FRACTAL ROUGHNESS REPRESENTATION IN A STOCHASTIC ONE-DIMENSIONAL TURBULENCE MODELING APPROACH

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ABSTRACT

The present study aims to capture some key hydrodynamic effects resulting from turbulent flows in wall-bounded configurations with homogeneous roughness. We formulate a Darcy-like 1-D model that enables a dimensionally reduced dynamical representation of roughness effects for the wall-normal coordinate in boundary-layer-type flows. Similar roughness modeling approaches have been pursued in the past. Here, we rely on a conceptualization of the roughness as a fractal set of roughness elements. Our goal is to incorporate the effect of the turbulence in the modeling framework, which has not been considered in the past for similar reduced order models. To that extent, a map-based stochastic 1-D turbulence model is also used in order to numerically simulate the turbulent flows over rough walls. In this way, full-scale resolution of wall-normal transport processes is achieved without requiring the explicit representation of the roughness topography. Preliminary results reveal that the model calibration is uniquely determined for large Reynolds numbers over smooth walls. We use an ad-hoc parameterization for the roughness-drag to demonstrate that the smooth wall calibration also holds for rough walls. To that extent, we show that simulated roughness-induced drag and wall-normal stress contributions are comparable to available direct numerical simulation (DNS) data.

VOLUME-AVERAGING FRAMEWORK

We consider a representative averaging volume (RAV), which has the form of a thick plane of surface area LW, where L is the length in streamwise x direction, and W is the width in spanwise z direction. This plane has thickness Δy, which is a thickness equal to a wall-normal resolution capable of resolving all turbulent flow scales, see Figure 1. We consider that for a given order of accuracy, any flow variable ψ can be resolved in y direction with resolution Δy. Following the volume-averaging theory (VAT) Whitaker (1996), ψ is averaged in the RAV $LW\,\Delta y$. This leads to the definition of superficial and intrinsic averages, $\langle\psi\rangle(y)$ and $\langle\psi\rangle^{\beta}(y)$, respectively, which are related to the porosity ε by $\langle\psi\rangle = \varepsilon\langle\psi\rangle^{\beta}$. Following Whitaker (1996), a volume-average can be applied to both the continuity and the Navier-Stokes momentum equation. Taking the velocity vector components as u_i ($i = 1,2,3$), or conversely u, v, w, the volume-averaged equation for u_i is

$$\rho\frac{\partial\langle u_i\rangle}{\partial t} + \rho\mathscr{M}_i = -\frac{\partial\langle p\rangle}{\partial x_i} + \mu\frac{\partial^2\langle u_i\rangle}{\partial x_j^2} - \mu\varepsilon K^{-1}_{\mathrm{T},ij}\langle u_j\rangle \quad (1)$$

Here, ρ and μ are the (constant) fluid density and dynamic viscosity, respectively, and p is the (hydrodynamic) pressure. $K^{-1}_{\mathrm{T},ij}$ ($j = 1,2,3$) is the inverse of the total permeability tensor, which involves a higher-order polynomial of $\langle u_i\rangle$. This comprises all linear contributions to the subgrid-scale Reynolds stress tensor (SGS-RST), residual pressure gradients, wall-normal porosity gradients, and fluid-roughness interfacial forces. This parameterization acts as a closure similar to that addressed by Whitaker (1996). The difference is that the unsteady term and $\mathscr{M}_i$ model all time-dependent and nonlinear SGS-RST contributions. Considering $K_{\mathrm{T},ij} = K_{T,xx}\delta_{ij}$, which is an isotropic tensor in the case of steady and homogeneous laminar channel flow, we obtain the viscosity-pressure-based drag model $\varepsilon\widetilde{D}^2 K^{-1}_{T,xx} = \varepsilon G + B_0$. Here, $\varepsilon G + B_0$ is a nondimensional parameterization in terms of a local pore Reynolds number, $Re_{\mathrm{p}}(y) = \rho\langle u\rangle(y)\widetilde{D}(y)/\mu$, where $\widetilde{D}$ is the pore diameter for the RAV. In this work, we use a cubic regression for G from Khalifa *et al.* (2020), extended with the same data to include a linear dependence on ε. Notice that the linear truncation of the cubic fit of Khalifa *et al.* (2020) results in the classical Darcy-Forchheimer's law for porous media. Notice further that the RAV is here associated with a disordered porous medium in which, although a periodic velocity field is observed, the measured pressure difference along x is characteristic of a time-averaged uniform mean pressure gradient $\mathrm{d}\overline{p}/\mathrm{d}x = (p_{x=L,y,z} - p_{x=0,y,z})/L$, a constant in Eq. (1). In order to allow nonzero wall-normal motion needed for our chosen model for $\mathscr{M}_i$ (see the section One-Dimensional Turbulence), we ignore the volume-averaged (1-D) incompressible continuity equation and the associated $\partial\langle p\rangle/\partial y$. The governing equations for $\langle u_i\rangle$ can now be written in short-hand form as

$$\rho\frac{\partial\langle u_i\rangle}{\partial t} + \rho\mathscr{M}_i = -\frac{\mathrm{d}\overline{p}}{\mathrm{d}x}\delta_{i1} + \mu\frac{\partial^2\langle u_i\rangle}{\partial y^2} - \mu\varepsilon K^{-1}_{T,xx}\langle u_{i\neq 2}\rangle \quad (2)$$

Here, $K^{-1}_{T,xx}$ is related to the model $\varepsilon G + B_0$. The drag term is not applied to $\langle u_2\rangle$ due to our model choice for $\mathscr{M}_i$. We note that Forooghi *et al.* (2018) proposed a simpler ad-hoc form for the roughness drag, without using VAT, which inserts an additional term in the (non-averaged) Navier-Stokes momentum equation. This is a parametric forcing approach (PFA) for the roughness drag, which avoids the need of a full DNS with roughness topology. The additional term in Forooghi *et al.* (2018) for the Navier-Stokes momentum equation has the form $-\rho C_{\mathrm{viscous}}(y)u_i - \rho C_{\mathrm{inertial}}(y)\,|u_i|\,u_i$. This parameterization is used here as an alternative way to generate preliminary results.

FRACTAL ROUGHNESS REPRESENTATION

The homogeneous roughness topography allows a simple characterization of $\varepsilon(y)$ and $\widetilde{D}(y)$. Specifically, $\widetilde{D}(y) = D_k(y)$ is the expected value of the diameter of roughness spots (each one associated with a roughness height k) at a given y coordinate. Utilizing fractal geometry, the fractal dimension associated with roughness spots can be defined as in Yang *et al.* (2014). It is also possible to calculate the total number N_T of roughness elements with associated diameters in the interval $[D_{k,\min}, D_{k,\max}]$. The fractal characterization of the roughness requires to relate the diameter D_k of a discrete roughness element to its height k so that $D_k = \gamma k$, where γ is a multiplying factor, see Fig. 1. The cone-shaped roughness elements in Forooghi *et al.* (2017) can be used as an example. Following this, the calculation of the first statistical moment of a probability density function $f(D_k)$ over $[D_{k,\min}, D_{k,\max}]$ using fractal theory (similar to the rationale in Yang *et al.* (2014)) leads to an expression for $\widetilde{D}(y)$. This in turn allows the definition of a porosity profile $\varepsilon(y)$. Both $\widetilde{D}(y)$ and $\varepsilon(y)$ can then be used to determine a local viscosity-pressure-based drag model using the nondimensional regression $\varepsilon G + B_0$.

ONE-DIMENSIONAL TURBULENCE

Our goal is to simulate the fully wall-normal resolved, plane-averaged macro-scale equations. A key issue for the present case is the representation of the direct kinetic energy cascade and anisotropic mixing in boundary-layer-type flows. The one-dimensional turbulence (ODT) model (see Kerstein (1999)) aims to represent fundamental properties of wall-bounded turbulence in a highly reduced manner. The nonlinear advection terms $\mathscr{M}_i$ in Eq. (2) symbolize a stochastic process formulated with the aid of spatial mapping events. For a channel flow with homogeneous roughness as sketched in Figure 2, only wall-normal gradients are present in the macro-scale equations. The flow can then be simulated with full-scale wall-normal resolution. The Lagrangian mapping used to model turbulent advection maps the 1-D scalar profiles of $\langle u_i \rangle$ in an intermittent way, parallel to the numerical time integration in the 1-D domain. This implies that every model-represented turbulent eddy by a mapping, also has an associated eddy turnover time. Following this, a stochastic process can be formulated to sample mappings based on their individually calculated rate (flow state). An ensemble averaging of these mapping events with their associated time integration (time-averaging) yields wall-normal flow statistics compatible with the VAT-based roughness representation. As a turbulence model, ODT naturally relies on the use of model parameters. These play a role in the sampling process, specifically in the calculation of the individual eddy turnover time. See Lignell *et al.* (2013) for details.

NUMERICAL SIMULATION RESULTS

The preliminary results discussed below show ODT simulation results for an ad-hoc PFA representation of roughness based on Forooghi *et al.* (2018). To that extent, Fig. 5 shows the form of the forcing coefficients $C_{\text{viscous}}(y)$ and $C_{\text{inertial}}(y)$ in viscous units for two different types of homogeneous roughness. The coefficients are associated to the effects of viscous and pressure-based (form) drag, respectively. One ODT simulation is performed for each type of roughness in a channel flow at friction Reynolds number $Re_\tau \approx 500$. Fig. 3 shows the ODT simulation results obtained for the mean velocity profiles. Channel flow results are obtained utilizing parallel walls with typical no-slip conditions in the 1-D domain (closed channel). Open channel flow results utilize the zero gradient condition for u and w ($i = 1, 3$) and the impermeability condition $v = 0$ ($i = 2$) at the upper domain boundary. The roughness forcing term is adapted in a corresponding way in order to simulate the closed or the open channel flow. Finally, Fig. 4 shows the stress contributions and turbulence kinetic energy (TKE) production for a selected case.

CONCLUSION

A stochastic 1-D modeling approach to fractal roughness has been proposed and fundamentally validated for a parametric forcing approach (PFA). The reduced order framework is compatible with volume-averaging theory (VAT). The model provides means for a detailed analysis of boundary-layer turbulence in response to surface roughness in terms of boundary-layer structure and stress balance, among others. The approach circumvents both the need to resolve roughness and flow structures in 3-D if only bulk properties and 1-D boundary layer statistics are of interest. The ad-hoc PFA approach is, nonetheless, inconvenient, given that the parameterized forcing can be different in the 1-D model and the 3-D DNS. This is the reason why a more rigorous framework utilizing VAT to fractal roughness is pursued. The latter should not require additional parameter coefficients for the roughness drag representation. To remain compatible with this goal, we will demonstrate that the one-dimensional model formulation exhibits a uniquely determined and physically justified calibration over smooth and rough walls (within the PFA approach) for large asymptotic Reynolds numbers. Furthermore, we will show that the mean velocity, drag contributions, and wall-normal stresses are reasonably captured well across the roughness region so that the model enables investigations for an extended parameter space. Last, we will utilize fractal theory to estimate $\widetilde{D}(y)$ and $\varepsilon(y)$. The fractal roughness profiles for $\varepsilon(y)$ will be compared to those used in the PFA of Forooghi *et al.* (2018) for validation.

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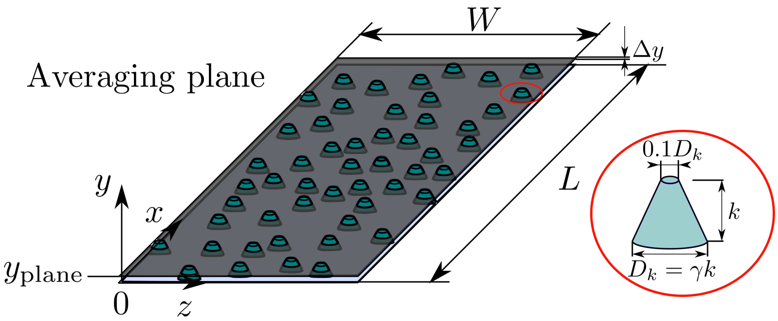

Figure 1. RAV and representative roughness element.

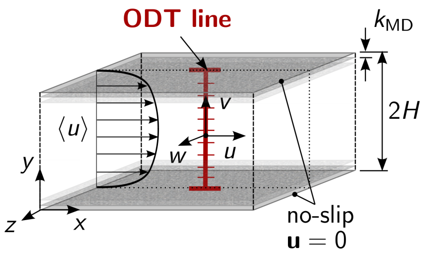

Figure 2. Flow configuration sketch.

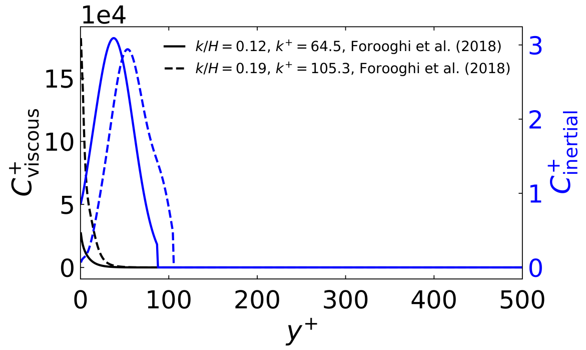

Figure 3. PFA coefficients as in Forooghi *et al.* (2018).

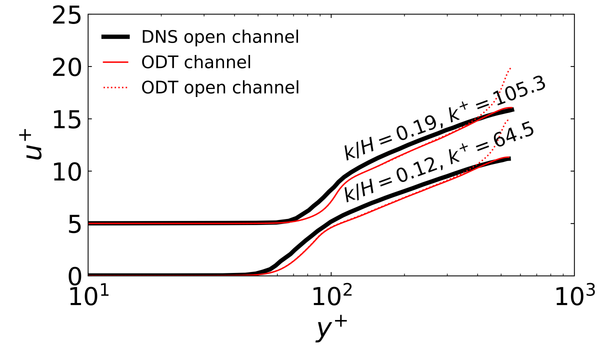

Figure 4. Mean velocity profiles.

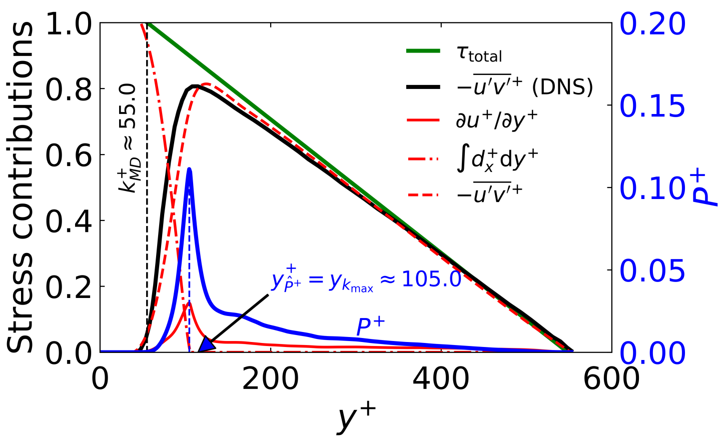

Figure 5. Stress contributions and TKE production for homogeneous roughness case with nondimensional mean roughness height $k/H = 0.19$ and $k^+ = 105.3$. Viscous units are obtained for the velocity and the y coordinate by dividing by the friction velocity u_τ and the viscous length scale η, respectively. The contributions listed in the legend are, from top to bottom, the total stress, the DNS Reynolds stress as in Forooghi *et al.* (2018), the mean velocity gradient, the parameterized drag integral contribution, and the calculated ODT Reynolds stress. The TKE production is shown with the blue line (right axis). The Fig. also shows the melt-down height position k^+_{MD} and the coordinate of peak production $y^+_{\hat{P}^+}$, which in this case coincides with the zero-crossing of the drag integral as in Yuan & Piomelli (2014). The peak production coordinate also coincides here with the maximum roughness height. The viscous scaling corresponds to channel flow simulations with friction Reynolds number $Re_\tau \approx 500$, which is defined based on the available channel height $2H$, subtracting the melt-down height $2k_{\mathrm{MD}}$, see also Fig. 2.