Investigating Prandtl number effects in heated concentric coaxial pipe flow at high Reynolds number

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Turbulent heat transfer in concentric coaxial (annular) pipe flows holds significance across various engineering applications, such as heat exchangers and liquid metal fast reactors, where variables such as the Prandtl number and Reynolds number play important roles. In the present study, a heated annular pipe flow is numerically investigated for friction Reynolds numbers up to $Re_{\tau} \approx 10,000$ are simulated for Prandtl number Pr = 0.71(air) and Pr = 0.025 (mercury), where the temperature is treated as a passive scalar. A stochastic approach, the so-called one-dimensional turbulence model¹ (ODT) formulated for cylindrical geometry², is applied here as a modeling tool. ODT is a dimensionally reduced flow model that makes it feasible to resolve all relevant turbulent flow scales along a single physical coordinate. In this case, the radial direction spanning from the inner to outer cylinder walls has been chosen. The past simulation study³ of annular pipe flow using ODT model showed that the ODT model provided reliable prediction with significantly lower computational cost in investigating momentum transfer properties. Besides, classical boundary-layer and mixing-length theory were utilized there to verify the model capabilities. We extended those theoretical analyses to the passive scalar transfer in the present study. The thermal boundary layer, which divides into a conduction-dominated region and mixinglength-dominated region, at the inner cylinder wall, is discussed. Figure 1 (a) shows law-of-the-wall plots over the inner wall for various Pr and bulk Reynolds number Re_b ; comparing with the mixing length theory, the legend shows the equation closed by Pr_t . Following Figure 1 (b) and (c), in the mixing-length-dominated region, ODT suggests turbulent Prandtl number $Pr_t \simeq 0.95$ for Pr = 0.71, and $Pr_t \simeq 1.15$ for Pr = 0.025. The mixing length model reveals presence of nonlocal curvature effects, which influence the conductive sublayer and the logarithmic region of the thermal boundary layer over the inner wall in contrast to the outer wall due to its much larger radius. Mixing length theory provides a reasonable approximation of the boundary layer profiles at high Reynolds numbers and low Prandtl numbers. In this contribution, we will address the ODT-based modeling of weakly heated coaxial pipe flows, emphasizing the dependence of the heat transfer on the Prandtl number and addressing heat flux fluctuation to gain a more comprehensive understanding of the turbulence properties.



Figure 1: Law-of-the-wall plots of the numerically and semi-analytically predicted thermal boundary layer. (a) Normalized mean perturbation temperature $\langle \Theta \rangle^+(r^+)$ over the circular inner wall for various Pr in a wide-gap configuration (radius ratio $\eta = 0.1$). ODT model results are given in color (dark to light) for $Re_b = 17,700$, 10^5 , and 10^6 . Predictions based on boundary-layer theory and a mixing-length approach are given in black for $Re_b = 10^6$. (b, c) $Pr_t(r^+)$ over the circular inner wall for $Re_b = 10^5$ for Pr = 0.71 and 0.025.

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¹Kerstein, J. Fluid Mech. 392, 277-334 (1999).

²Lignell et al., *Theor. Comput. Fluid Dyn.* **32**, 495 (2018).

³Tsai et al., *PAMM* **23(4)**, e202300167 (2023).