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Towards a One-Dimensional Turbulence Approach for Electrohydrodynamic Flows

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Understanding EHD flows

Electrostatic precipitation is highly appealing in the industry due to its uses in flue gas purification or chemical manufacture. This work is an attempt to model EHD flows using the stochastic One-Dimensional Turbulence (ODT) model. Turbulence is modeled based on the length and time scales arising from the dissipative energy cascade of turbulent flows and an external energy input. Considering statistical stationarity, the flux of energy for an incompressible turbulent channel flow is the balance between an input (e.g. pumping) power $\Pi_p = GU_b$, and an energy dissipation rate of the time-averaged velocity field Φ and the fluctuating velocity field ϵ [3],

$$GU_b = \Phi + \epsilon + \Phi_{EHD}. \tag{1}$$

Here, G is the hydrodynamic pressure gradient and U_b is the bulk velocity in the channel. For EHD flows, the electrical power dissipation Φ_{EHD} needs to be added to the RHS of Eq. (1). For such cases, if the flow is driven by a Constant Pressure Gradient (CPG), we expect drag reduction effects proportional with the increase in power dissipation [5].

ODT model for one-way coupled EHD flows

The ODT model implementation in a wire-plate (channel) electrostatic precipitator (ESP) is shown in Figure 1. It shows an eddy taking place at a location x_{eddy}. Wire-electrodes are located in a periodic array configuration at a wire-to-wire distance of 2δ. The ODT line spans the cross-width h of the channel.

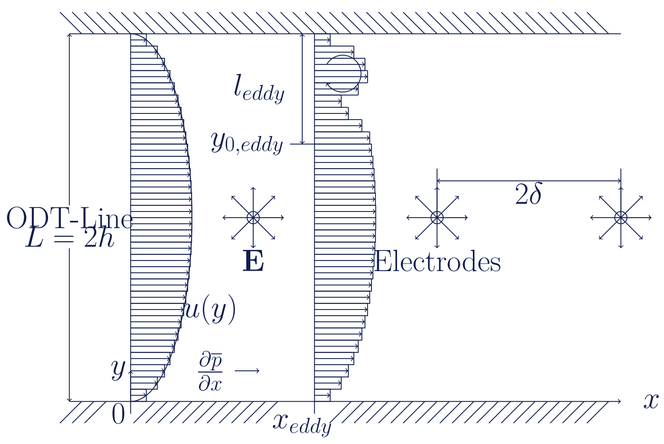

Figure 1: Description of an eddy event taking place at a certain position x_{eddy} in the flow within an ESP.

Eddy events, which represent effects of turbulent advection on a 1-D domain by a triplet map transformation $f(y)$, are selected following a statistical Poisson process involving the position and the length of an eddy, y_0 and l, respectively. Every eddy event deemed energetically plausible by its calculated rate λ is followed by a deterministic advancement process [4]. This operation involves the streamwise advancement of the ODT governing equations (S-ODT). The advancement takes place from a starting position x_0 to a position $x_0 + \sum \Delta x_{sampling}$, where $\sum \Delta x_{sampling}$ is a sequence of streamwise sampling ratios resulting in an eddy event.

$$\frac{\partial \rho u^2}{\partial x} = \rho G + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \tag{2}$$

Eq. (2) is the streamwise momentum flux balance used in the S-ODT deterministic advancement. This is solved together with a trivial zero gradient condition for the crosswise velocity component (incompressibility condition in 1-D). Here, ρ is the density and μ the dynamic viscosity. For our one-way coupled electric fields, the electrostatic body force is calculated by solving a 2-D Boundary Value Problem (Maxwell equations). If such a term is inserted into Eq. (2), this would induce an elliptic character on the flow, which contradicts the parabolic character intended by S-ODT equations [1]. Thus, only the rotational contribution to the current density, responsible for turbulence generation [2], is considered as an electrostatic potential energy contribution in ODT. Here, we assume a true current density given by the collision of charged particles and the drift velocity of ions in air,

$$\mathbf{J} = b\rho_f \mathbf{E} = \mathbf{J}_{ODT,rot} + \mathbf{J}_{irrot}. \tag{3}$$

Here, ρ_f is the charge density, $\mathbf{E}$ the electric field and b the ionic mobility. $\mathbf{J}_{ODT,rot}$ is the rotational component of the current density projected into the ODT domain (1-D), effectively complying with $\nabla \cdot \mathbf{J}_{ODT,rot} = 0$, while $\mathbf{J}_{irrot}$ is an irrotational component, which is discarded during eddy event sampling and selection.

These considerations allow a formulation of the electrostatic potential energy flux in ODT,

$$E_{EHD} \sim \int_l \rho u \left(\int_\Delta \frac{\rho_f E_y}{\rho} \mathrm{d}y \right) \mathrm{d}y \sim \int_l u \left(\frac{J'_{ODT,rot,f(y)}}{b} \right) K \mathrm{d}y. \tag{4}$$

Here, Δ is a disturbance in the rotational current density due to the eddy event evaluated by $J'_{ODT,rot,f(y)}$ and $K = y - f(y)$, where the latter is the ODT kernel function. Energy fluxes are used for the calculation of the eddy turnover distance ξ, which is later dimensionally related to the eddy rate λ [4, 1].

Low Reynolds number EHD channel flow

The investigated flow is a channel flow forced by a mean pressure gradient, which is subject to an external electrical body force, in a one-way-coupling dynamic (the electric field remains constant during the simulation). 3 cases were analyzed in this study: electrodes operating at ground potential (case 0), at low voltage (case A) and high voltage (case B). Figure 2(a) shows the 2-D electrostatic potential distribution, which remains fixed during the ODT simulations. Figures 2(b) and (c) show a comparison of eddy events in ODT for the electrode array at ground voltage (turbulent channel flow without EHD effects) and at the high voltage case. A larger density of events can be seen in the regions closer to the electrodes, which is our representation of the EHD instability mechanism produced by disturbances in the charge distribution due to the flow velocities, i.e. Eq. (4).

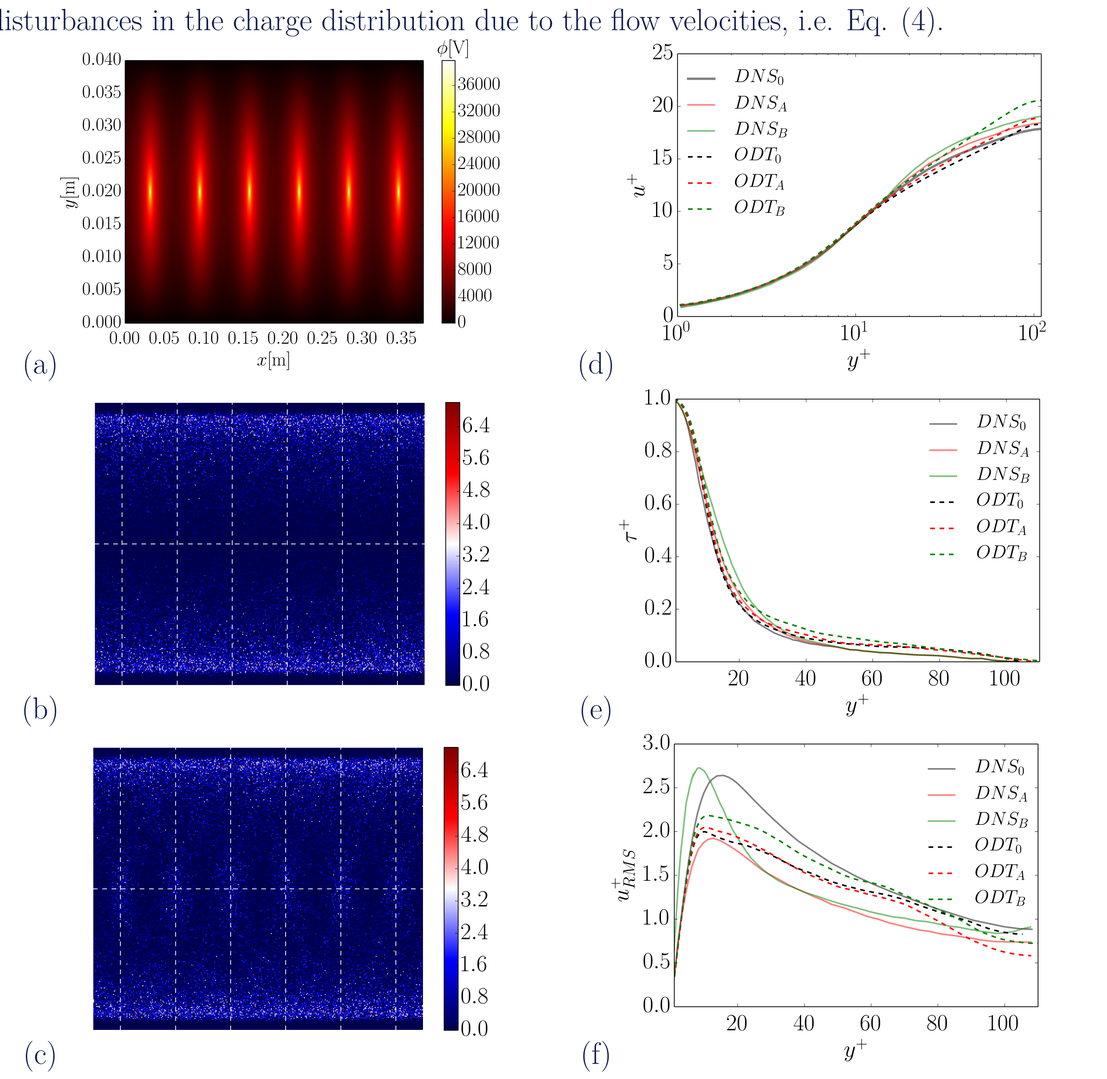

Figure 2: (a): Electrostatic potential distribution in the ESP. (b), (c): Eddy event distribution in the ESP for the electrode at ground potential and at the high voltage case, respectively. (d)-(f): Wall-normal profiles of the mean streamwise velocity (d), shear stress (e) and RMS streamwise velocity (f) at the ground potential, low voltage and high voltage cases.

Statistics obtained in the three simulated cases are faithful in the viscous layer and inner buffer layer when comparing ODT and DNS results [5], see Figures 2(d)-(e). In the outer buffer layer and bulk region of the flow, the deviation between ODT and DNS results increases proportionally with the voltage, although the qualitative trends are maintained. The ODT streamwise RMS velocity profile in Figure 2(f) is able to match the DNS results regarding the turbulence intensity reduction in the region close to the wall for the low voltage case, and the subsequent increase for the high voltage case. Nonetheless, it is important to stress that the problem sketched in Fig. 1 is fully elliptic. The resulting flow shows very significant effects of recirculation in the DNS solution, which can not be captured by our S-ODT parabolic formulation. Future studies will consider the cylindrical geometry of pipe ESPs, where the current density vector is fully irrotational (1-D).

Acknowledgements

The authors acknowledge the financial support from the European Regional Development Fund (EFRE-Brandenburg), grant number StaF 23035000. We also thank Nicolas Ballarini of the Medical University of Vienna for making the LaTeXtemplate used in this poster freely available online.

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