEO) on the forced response of the fundamental flap mode [14], [15]. The second prototype corresponds to the tuned design intention with preferably identical blades.

Again aiming to provide a basis of comparison for validation purposes, blade by blade mistuning tests are carried out by employing the method described in the previous paragraph (Fig. 12). Results of these reference solutions (Fig. 13, square symbolled curves) are revealing deviations from design intentions (circle symbolled) since additional random mistuning is unavoidable. Nevertheless, the principle characteristics of the intended mistuning pattern (Fig. 13a) are still recognizable. Apart from Blades 6, 7, and 8 an extraordinary successful manufacturing could be accomplished in case of the ‘tuned’ blisk (Fig. 13b). Since the first blade mode family (BF 1) considered here is well isolated, the experimental results obtained are regarded to be highly reliable.


Fig. 12 Mistuning identification of axial turbine blisk (blade by blade)

After removing the additional mass detuning applied for the mistuning test (Fig. 12) the following experimental determination of FRFs is accomplished in analogous manner as described before for the impeller. The FRFs shown in Fig. 14 reveal a substantial modal coupling and thus a high modal density within BF 1. Figure 14b exemplarily shows the FRF of Blade 19, which clearly indicates the appearance of more than 30 resonances in a narrow frequency band starting from a normalized frequency of 1. Hence, extracting modal quantities such as modal damping ratios proves to be an extremely challenging task by using a classical MDOF-approach.

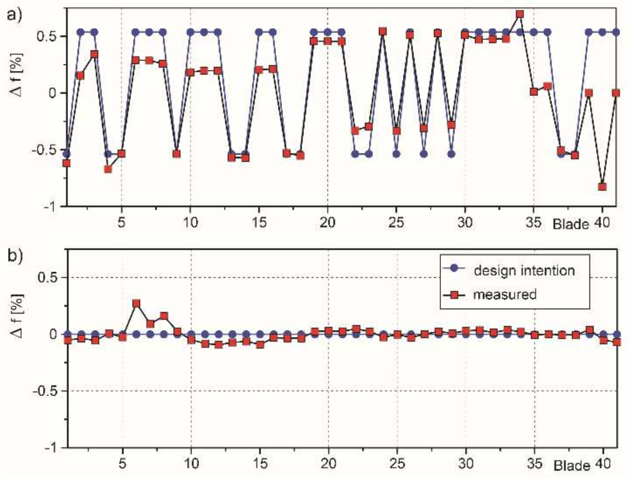

Fig. 13 Frequency mistuning of mode 1, a) intentionally mistuned and b) tuned [13]

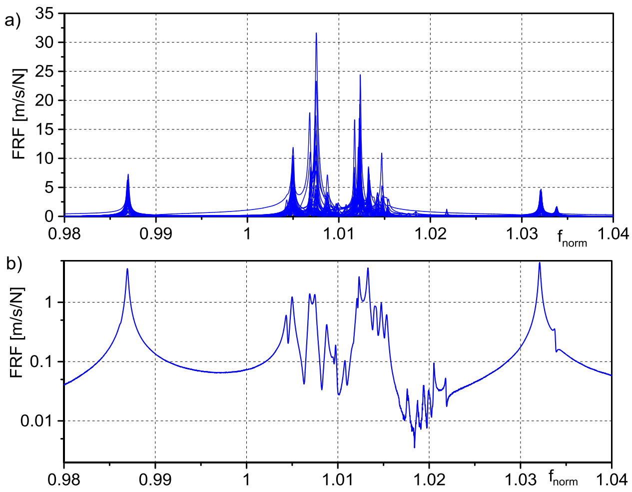

Fig. 14 Magnitudes of FRF (BF 1, intentionally mistuned axial turbine blisk), a) all blades, and b) Blade 19

APPLICATION OF INVERSE APPROACH

Identification of Turbine Impeller Parameters

Subsequently, the data basis measured before serves as input for the new inverse approach. There is no need to use moving or weighting technologies as suggested in [9] due to the high quality of the experimental data. Starting with the frequency mistuning pattern of the turbine impeller, a satisfying match for mode 1 (Fig. 15) could be found by comparison of the methods. Moderate deviations e. g. at blade 6 could be a consequence of the fact, that first the inherently well established blade by blade method is more prone to possibly remaining modal coupling during the test. Second, both testing procedures took place in a three-year interval, so that ambient conditions were not exactly the same.

Regarding the computation of tuned reference system's natural frequencies (Fig. 16a) the good coincidence of results is confirmed. Only a small discrepancy occurs in case of the umbrella mode (CSM 0) which results from the influence of boundary conditions during the measurement. On the contrary, the comparison of modal damping ratios (Fig. 16b) unsurprisingly yields differences with respect to exact values, however, the order of magnitude agrees very well. The results of the inverse approach are representing the modal damping ratios of the tuned reference where each TWM is dedicated to a specific damping ratio. In contrast, the basis of comparison is derived by employing an MDOF-fit approach of a mistuned wheel. Strong modal couplings are arising and consequently, separated TWMs do not exist anymore so that a superposition of different TWMs is dedicated to each peak in the FRF. Solely CSM 0 or TWM 0 respectively, represents an exception, since the assigned mode hardly couples with the others. Hence, the modal damping ratios for this mode are almost identical.

Figure 17 reveals the normalized forcing contribution to every CSM resulting from the impact excitation used in the experimental analyses. Expectably, the major part of energy is transferred into CSM 0 and 1. In contrast CSM 2, 3, 4, 5 and 6 are subjected to strong modal coupling (Fig. 16a) and less disk participation. The latter is confirmed by a disk to overall strain ratio taking a value close to zero (Fig. 18). The comparatively high values of $\tilde{r}_m$ for CSM 0 and 0 indicate a concentration of strain energy in the shaft extension welded to the bottom of the disk [12]

The last discussion is addressing an exemplary but representative comparison of FRFs obtained from inverse approach and experiment (Fig. 19). Although minor deviations become visible, the conductivity of the inverse approach is confirmed considering the overall matching of curves. This particularly applies in the light of the extreme low level of damping in the present case.

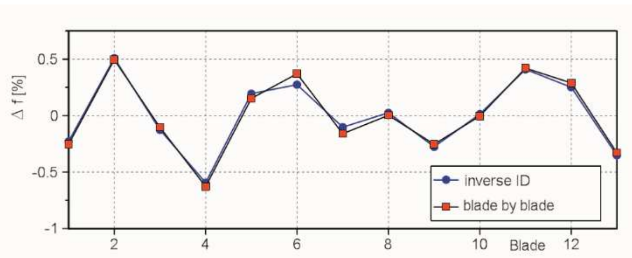

Fig. 15 Frequency mistuning (Mode 1, impeller)

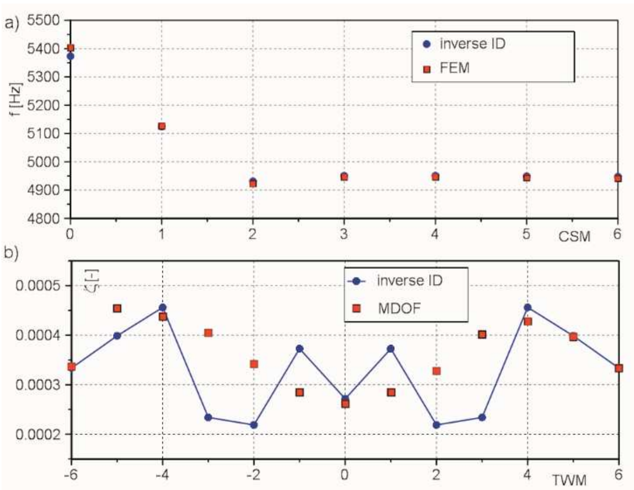

Fig. 16 a) Natural frequencies and, b) modal damping ratios (mode 1) of tuned reference

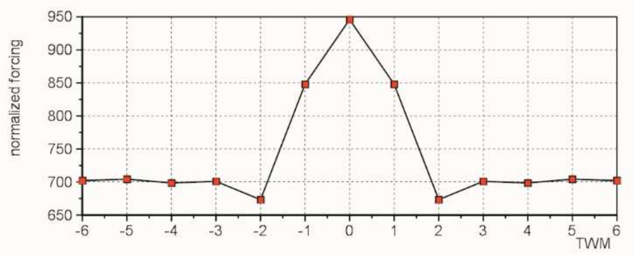

Fig. 17 Forcing identified with inverse approach

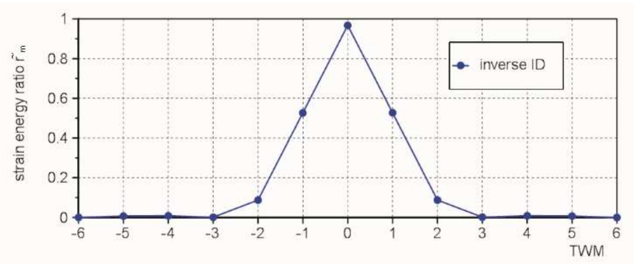

Fig. 18 Identified disk/shaft to overall strain energy ratio

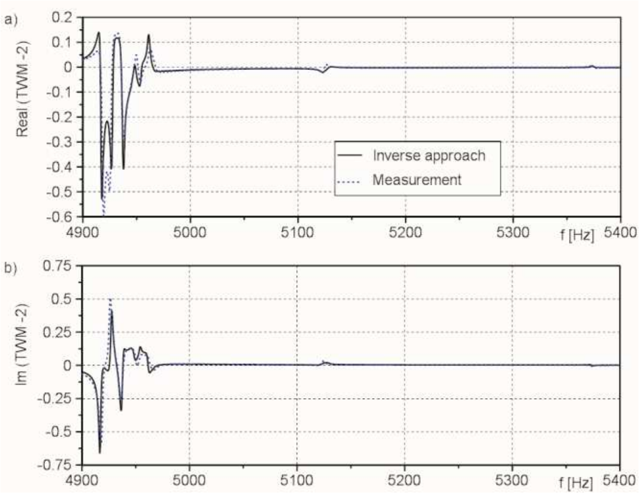

Fig. 19 FRF (BF 1) in TWM coordinates: a) Real part of and, b) imaginary part of TWM -2

Identification of Axial Turbine Blisk Parameters

Excellent matchings are achieved with respect to identified frequency mistuning patterns of the second wheel type, the axial turbine blisk (Fig. 20). In particular the validation bases, namely the blade by blade tests benefit from the well isolated character of BF 1 with negligible impact of adjacent BFs.

However, small discrepancies of real disk geometry and the CAD-geometry used for the FE-mesh result in minor deviations of FE-results from identified natural frequencies (Fig. 21). In contrast, the inverse approach applied to the two different blisks expectably yield approximately identical results.

The comparison of modal damping ratios indicates promising results for the regular turbine blisk with moderate mistuning (Fig. 22a) for two reasons: first, the inverse approach identifies modal damping ratios of the tuned counterpart, second, a clear assignment of modal damping ratios derived from an MDOF-fit to a particular TWM is rather possible than for the blisk with IM. Nevertheless, even for the regular blisk the assignment fails for a number of TWM, in particular high TWMs. Independently, those that are assignable, indicate that the modal damping ratios identified by the inverse approach are plausible. In contrast, for the blisk with large intentional mistuning the dedication of damping ratios extracted from MDOF-fit is limited to TWM 0 and 2, with reservations TWM 1 and 3 can be assigned. Consequently, the validation basis remains small. Notwithstanding the above, TWM 0 shows an excellent match and the order of magnitude for TWM 1, 2, and 3 fits as well.

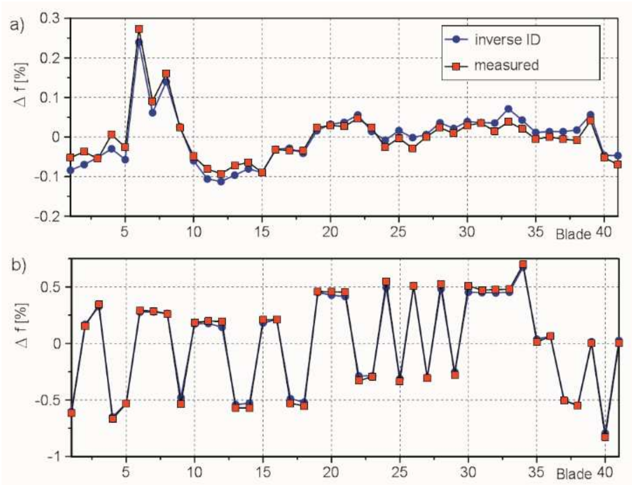

Fig. 20 Frequency mistuning (Mode 1, axial turbine), a) regular blisk, and b) IM blisk

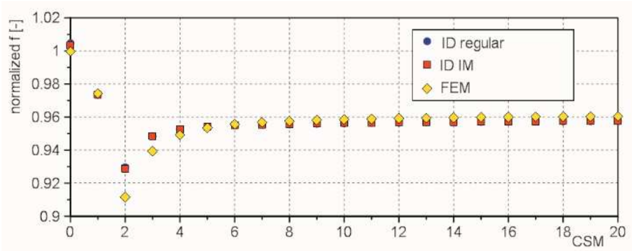

Fig. 21 Natural frequencies of tuned reference (BF 1)

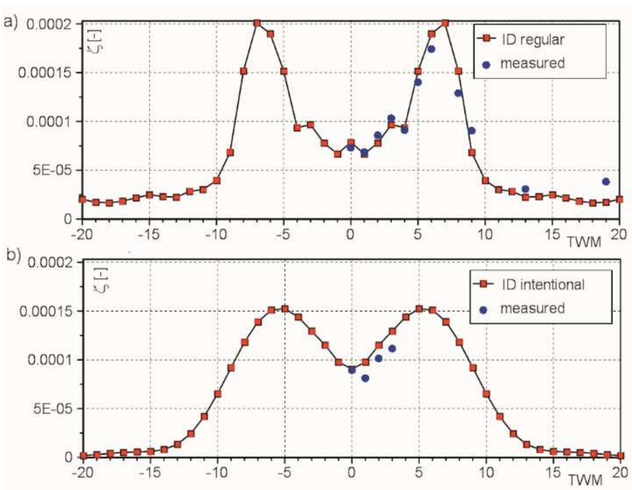

Fig. 22 Modal damping ratios: a) regular blisk, b) blisk with IM (ID: tuned counterpart, measured: MDOF-fit)

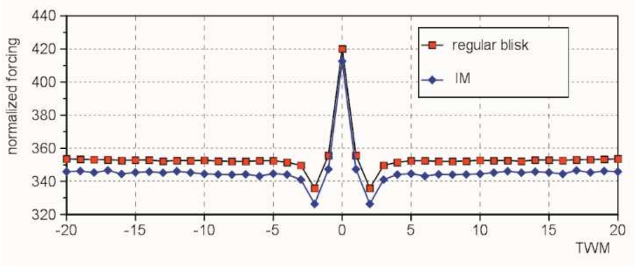

Fig. 23 Identified forcing of axial turbine blisks

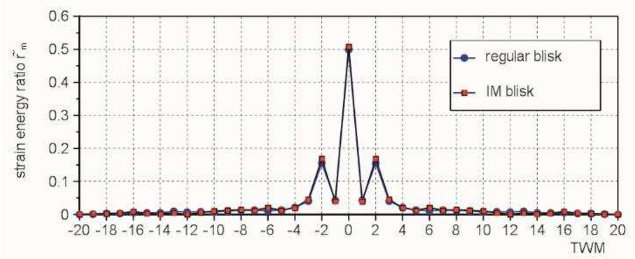

Fig. 24 Identified disk to overall strain energy ratio

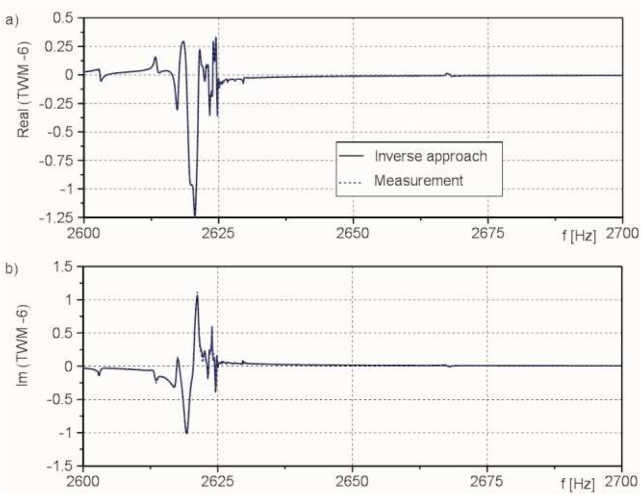

Fig. 25 FRF (BF 1) in TWM coordinates: a) Real part of and, b) imaginary part of TWM -6 (regular blisk)

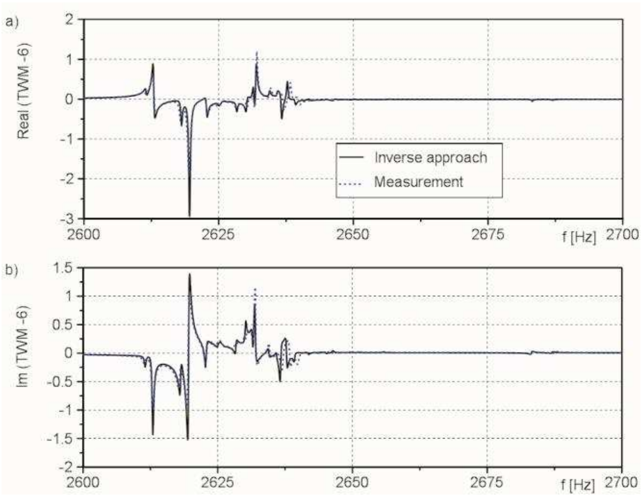

Fig. 26 FRF (BF 1) in TWM coordinates: a) Real part of and, b) imaginary part of TWM -6 (blisk with IM)

The identification of forcing confirms that the TWM 0 is most excited due to hammer impact (Fig. 23). This applies to both the regular and the IM blisk. The two wave TWM-2 and 2 reveal the second largest deviation from the majority of the results. Again, more strain energy is concentrated in the disk (Fig. 24) and less modal coupling occurs for these modes.

As long as the regular blisk is considered, the comparison of FRFs reveals approximately perfect results as exemplarily given in Fig. 25 for a TWM -6. In case of the intentionally mistuned blisk minor deviations are becoming apparent, but nevertheless the overall run of the identified curves remain perceptible (Fig. 26).

SUMMARY AND CONCLUSION

A new methodology has been presented which enables to identify the mechanical system of blisks, namely blade to blade mistuning, modal damping ratios, natural frequencies of the tuned counterpart, the forcing, and indirectly even the disk to overall blisk strain energy ratio. The approach is based on solving an over-determined system of linear equations. It has been derived in the first part of this paper [9] with a focus on rotating structures. In those cases experimental data e.g. gathered within engine test runs are required as input information. Aiming at its application to resting blisks an adjustment of the theory is provided in the present paper. Experimental frequency response data is employed to build up the system of equations, which is finally solved with respect to the quantities mentioned before by using a least squares approach.

By reference to a number of examples it could be shown that the method is well suited for identifying the mechanical system of blisks at standstill conditions in high quality. Starting with a numerical experiment of an axial turbine blisk it could be shown that the new inverse approach yields perfect results if simulated FRFs are employed as input data and even large mistuning is admitted. After that, real hardware of the same blisk type is considered again. In this context, two blisks with different blade mistuning patterns are processed using experimentally determined FRFs as input. In case of small mistuning reconstructed and experimentally determined FRFs match accurately. Marginal deviations occur if large mistuning is considered. The same generally applies to identified frequency mistuning and modal damping ratios which constitutes itself with inaccuracies of the experimental basis of comparison, namely the blade by blade mistuning ID procedure and the MDOF-fit for identifying modal damping ratios. Another practical example is addressing a turbine impeller wheel, which results in promising results as well.

Generally, the identification of mistuning works much more accurate compared to the method presented in [8], which partly fails due to the strong coupling of blade and weak disk motion. In addition, modal damping ratios, natural frequencies of the tuned counterpart, and the forcing are identified. Hence, more valuable information is gained about the structure in terms of updating high fidelity models and later forced response predictions.

ABBRIVIATIONS

BF	blade mode family
CSM	cyclic symmetry mode
FE	finite element
FMM	fundamental mistuning model
FRF	frequency response function
IM	intentional mistuning
LEO	low engine order excitation
MDOF	multi degree of freedom
SNM	subset of nominal system modes
TWM	travelling wave mode

NOMENCLATURE

a_l	amplitude of l-th harmonic
f	frequency
$\underline{\hat{f}}_m^{\text{modal}}$	modal force of the m-th mode
$\underline{\hat{f}}_m'$	generalized modal force (m-th mode)
j	blade index
k	nodal diameter index
l	harmonic index
m	mode index
$\tilde{r}_m$	indicator function, strain energy ratio
N	number of blades
N_H	number of harmonics
δ_m	decay factor of m-th mode
ζ_m	modal damping ratio of m-th mode
ω_0	angular natural frequency of tuned reference
ρ_m	mode shape scaling factor
$\boldsymbol{n}$	normal vector
$\boldsymbol{\phi}_{m,j}$	vector of m-th system mode on j-th blade
$\hat{\boldsymbol{\psi}}_m$	m-th mode shape

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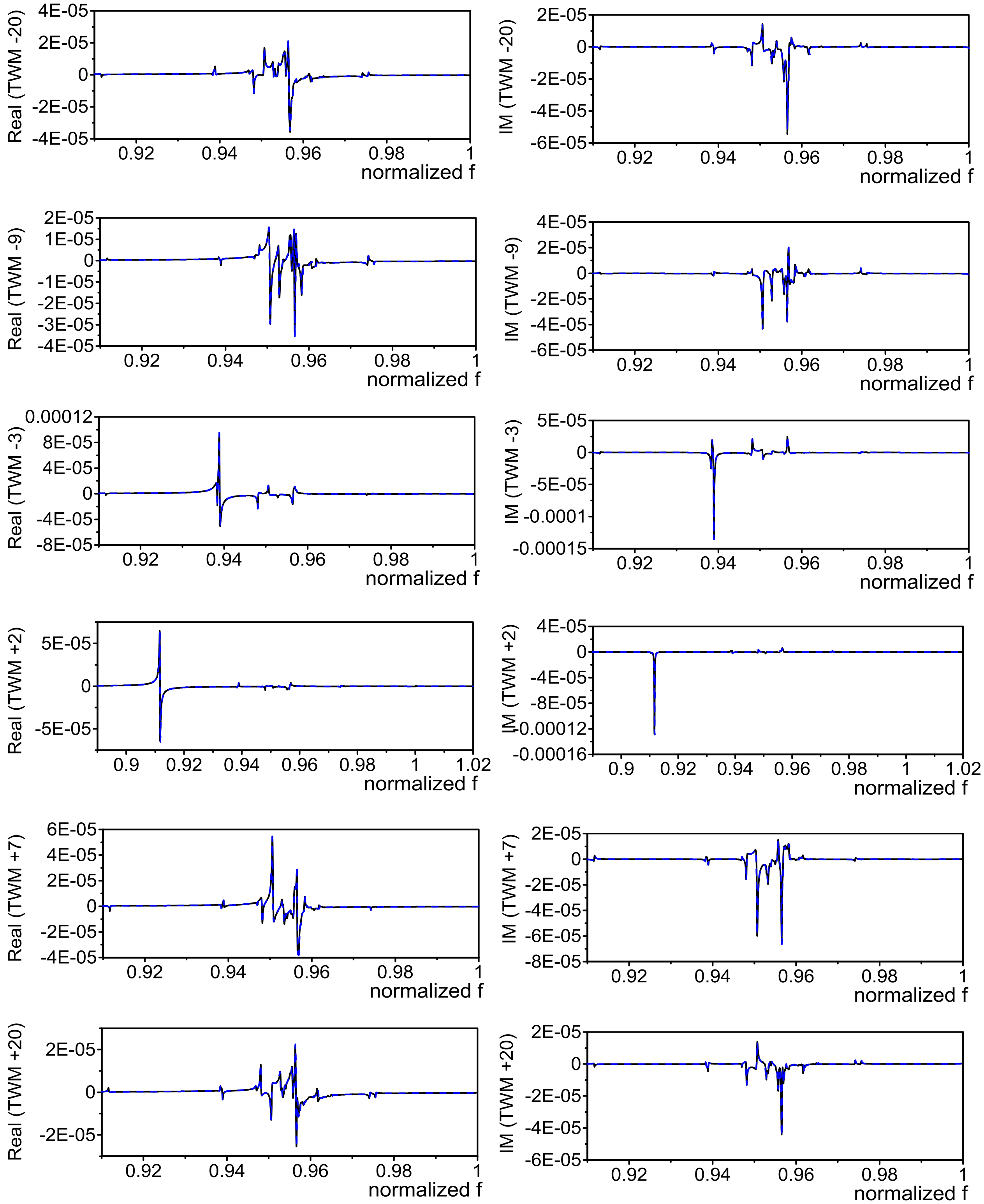
ANNEX A

FREQUENCY RESPONSE FUNCTIONS OF NUMERICAL EXPERIMENT (SELECTION)



Black: inverse ID, blue: SNM-computation