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Stochastic modeling of asymmetric turbulent boundary layers in annular pipe flow

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Introduction

Concentric coaxial (annular) pipe flows with turbulent transfers are widely used in numerous engineering applications. Various cross-sections of annular geometry relevant to applications are shown in Figure 4(a). Larger radius ratios, $0.2 < \eta < 0.9$, are typically encountered in heat exchangers [1], as shown in Figure 1(a), providing better heat transfer properties than a regular pipe due to the increased surface area. Small radius ratios, $\eta < 0.2$, are found in electrostatic precipitators used in gas cleaning [2], as shown in Figure 1(b). In contrast to the flow through a plane channel or circular pipe, annular pipe flow exhibits a more complex geometry that encompasses additional internal curvature imposed as boundary condition to the flow. Consequently, performing numerical simulations of annular pipe flow becomes a challenge.

Challenges for numerical modeling of annular pipe flow :

- Wall effects (curvature effects, curvature-induced instability)
- Computational cost (mesh generation, grid resolution)
- Limited availability of validation and experimental data

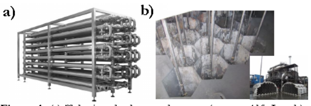


Figure 1: (a) Tube-in-tube heat exchangers (source: Alfa Laval) (b) Electrostatic precipitator (source: Lukasz Katlewa)

Main objectives

- Utilize the stand-alone **one-dimensional turbulence (ODT) model** [3] for $\eta \leq 0.1$ cases
- Comprehensive analysis of how the **radius ratio** influences features of the boundary layer at inner cylinder wall
- Influence of **high Reynolds number** ($Re_b = 2\times10^3 \sim 2\times10^7$) on the boundary layers

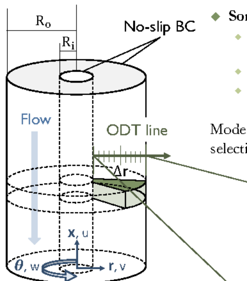
Overview of ODT model formulation and calibration

ODT is a dimensionally reduced flow model that can be economically utilized as a stand-alone tool. It aims to resolve all relevant scales along a one-dimensional physical coordinate, the so-called ODT line. In this case, ODT line is taken as the wall-normal direction and aligned with the radial direction, shown in Figure 2. No-slip boundary conditions are prescribed at the cylinder walls, and an axial mean pressure gradient force is imposed to drive the flow.

- ODT governing equations for the cylindrical [4] velocity components $\vec{u}_i$ is

$$\frac{\partial \vec{u}_i}{\partial t} + \sum_{t_e} \varepsilon_i(\alpha, C, Z)\,\delta(t-t_e) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\nu\frac{\partial \vec{u}_i}{\partial r}\right) - \frac{1}{\rho}\frac{dP}{dx}\delta_{1i} \quad -(1)$$

where t_e represents the stochastically sampled times of eddy event $\varepsilon_i(i = x, r, \theta)$ occurrences, $\delta(t-t_e)$ the Dirac distribution function. ν is the kinematic viscosity, ρ the mass density of the fluid and dP/dx the prescribed uniform mean pressure gradient force. Noted that a stochastic sequence of discrete eddy events, as represented by the sum, has formally replaced the advection term (and fluctuating pressure term) of the Navier–Stokes momentum equation.



- **Some important ODT model parameters [5] :**
  - α : **Energy Distribution parameter**, controls the efficiency of the inter-component energy transfer induced by modeled fluctuating pressure gradient forces.
  - C : **Eddy frequency parameter**, governs the overall turbulence intensity
  - Z : **Viscous penalty parameter**, influences the suppression of unphysically small eddy events by a viscous energetic penalty condition

Model calibration had been done in previous study [6], in this case, the selection of model parameter is $(\alpha, C, Z) = (2/3, 6, 300)$

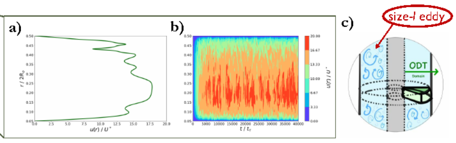


Figure 2: Schematic of the annular pipe flow configuration investigated. The wedge-like radially oriented ODT (line) domain is sketched. No-slip boundary conditions are prescribed at the annulus walls as indicated.

Figure 3: (a) The instantaneous radial profile of the axial velocity component. (b) Space-time diagram of a turbulent ODT solution for an annular pipe flow with $\eta = 0.1$ and $Re^* = 600$ with the indication of the selected flow profile $\vec{u}(r,t)$. U^* and t_τ denote friction velocity and time scales used for normalization. (c) An illustration of eddies in flow field.

Mean velocity and boundary layer profiles at outer cylinder

The mean axial velocity profiles of annular pipe flow with $\eta = 0.1, 0.04, 0.02$ in comparison to reference DNS [7] is shown in Figure 4(b). It is seen that the influence of wall curvature is more pronounced at the inner cylinder wall than at the outer cylinder wall, and this effect is more significant in the cases with smaller radius ratios, where a greater velocity gradient is observed. ODT slightly misestimates the velocity field near the center of the annular gap, nevertheless, it can capture the main features of the mean velocity field, especially the location of the maximum velocity. Figure 4(c) shows the velocity boundary layer profiles at the outer cylinder wall compared with DNS results for pipe flow [9] and channel flow [10]. First, it demonstrates that the ODT model provides full-scale flow resolution down to $r^+ < 1$. Second, the well-collapsed data indicates that outer cylinder boundary layer similarity is established. Last, it is demonstrated that the ODT exhibits the law-of-the-wall, composed of the viscous sublayer and the logarithmic region, which phenomenologically reproduces reference DNS [7].

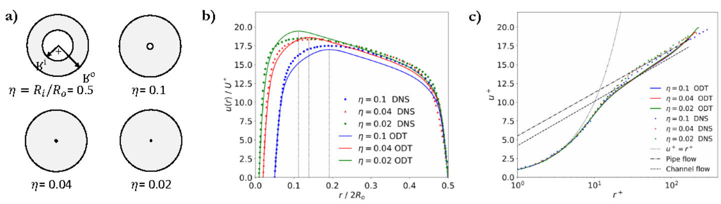


Figure 4: (a) To-scale sketches of different annular pipe geometry. (b) Normalized mean velocity profiles for $\eta = 0.1, 0.04, 0.02$ with $Re^* = 600$ in comparison to reference DNS [7]. The dotted line shows the expected location of the maximum velocity according to the experimental results [8]. For scaling purposes, the velocity scale $U^* = \sqrt{-(R_o/2\rho)(dP/dx)}$ and the corresponding Reynolds number $Re^* = 2U^*R_o/\nu$. (c) Boundary-layer profiles of the mean axial velocity at the outer cylinder wall using the local friction velocity for normalization. Corresponding reference DNS are from [7]

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Velocity boundary layer profiles at inner cylinder

The velocity boundary layer profiles captured by ODT at the inner cylinder is shown in Figure 5(a). The results do not completely agree with the reference DNS [7]. The disagreement in the large r^+ region is expected due to the misestimation of the value of maximum mean velocity. In order to quantify the non-monotonic effect with respect to η, we utilize the boundary-layer analysis for the model results in the same way as this was done previously for the reference DNS [7]. The boundary layer over the inner cylinder is hence separated into a **viscosity-dominated region** and a **mixing-length-dominated region.**

The viscosity-dominated region :

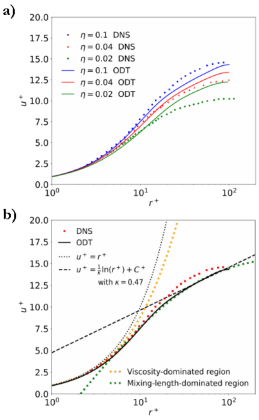
It is assumed that the mean flow is fully developed and symmetric to the axis. After applying the temporal average to the momentum equation (1), re-arranging, and integration over r yields the shear-stress balance equation as

$$\overline{u'v'} - \nu\frac{\partial \bar{u}}{\partial r} = -\frac{r}{2}\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + \frac{C_1}{r}, \text{ where } C_1 = -\frac{1}{\rho}\frac{\tau_o R_i - \tau_i R_o}{R_i/R_o - R_o/R_i} \quad -(2)$$

where C_1 is an integration constant that depends on the flow solution by the wall-shear stress, and $\overline{u'v'}$ is used to represent the ensemble effect of the turbulent eddy events as a model analog of the Reynolds stress.

In the region very close to the inner cylinder, where $r < \delta_v$ (viscous length scale $\delta_v = \nu/u_\tau$), it is assumed that the viscous stress dominates , that is, $\overline{u'v'} \ll \nu(du/dr)$. For small radius ratios $\eta \to 0$, equation (2) reduces to

$$u^+(r^+) = R_i^+ \ln\left(\frac{r^+}{R_i^+}\right), \text{ where } R_i^+ = \frac{R_i u_{\tau,i}}{\nu} \quad -(3)$$

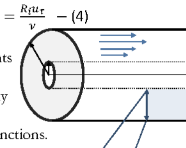


The mixing-length-dominated region :

For the mixing-length-dominated region, it is assumed that the total stress is carried by the Reynolds stress so that $\nu(du/dr) \ll \overline{u'v'}$. Fellow the standard eddy viscosity approach and model the eddy viscosity ν_t with mixing length model [7], equation (2) reduces to

$$u^+(r^+) = \frac{1}{\beta^+}\ln\left[\frac{r^+ - R_i^+}{r^+}\right] + D^+, \text{ where } R_i^+ = \frac{R_i u_\tau}{\nu} \quad -(4)$$

where β^+ and D^+ are parametrization coefficients that can be determined with flow profile data. Figure 5(b) presents a comparison of the velocity boundary layer obtained with ODT and DNS together with conventional and modified wall functions.



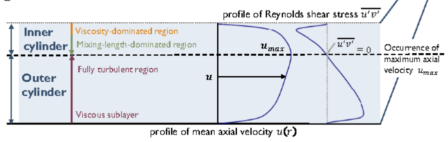
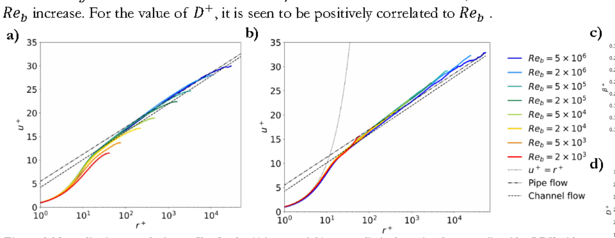


Figure 5: (a) Boundary-layer profiles of the mean axial velocity at the inner cylinder wall normalized with local friction velocity and compared with DNS [7]. (b) Velocity boundary layer obtained with ODT together with the fitted semi-analytic profiles given by equations (3) and (4) for $\eta = 0.1$ and $Re^* = 600$.

Influence of high Reynolds number

Figure 6 shows boundary layer profiles at the (a) inner and (b) outer cylinder for radius ratio $\eta = 0.1$ with various Reynolds numbers. The outer cylinder's collapse is maintained across an extended logarithmic region. For the inner cylinder, as the Reynolds number increases, the impact of curvature diminishes, and the boundary layer profiles tend to collapse better. ODT predicts that for asymptotically large Re_b , the logarithmic law from pipe flow is also approached at the inner cylinder. Figures (c) and (d) give the parameters β^+ and D^+ from equation (4) with different Re_b . It is found that the value of β^+ tends to be a fixed value, whereas the increment becomes less as Re_b increase. For the value of D^+, it is seen to be positively correlated to Re_b .



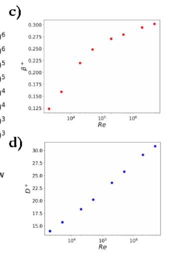


Figure 6: Normalized mean velocity profiles for the (a) inner and (b) outer cylinder boundary layer, predicted by ODT with $\eta = 0.1$ for various bulk Reynolds number Re_b from the range 5×10^3 to 2×10^6, where $Re_b = 2(R_o - R_i)U_b/\nu$ and U_b is the cross-sectional-averaged bulk velocity. (c) and (d) present the parameters β^+ and D^+ vary with Re_b .

Conclusions

- The standalone **one-dimensional turbulence (ODT)** model provides an economical and simplified representation of the complex boundary layer dynamics.
- A theoretical analysis that utilizes boundary-layer and mixing-length theory was conducted, and it is capable to predict the main characteristics of the velocity boundary layer adjacent to the inner cylinder.
- The results demonstrate that spanwise **curvature effects** cannot be ignored, especially in small radius ratio cases.
- The **curvature effects** persist up to high Reynolds numbers across the boundary layer for finite radius ratio.

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