# A dusty gas approach for electrostatic precipitation of monodisperse aerosols **b-tu** Brandenburg University of Technology Cottbus - Senftenberg

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This work presents an economical alternative for the numerical simulation of turbulent flows in Electrostatic Precipitators (ESPs). Simulations are carried out using a stochastic reduced order model. Results for the mass-related penetration in an industrial-like ESP (Re  $\approx 21600$ ) signalize the importance of the charging state of the particle field when comparing with experimental measurements [1]. Numerical simulations of industrial-like ESPs have been so far limited to 0-D or filtered-like approaches, making this study one of the few scale-resolving investigations with implications on the potential elucidation of new physical insights.



### **Background and modeling insights**

Unlike in filter-based turbulence models, One-Dimensional Turbulence (ODT) does not require any kind of wall-modeling or subgrid scale modeling [2]. This is interesting for electrohydrodynamic (EHD) flows, where most related fundamental research has been focused on the analysis of integral quantities [3, 4, 5]. Few Direct Numerical Simulations (DNSs) have addressed EHD flows [6], and thus, little information is available for the formulation of subgrid scale models. In ODT, small scale advection is modelled by mappings on a 1-D scalar profile, see Figure 1(a)-(b). This reproduces on an ensemble average basis the effects of the turbulent scalar flux. With a direct 1-D solution of additional transport effects, e.g., viscous transport, and under assumption of parabolic-like flows to allow marching schemes, e.g., in streamwise direction (S-ODT), all relevant length and time scales of the turbulent flow can be resolved in the 1-D domain. Pressure scrambling, variable density, and EHD effects can also be incorporated in the model, see [7].



Figure 2: Inlet conditions for S-ODT simulations. (a) Snapshots and ensemble averaged velocity profiles. (b) Charge density and particle (mass) density profiles.

Figure 3 shows radial particle (mass) density profiles at three different streamwise positions  $(z/D \in \{0, 10, 20\})$  in the ESP. ODT simulation results are compared to quasi-laminar 1-D solutions, in which, Eq. (2) is advanced without the implementation of mappings, effectively neglecting the turbulent transport. In Figure 3(a), an a priori choice of an elementary charge  $q_e$  per particle is considered, while (b) shows the effects due to particles charged up to the saturation limit  $q_{\text{sat}}$  [8]. The charge state is assumed constant. Clearly,  $q_{\text{sat}}$  results in larger precipitation. It is noted that the turbulent transport greatly modifies the form of the profiles, enhancing precipitation with respect to the quasi-laminar flow at the  $q_e$  state, and delaying the drift towards the collector wall at the  $q_{\text{sat}}$  state.



Figure 1: The effect of a cylindrical triplet map on a discrete density field exhibiting variations due to a temperature gradient  $T_H - T_L$ . (a) Before mapping. (b) After mapping. The mapping is determined by a sampled position and length,  $r_0$  and l. A flow configuration sketch for the numerical simulations is shown in (c).

We consider a particle phase in the ESP, see Figure 1(c), as well as the feedback of the flow on the electroquasistatic fields by a modification of the 1-D electric field E due to Gauss' law as

$$\frac{1\partial \left(rE\right)}{r\partial r} = \frac{\left(\rho_{f,i} + \rho_{f,p}\right)}{\epsilon_0} \tag{1}$$

Here, r is the ESP radial coordinate,  $\rho_{f,i}$  is the free (ionic) charge density,  $\rho_{f,p}$  is the particle charge density, and  $\epsilon_0$  is the vacuum permittivity. The particle phase is considered with low inertia, such that an Eulerian dusty gas approach can be assumed. The flow feedback on E is due to the dynamic change of  $\rho_{f,p}$  due to advection, drift and diffusion. Using a material derivative considering advection by a mass-averaged velocity, a partial differential equation (PDE) can be obtained for



Figure 3: Radial particle (mass) density profiles at different streamwise positions using (a) a  $q_e$  state for each particle in the particle field, and (b), a  $q_{sat}$  state.

#### Conclusions

The simulations allowed the evaluation of statistics for the particle density field, as in Figure 4(a). The charge state of the particles has a large influence on the massrelated penetration, see Figure 4(b). The actual particle charge should be somewhat lower than  $q_{\text{sat}}$  in order to match the experimental measurements of [1]. The a priori charge state assumption will be dropped in future work by implementing a charging model. We show that turbulent transport may play an important role in the obtained precipitation as seen in Figure 3. Overall, we demonstrate the advantage of the use of a reduced order model such as ODT, in the potential elucidation of new physical insights from pioneering experimental work.



the particle (mass) density  $\rho_p$  in the 1-D system

$$\frac{\mathrm{d}\rho_p}{\mathrm{d}t} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\rho_p\beta_p E - rD_p\frac{\partial\rho_p}{\partial r}\right) = 0 \tag{2}$$

The mass diffusivity  $D_p$  and the particle mobility  $\beta_p$  are assumed constant and uniform. It is noted that  $D_p$  and  $\beta_p$  are related by the Einstein-Smoluchowski equation, such that  $\beta_p$  effectively depends on the particle charge. Further details of the model formulation follow [7].

#### Numerical simulation results

Fully developed inlet conditions corresponding to turbulent flow obtained with a constant properties and vector ODT formulation, as well as quasi-laminar 1-D inlet conditions, are shown in Figure 2(a). The inlet Reynolds number is Re  $\approx 21600$ . Figure 2(b) shows the  $\rho_{f,i}$ ,  $\rho_{f,p}$  and  $\rho_p$  fields for an operating voltage condition of  $\phi_{el} = 35$  kV and a corresponding measured current of 0.001 A. We also evaluate operating voltages of 39.8 and 46.5 kV, in order to compare with experiments [1].

Figure 4: (a) Mean particle density field,  $35 \text{ kV} \& q_{\text{sat}}$ . (b) Penetration results; the Deutsch model [9] is also shown for reference.

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