Turbulence Still Surprises: Explorations Using a 1D Model

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MetStröm Short Course at BTU

Outline of presentation

- Overview of the modeling approach
- Unexpected large-scale effects in pipe mixing
- Stronger-than-expected differential molecular diffusion
- Spontaneous layer formation in buoyant stratified flow
- Counterintuitive dependence of jet spreading on molecular diffusivity
- Conclusions

The conventional representation of turbulent advection as enhanced diffusion omits important physics

Constant-property equations of motion (Navier-Stokes equation, scalar transport equation):

$$\mathbf{u}_t + \mathbf{u} \cdot \nabla \mathbf{u} = \nu \nabla^2 \mathbf{u} - (1/\rho) \nabla p \qquad \theta_t + \mathbf{u} \cdot \nabla \theta = \kappa \nabla^2 \theta$$

To obtain a turbulence model in 1D, apply the boundary-layer approximation and either

- **average and replace ν and κ by ν_e and κ_e (usual: represents advection by diffusion)**

or

- **replace** $u \cdot \nabla$ **by a different advection process (approach used here: no averaging)**

<u>Simple example</u>:

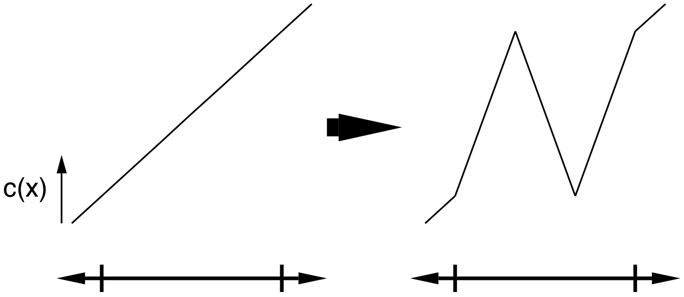
For time-developing unforced flow, obtain the following alternative modeling frameworks for the lateral (y) profile of streamwise velocity u and a passive scalar θ:

$u_t = \nu_e(y,t)\, u_{yy}$ $\qquad$ $\theta_t = \kappa_e(y,t)\, \theta_{yy}$ $\qquad$ Pr_e (or Sc_e) $= \nu_e(y,t) / \kappa_e(y,t)$

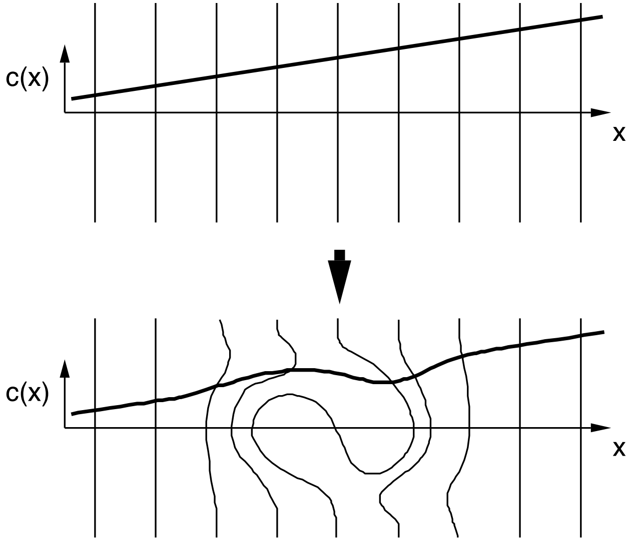
$u_t = \nu u_{yy}$ + ‘advection’ $\qquad$ $\theta_t = \kappa\theta_{yy}$ + ‘advection’ $\qquad$ Pr (or Sc) $= \nu / \kappa$

neither framework is complete as written

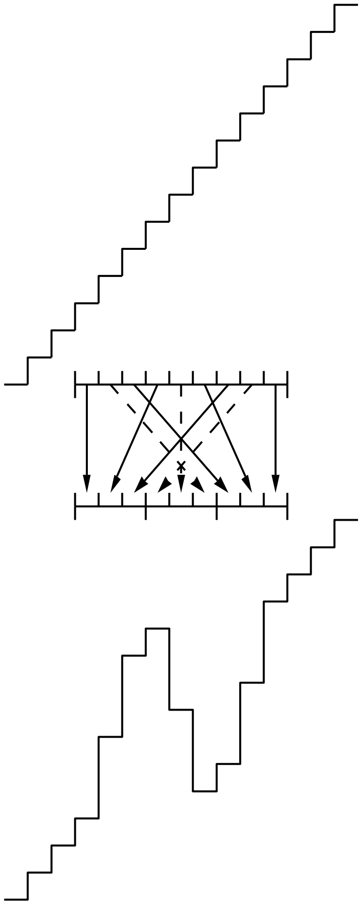
Advection is modeled as a sequence of *triplet maps*, which preserve desired advection properties



The triplet map captures compressive strain and rotational folding effects, and causes no property discontinuities



This procedure emulates the effect of a 3D eddy on property profiles along a line of sight



The triplet map is implemented numerically as a permutation of fluid cells (or on an adaptive mesh)

There are different ways to specify the map sequence during a simulation

- Linear-Eddy Model (LEM): Map occurrences and properties (size, location) are sampled from fixed distributions
  - Parameters determining these assignments based on the turbulent flow state at each location must be provided as input
  - LEM evolves scalar profiles but not velocity, hence is a turbulent mixing model, not a turbulence model

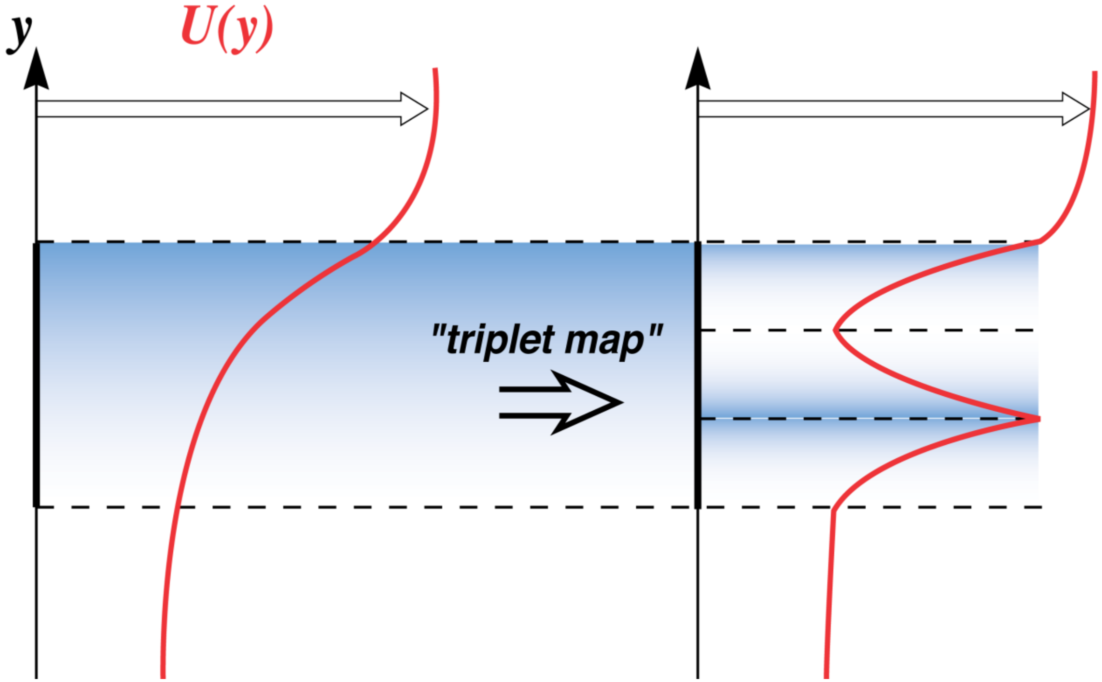
- One-Dimensional Turbulence (ODT): Eddy sampling is based on the flow state evolved by the model
  - After parameter adjustment, ODT predicts turbulence evolution
  - The required input is the flow configuration (ICs, BCs)

- In either model, the eddies (instantaneous maps) punctuate continuous-in-time advancement of molecular-diffusive transport, chemistry, etc. For example:

$$u_t = \nu u_{yy} + \text{`eddies'} \qquad \theta_t = \kappa \theta_{yy} + \text{`eddies'}$$

In ODT, the triplet map amplifies shear, inducing an *eddy cascade* (feedback mechanism)

- **The key to model performance is the eddy selection procedure**
- Eddy likelihood, in a random sampling procedure, is governed by local shear
- When an eddy occurs, the local shear is amplified, which modifies eddy likelihoods



High shear at small scales drives small eddies, leading to an eddy cascade

(In LEM, inertial-range-cascade scaling is hard-wired)

ODT eddy selection is based on the mixing-length concept, applied locally

- Each possible eddy, defined by eddy spatial location and size (S), is assigned a time scale τ based on local energetics (e.g., shear)
- This defines an eddy velocity S/τ and energy density $\rho\,(S/\tau)^2$
- The set of τ values determines an eddy rate distribution from which eddies are sampled
- Unlike conventional mixing-length theory, this procedure is local in space and time (no averaging) and is applied to all eddy sizes S (multi-scale) rather than a single selected S value (‘mixing length’)

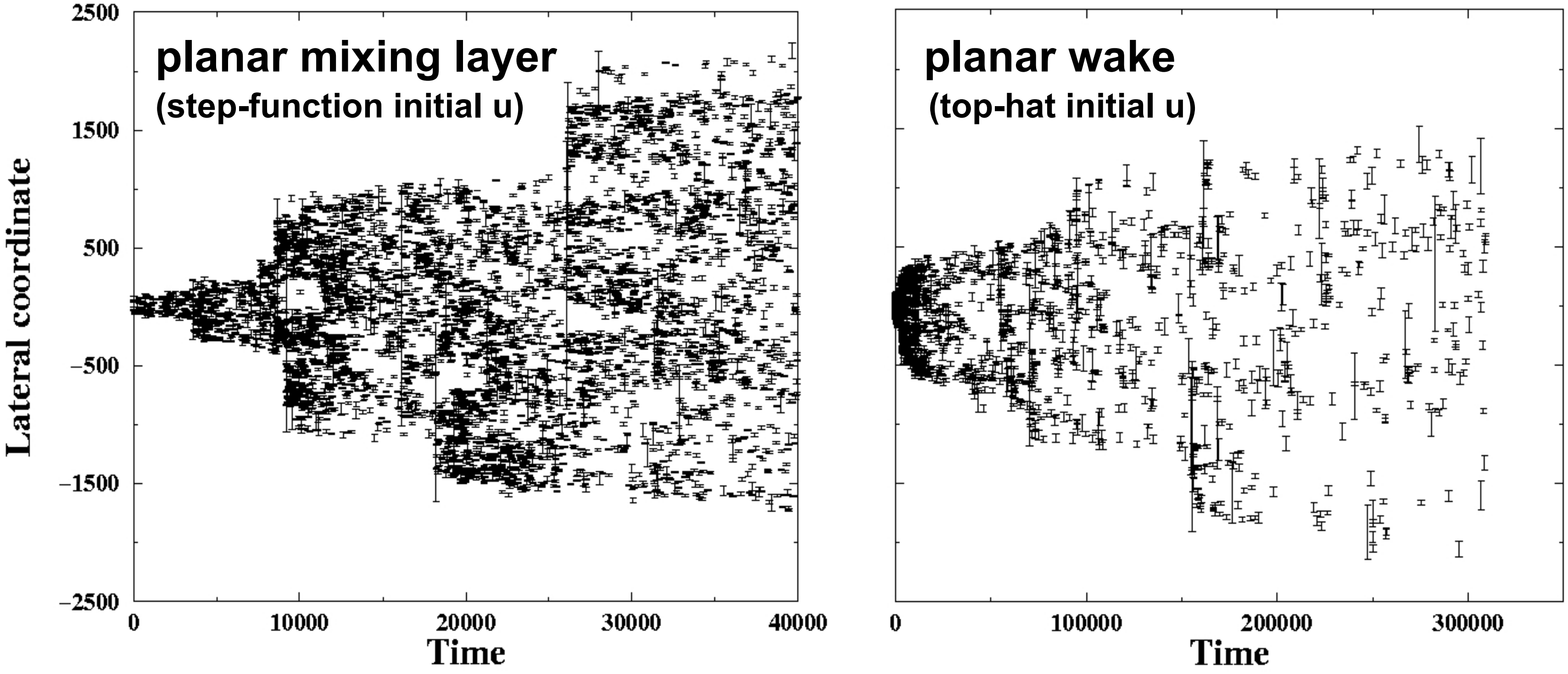
To capture energy transfers (e.g., buoyancy-induced), the ODT eddy time scale is based on an energy balance

- **Energy balance (schematic):** **S E = C (K – P – Z V)**
  - S is the eddy size
  - E = $\rho\,(S/\tau)^2$ is the eddy kinetic-energy density
  - K is the ‘available’ kinetic energy of velocity profiles within the eddy
  - P is the gravitational potential energy change caused by the eddy
  - V is a ‘viscous penalty’ (imposes a threshold eddy Reynolds number)
  - C and Z are free parameters
- **This relation determines the eddy time scale τ**
- **(u, v, w) velocity profiles are adjusted (wavelet method) so that total (kinetic plus potential) energy is conserved**

- **This framework accommodates various energy couplings, e.g. pressure scrambling, compressibility effects, surface tension**

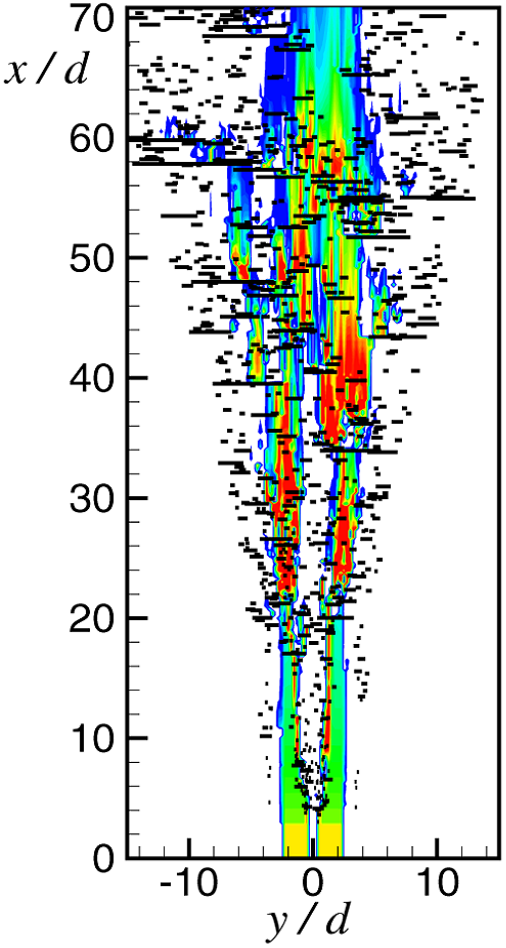
ODT simulations provide detailed flow-specific representations of turbulence

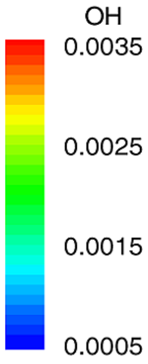
These simulations are based on time advancement of $u_t = \nu u_{yy}$ with flow-specific initial u profiles (see below), plus eddies



- **Each vertical line shows the spatial extent of an eddy**
- **Horizontal location is its time of occurrence**
- **Units are arbitrary**

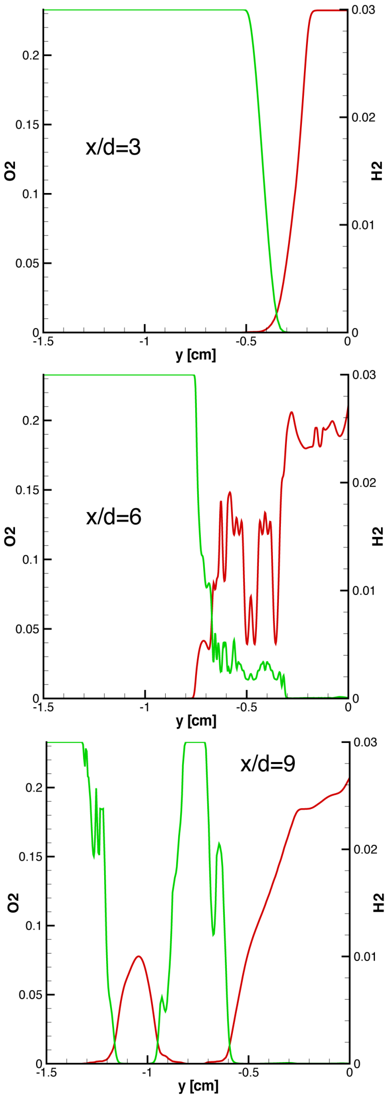
LEM and ODT resolve advective-diffusive-reactive couplings and hence all flame regimes





ODT simulation of a piloted methane-air jet diffusion flame (Sandia flame D)

O_2 and H_2 profiles from an ODT simulation of a syngas ($CO/H_2/N_2$) jet diffusion flame



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Simple configuration: one eddy size, sinusoidal initial scalar – what happens?

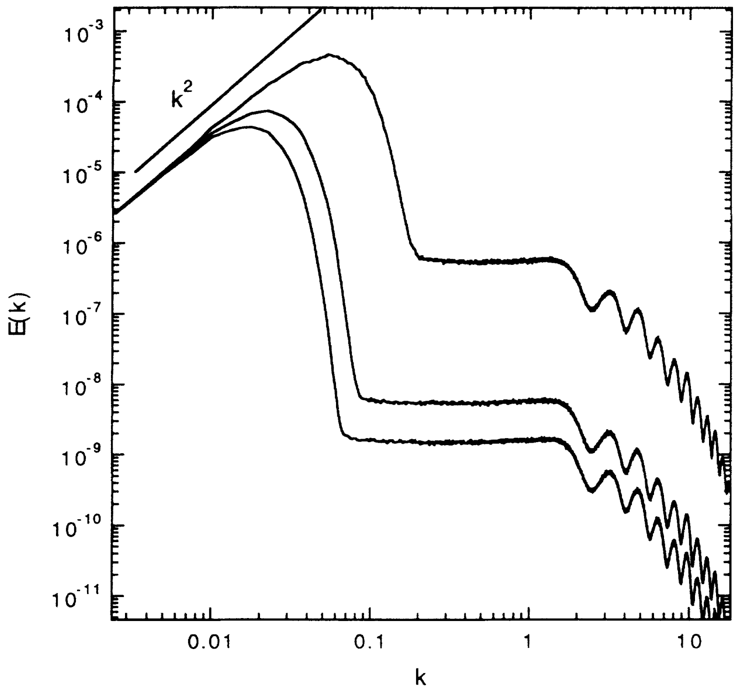
Evolve $\theta_t = \kappa \theta_{yy}$ + ‘eddies’ with

- θ (y,0) = sin(2πy/L)
- Randomly placed triplet maps, all size L
- High map frequency (eddy transport >> κ)
- Domain size >> L, periodic boundary conditions

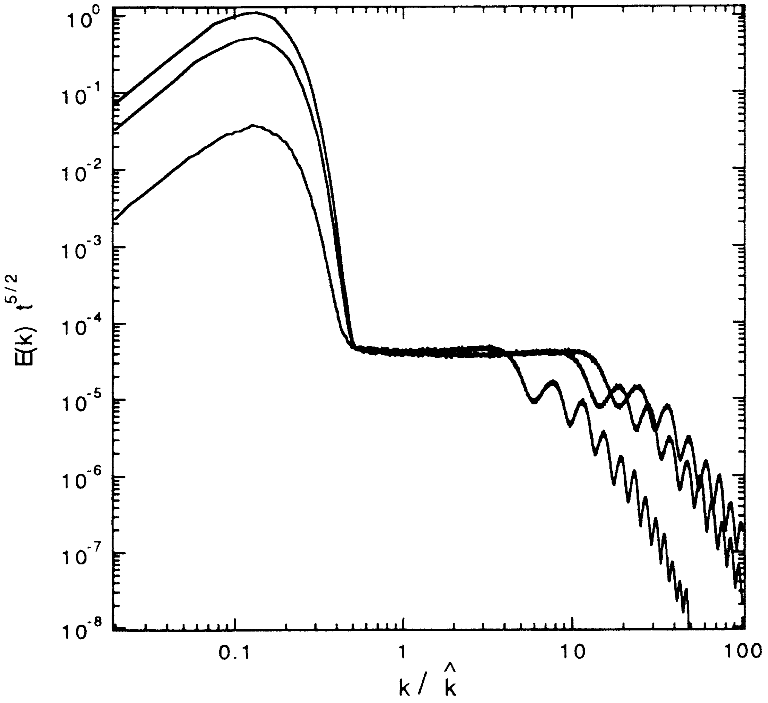
What is the time evolution of:

- Scalar variance?
- Scalar power spectra?

The result was surprising (amazing!) – then an explanation was found



top to bottom: increasing t



analysis predicts the collapse seen in this scaled plot

Pipe flow measurements motivated by these results illustrate the cause of this behavior

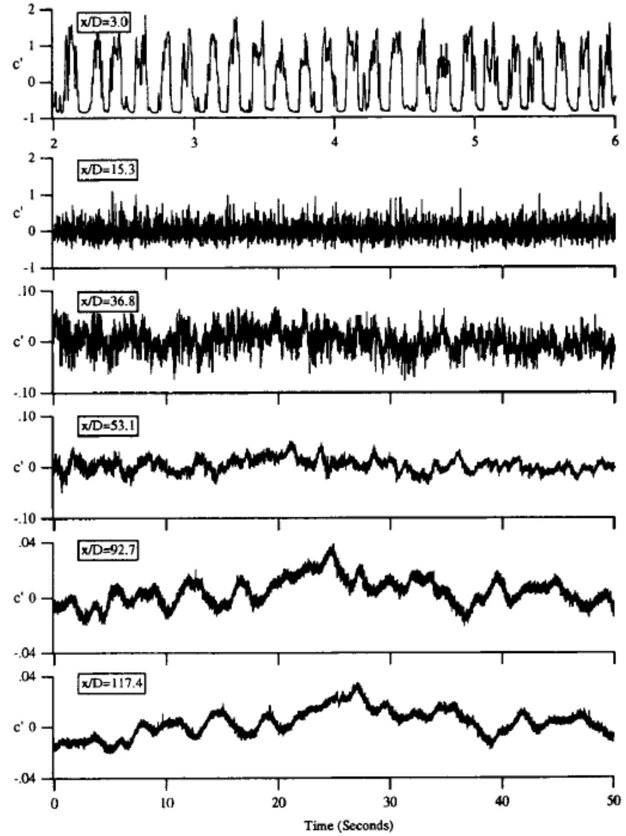
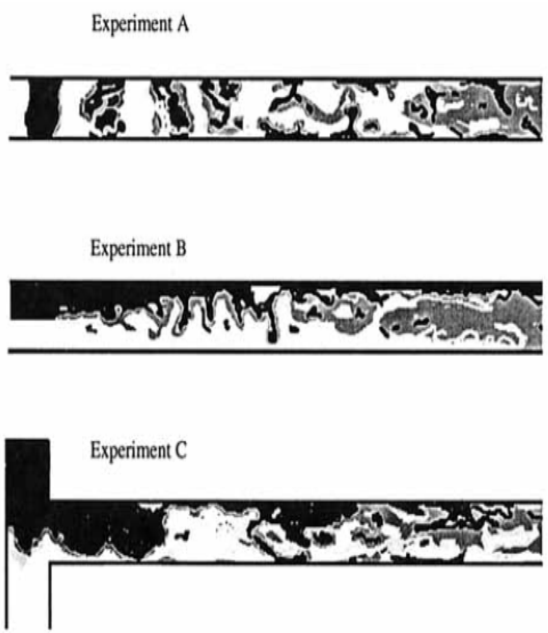


Figure 3. Time series of measured scalar field at the center of the pipe for experiment A.

A 4-s period is shown for $x/D = 3.0$ to show the idealized inlet condition achieved. At all other locations a 50-s time series is shown.



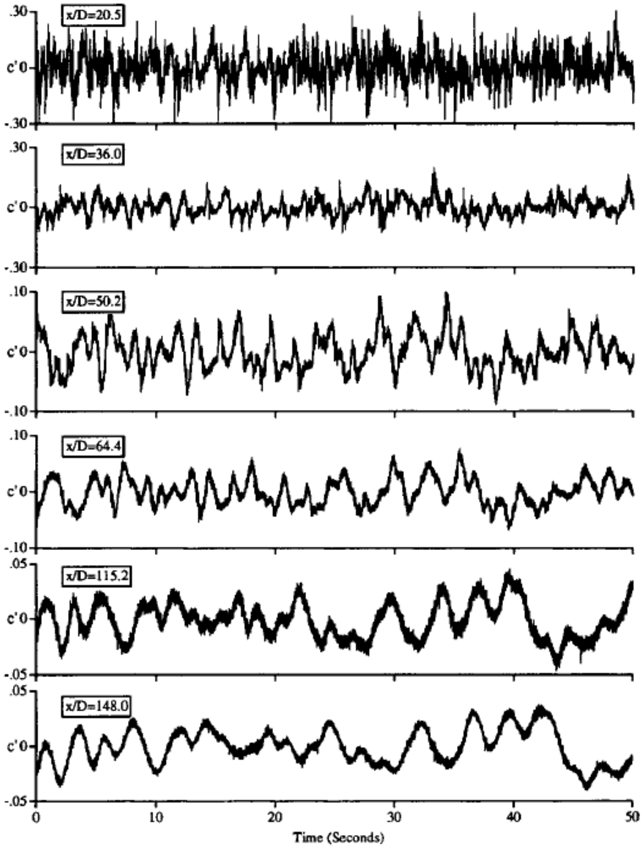
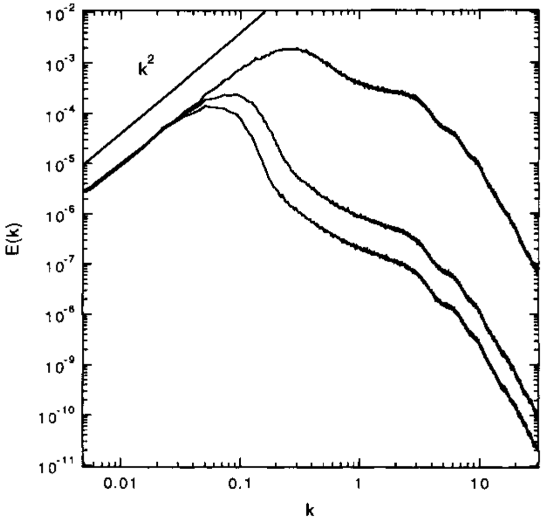
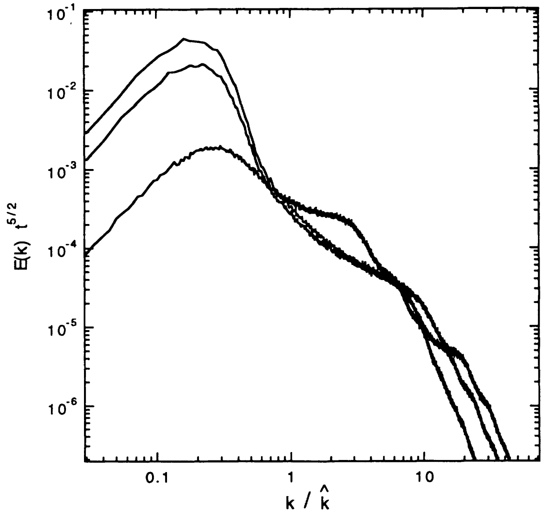


Figure 11. Time series of measured scalar field at the center of the pipe for experiment C.

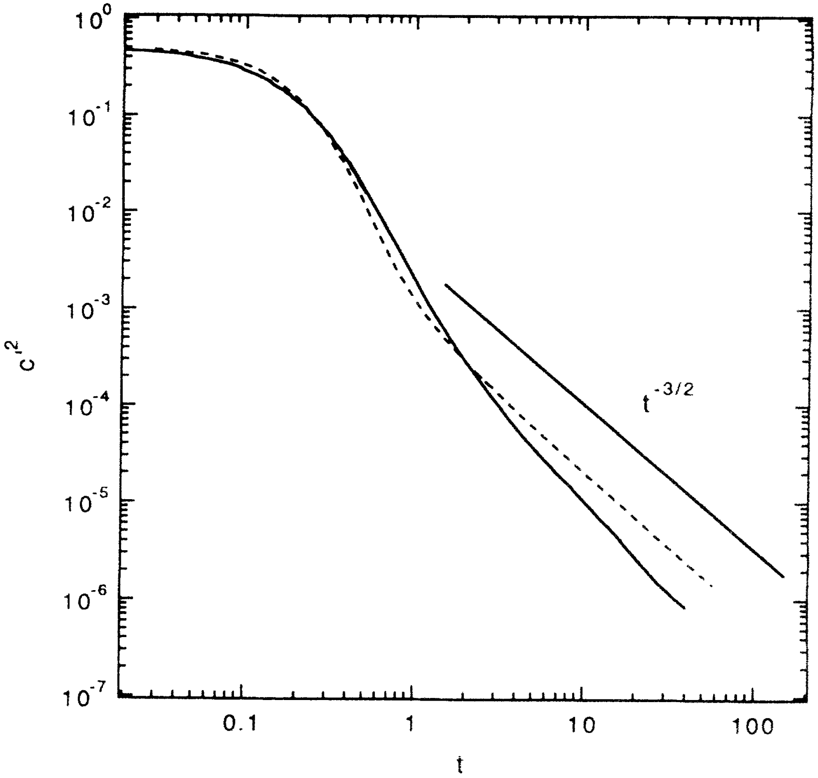
Guilkey, McMurtry, and Klewicki, 1997

Simulations were performed for a ‘pipe-like’ map-size distribution





Analysis predicts $t^{-3/2}$ scalar-variance decay



— — one map size

—— pipe-like size distribution

Scalar power-spectrum measurements exhibit the predicted features

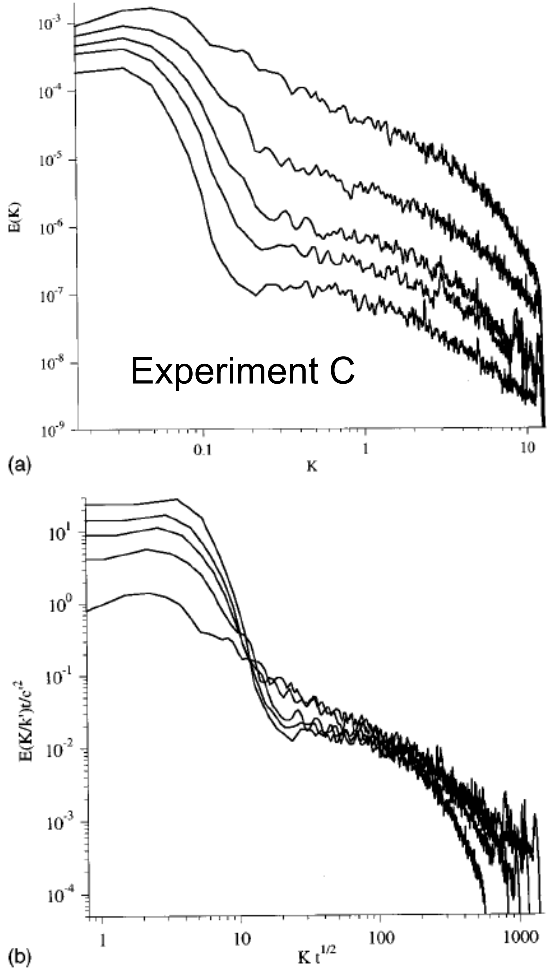


FIG. 5. (a) Power spectral densities of scalar fluctuations, experiment 2. Axial locations (from top to bottom) are $x/D = 20.5$, 36.0, 50.2, 64.4, and 90.3. (b) Spectra subject to ''equilibrium'' range scalings, indicating self-preserving behavior.

Pipe measurements show a transition from exponential to power-law variance decay

Brodkey, 1966, ‘confirmed’ exponential decay (Corrsin’s batch-reactor analysis) to x/D = 30

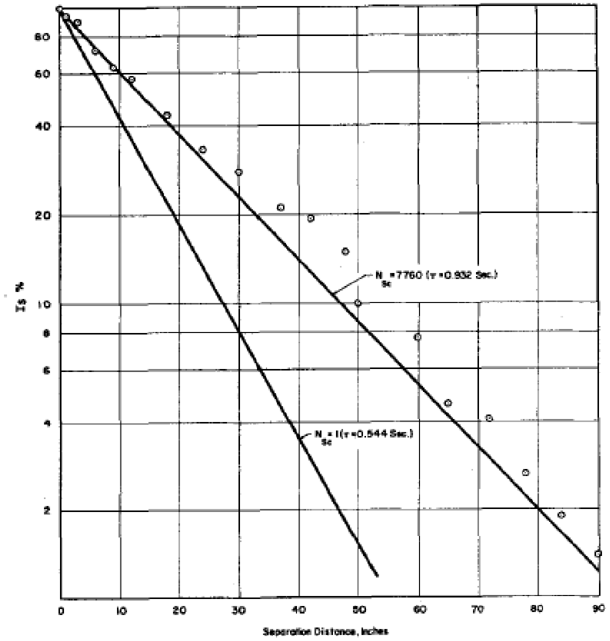
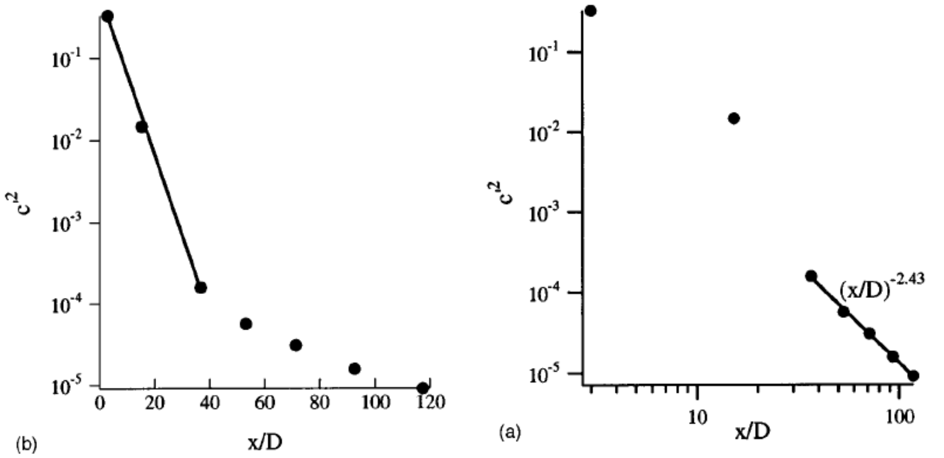
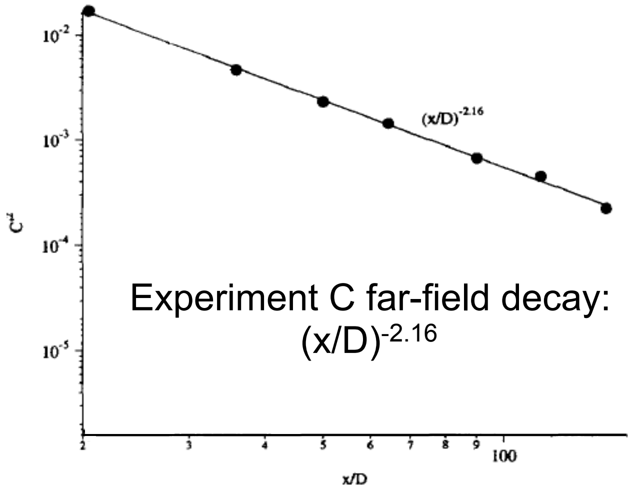


Fig. 1. Intensity of segregation.

Experiment A: near-field exponential, far-field $(x/D)^{-2.43}$





Experiment C far-field decay: $(x/D)^{-2.16}$

Near-field decay depends on initialization – <u>the only robust result is the far-field power law</u> (with a non-universal exponent)

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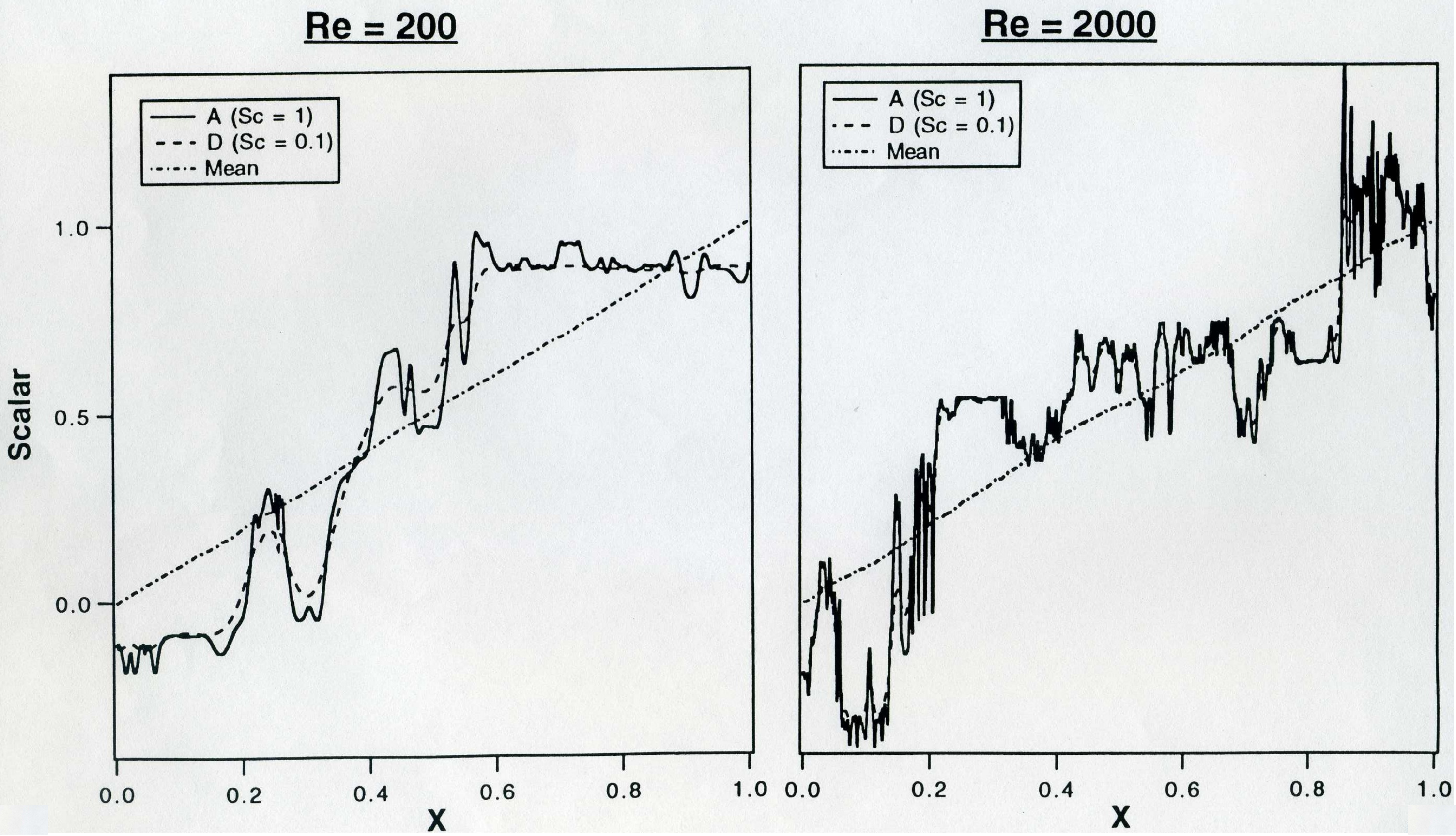
Linear-eddy model (LEM): distribution of eddy sizes, obeys inertial-range scalings

Evolve $\theta_t = \kappa\,\theta_{yy}$ + ‘eddies’ where

- The map distribution is spatially uniform (homogeneous turbulence)
- Map sizes range from η (Kolmogorov microscale) to L
- Map size PDF $f(\ell)$ is determined by $\kappa_e(\ell) \sim \ell\, v(\ell) \sim \ell^{4/3}$
- Need an input value of $\kappa_e(L)$ to set the overall map frequency
- Non-dimensional parameters: Re ~ $(L/\eta)^{4/3}$, Pe ~ $\kappa_e(L) / \kappa$
- Sc ~ Pe / Re, which implies a kinematic viscosity (though no velocity!)

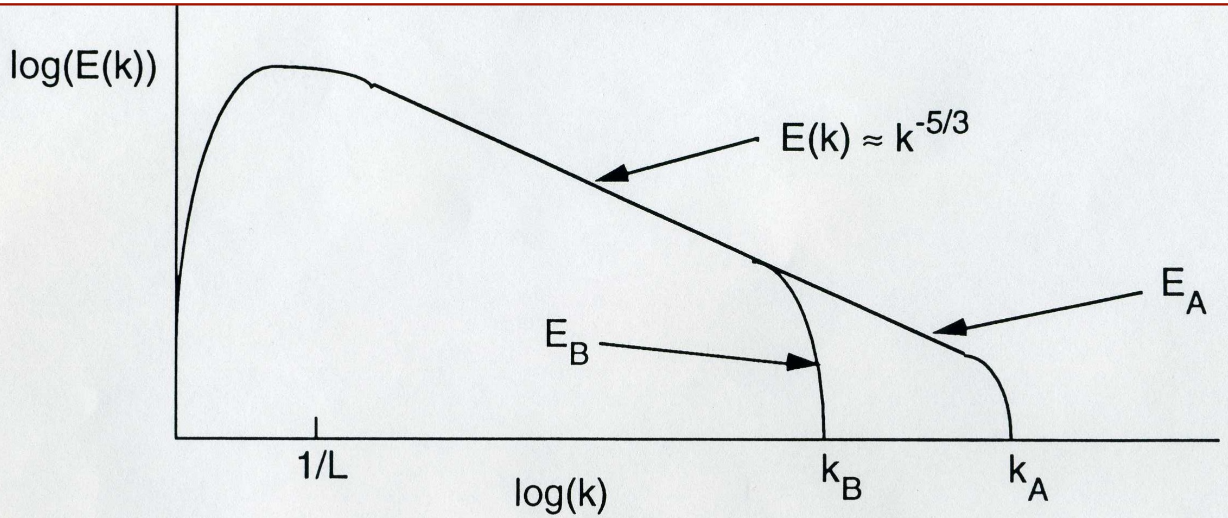
Goal: Analyze the time evolution of two scalars with identical initial spatial distributions but different diffusivities

Do differential molecular diffusion effects vanish with increasing Re as quickly as supposed?



Bilger and Dibble, 1982, proposed $z' \sim Re^{-1}$, where $z = c_A - c_D$

A spectral picture suggests slower falloff of z'



- Obukhov-Corrsin scale: $L_C \approx (Pe)^{-3/4}$
- $c'^2 = \int E(k)\,dk$
- Since A and B only separate in the wavenumber range $k_B < k < k_A$ we get:

$$z'^2 \sim \int_{k_B}^{k_A} k^{-5/3} dk \sim k_A^{-2/3} - k_B^{-2/3} \sim Pe_A^{-1/2} - Pe_B^{-1/2} \sim Re^{-1/2}(Sc_A^{-1/2} - Sc_B^{-1/2})$$

$$\Rightarrow z' \sim Re^{-1/4}$$

This was confirmed using LEM (Cremer, Kerstein, and McMurtry, 1995) and then using DNS (Nilsen and Kosaly, 1998)

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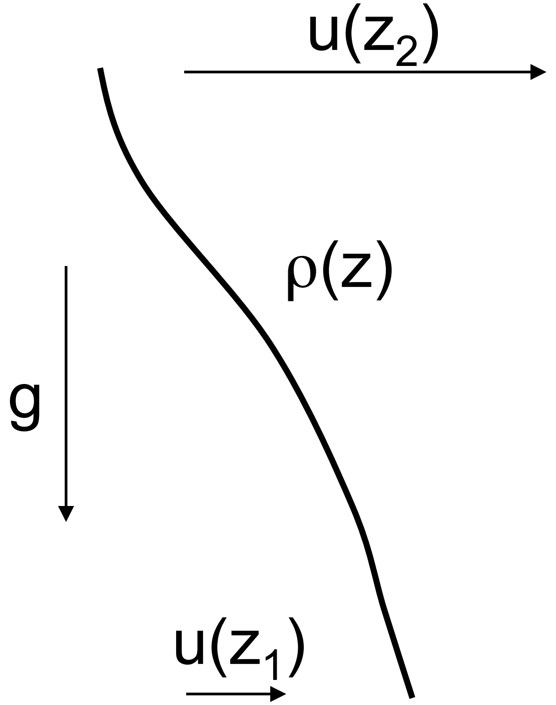
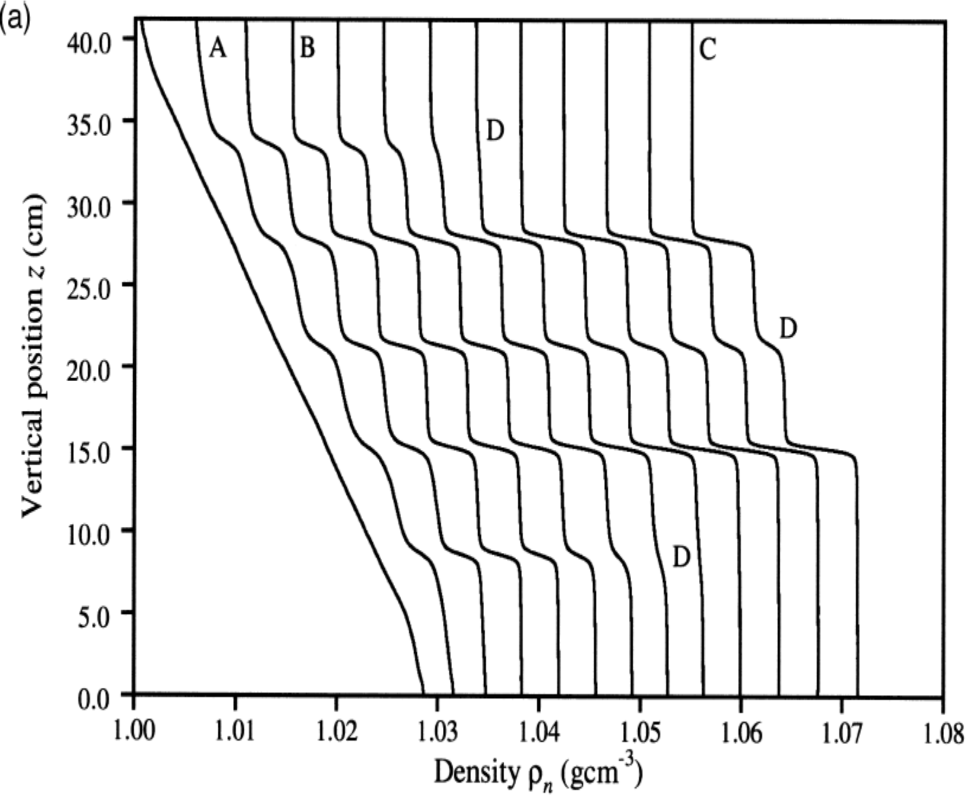
In a gravitationally stable fluid, apply enough shear to generate turbulence – what happens?

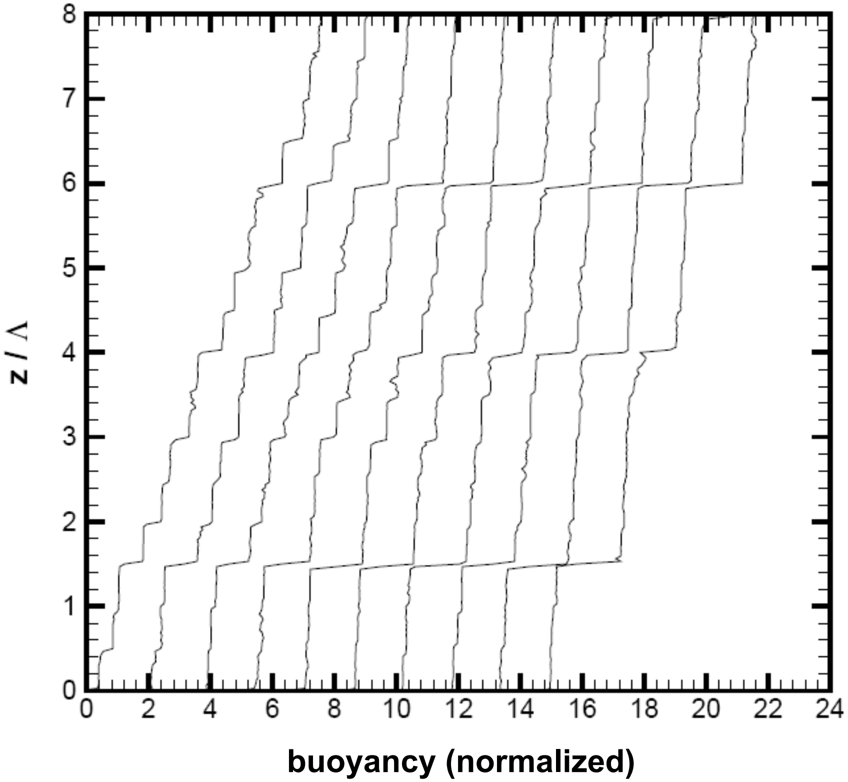


Layers form spontaneously!

Experiment: Holford and Linden, 1999



ODT: Wunsch and Kerstein, 2001



ODT parameter studies over a wider Pr range than is experimentally accessible led to new understanding and better collapse of data

A slow-diffusing stable species can cause layering of a convection process: *double-diffusive instability*

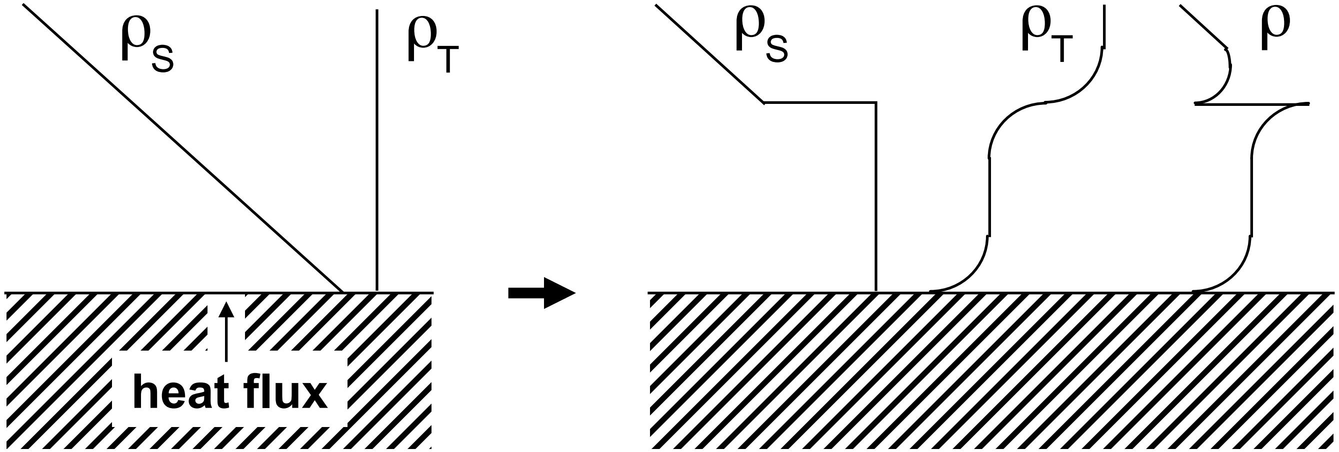
ρ_T is the density variation due to temperature variation

ρ_S is the density variation due to salinity variation

Initial state: constant temperature, salinity decreases with increasing height (stable, no motion)

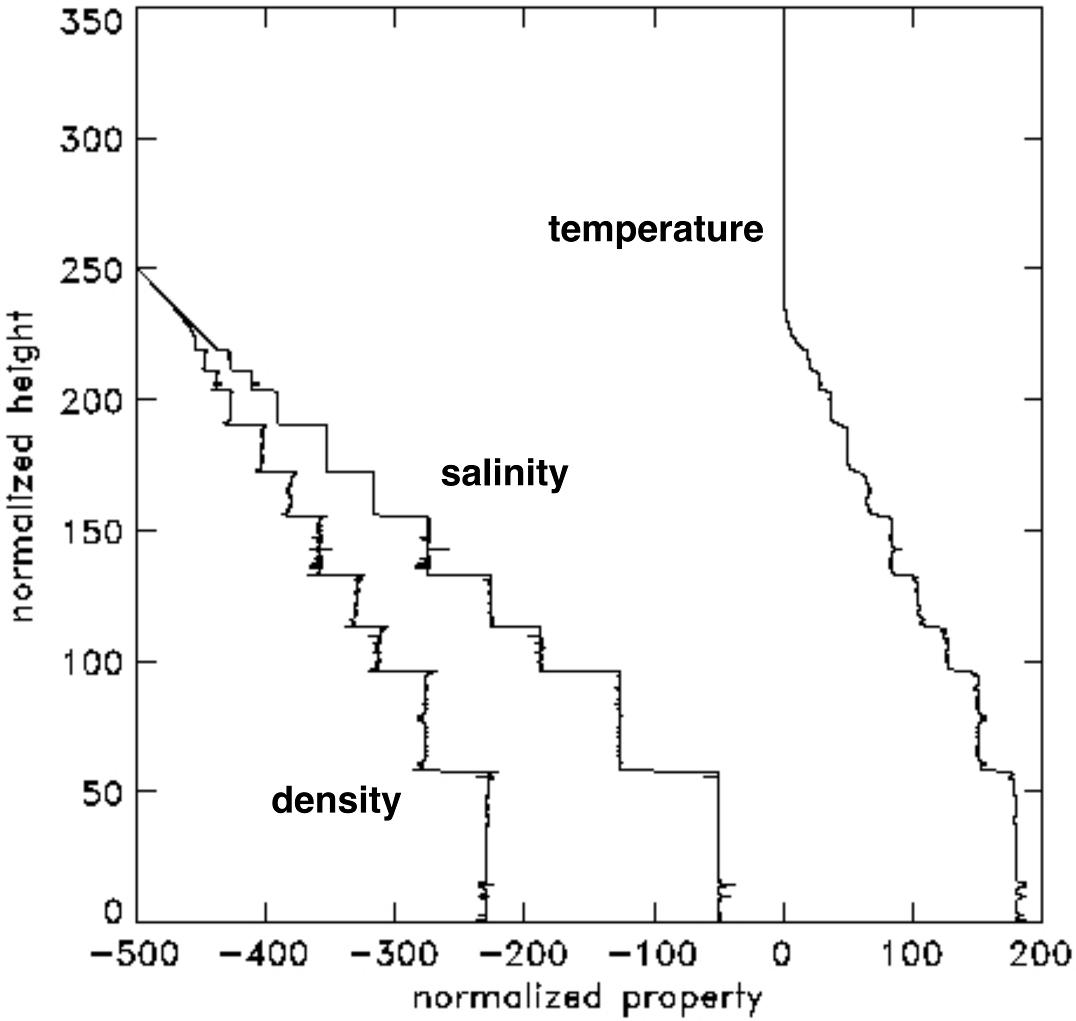
Forcing: heat from below causes gravitational instability leading to turbulent mixing

Role of molecular transport: salt diffusivity is negligible, so stable jump forms, but heat diffuses across, initiating a new turbulent layer above the jump

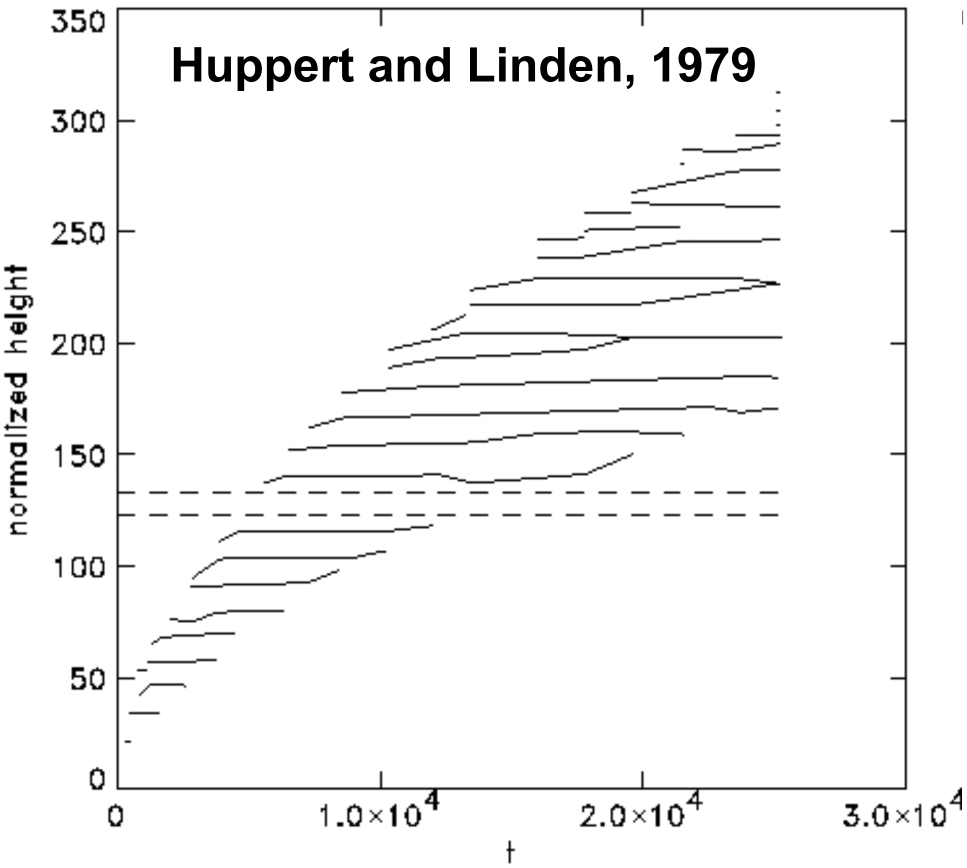


thermohaline staircase

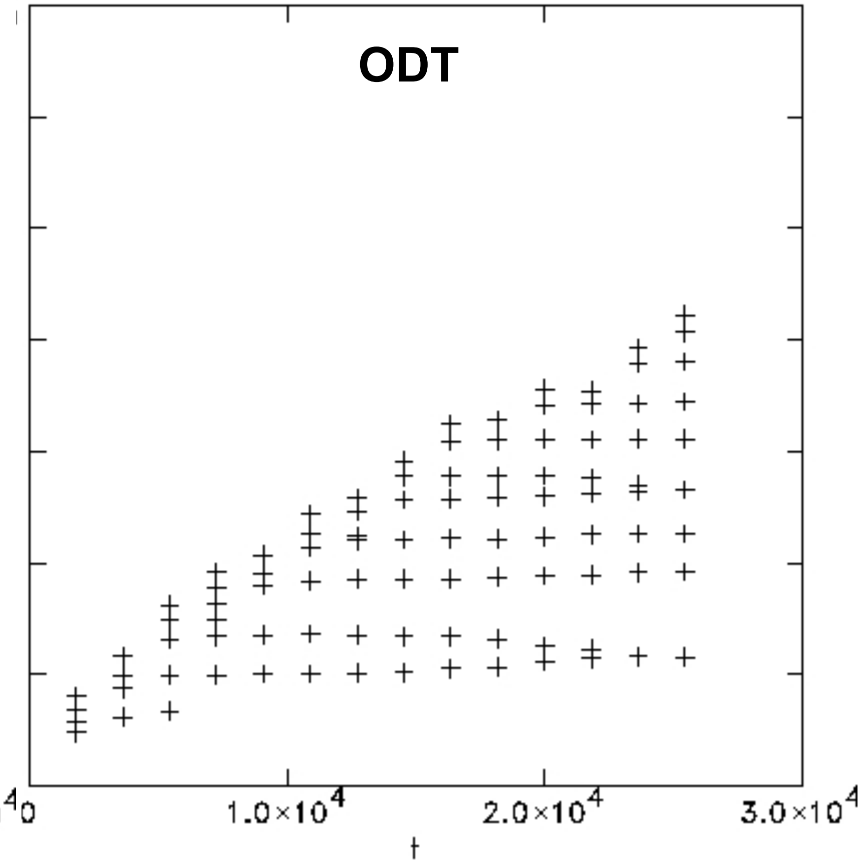
ODT captures the wide range of dynamically relevant time and length scales



The results are consistent with measurements

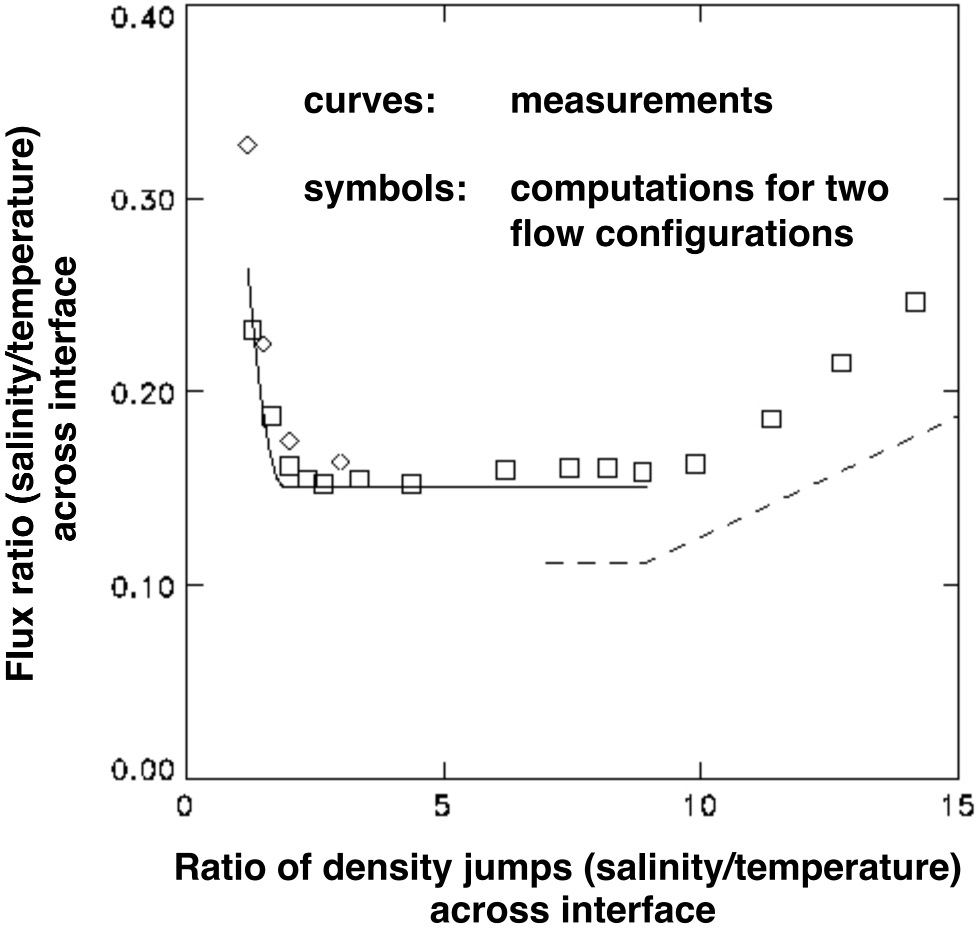


Huppert and Linden, 1979



ODT

ODT captures the observed regimes of diffusive interface structure



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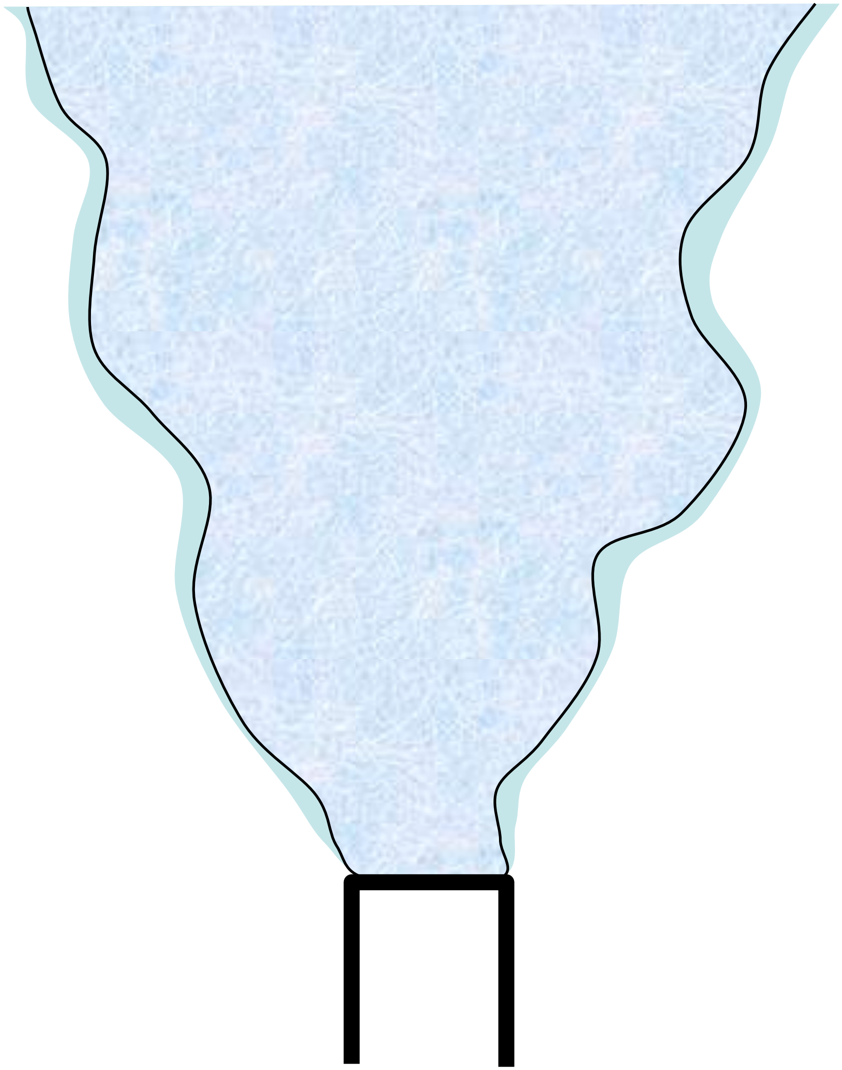
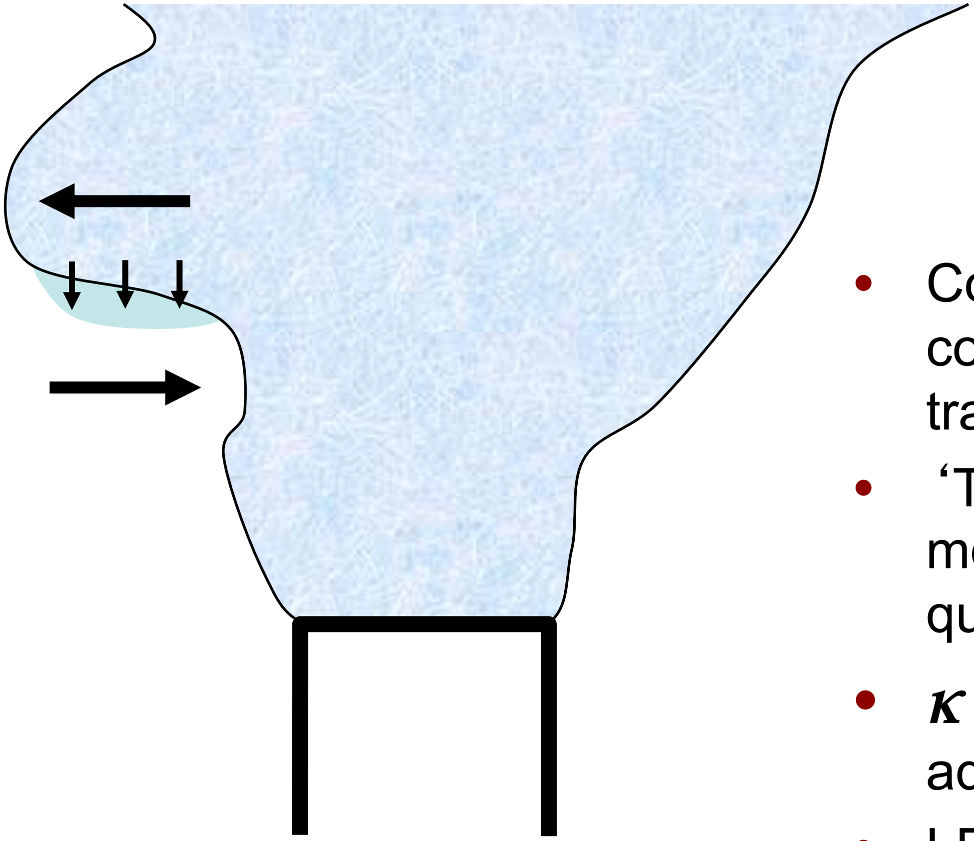
In a two-species turbulent jet, does the lighter or heavier species spread faster?



Measurements showed slower spread of the lighter species, then Saffman (1960) explained it



Key points:

- Continuity causes spatially correlated motion that affects transport
- ‘Turbulent transport’ and molecular transport are qualitatively different
- $\boldsymbol{\kappa}$ and $\boldsymbol{\kappa}_e$ are not necessarily additive, as commonly assumed
- LEM and ODT do not capture this inherently multidimensional effect

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More surprises await us

- There is much remaining to be discovered through further exploration of all turbulent flow regimes by every possible means
- LEM and ODT can make useful contributions, complementary to experiments and multi-dimensional simulations
- Practical as well as fundamental scientific insights will be gained through this exploration

Resources

- Available codes (fortran except where noted):
  - Public download at https://sites.google.com/site/odtresearch/codes :
    - BasicODT for channel flow
  - Repository (xp-dev.com, by invitation):
    - BasicODT extensions (e.g., buoyant flow, periodic BCs)
    - Adaptive-mesh ODT (c++), including an ODT wiki
    - ODTLES (ODT-based 3D flow simulation)
  - Various other special-purpose codes are available but not supported or documented
- Website: https://sites.google.com/site/odtresearch/
- Forum: https://groups.google.com/forum/#!forum/odt-research/