# Simulation of radiatively driven mixing in a smoke cloud using "one-dimensional turbulence"


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## Why 1D?

Is a 1D turbulence model for boundary layer atmospheric simulations still relevant in our 3D world? Yes, because:

- **1D is cheap!** Atmospheric-Reynolds-number resolution is not viable across the entire boundary layer in 3D LES and DNS models in the near future [1].
- **Vertical profiles are what we care about (often).** Consistent with the horizontal periodic boundary conditions in many 3D models.
- **Kolmogorov-scale resolution is important.** Down-to-centimetre-scale resolution is necessary to mitigate the broadening of the inversion layer by numerical diffusion [2].

## What is ODT?

The **O**ne-**D**imensional **T**urbulence Model (ODT) was first proposed by Kerstein [3].

- **ODT is 1D – surprise!** Flow variables (velocity components, temperature, tracers etc.) are only functions of height.
- **ODT resolves molecular diffusion explicitly.** Diffusion equations are solved for all transported flow variables.
- **ODT models turbulent advection.** Contrary to LES, the "supergrid-scale" transport is modelled by "eddy events".
- **ODT is stochastic.** The size and location of the eddy events are sampled stochastically based on the kinetic and potential energy of the current flow state. The **probability** to implement in a small time interval $\mathrm{d}t$ an eddy whose lower edge lies in $[z_0, z_0 + \mathrm{d}z_0]$ and whose length lies in $[l, l + \mathrm{d}l]$ is given by

$$\lambda(z_0, l)\,\mathrm{d}z_0\,\mathrm{d}l\,\mathrm{d}t := \frac{C}{l^2\tau}\,\mathrm{d}z_0\,\mathrm{d}l\,\mathrm{d}t = \frac{C'}{l^2}\sqrt{\frac{\mathcal{H} - ZE_{vp}}{l^3\rho_0}}\,\mathrm{d}z_0\,\mathrm{d}l\,\mathrm{d}t$$

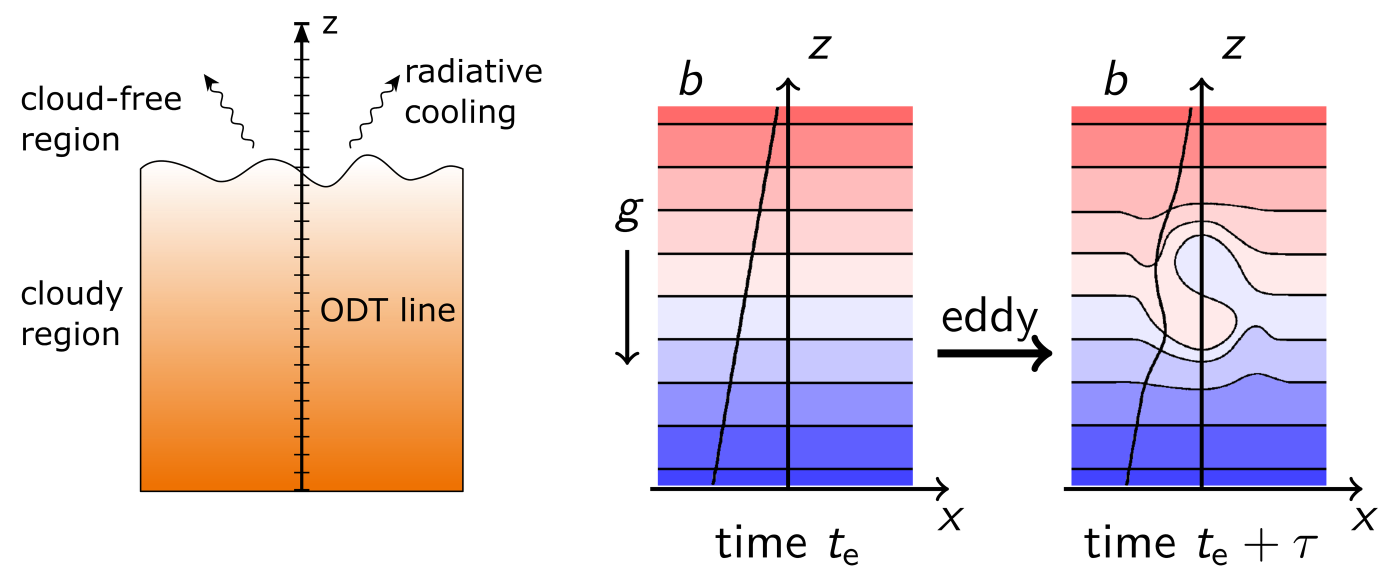


Figure 1. Left: A schematic of the ODT set-up. Right: A schematic of an eddy event in a stratified flow state.

## The governing equations

The horizontally averaged Boussinesq equations for smoke clouds with radiative cooling [2]:

$$\partial_t \overline{\boldsymbol{v}} + \partial_z \overline{w\boldsymbol{v}} = \nu \partial_z^2 \overline{\boldsymbol{v}} + (-\partial_z \overline{p} + \overline{b})\hat{\boldsymbol{k}}$$
$$\partial_t \overline{b} + \partial_z \overline{wb} = \kappa_t \partial_z^2 \overline{b} - \overline{F}$$
$$\partial_t \overline{f} + \partial_z \overline{wf} = \kappa_s \partial_z^2 \overline{f}$$

The corresponding ODT equations used in the model:

$$\partial_t \boldsymbol{v} + \boldsymbol{M}_v = \nu \partial_z^2 \boldsymbol{v} - \delta_z p \hat{\boldsymbol{k}}$$
$$\partial_t b + M_b = \kappa_t \partial_z^2 b - F$$
$$\partial_t f + M_f = \kappa_s \partial_z^2 f$$

The pressure correction term:

$$-\langle \delta_z p \rangle = -\partial_z \overline{p} + \overline{b} = \partial_z \overline{w^2}$$

## Profile comparison

In the following, a previous DNS study [2] is benchmarked against the ODT results. The bulk Reynolds number is chosen to be $\mathrm{Re}_0 = 1600$, and the initial stratification (i.e. the Richardson number) $\mathrm{Ri}_0 = 57$.

- **Fig. 2** and **Fig. 3** The general features of the buoyancy and smoke mean profiles from the DNS study [2] are reproduced by ODT.
- **Fig. 4** Without the pressure correction, cloud-top turbulent fluxes and thus the entrainment velocity are underestimated.

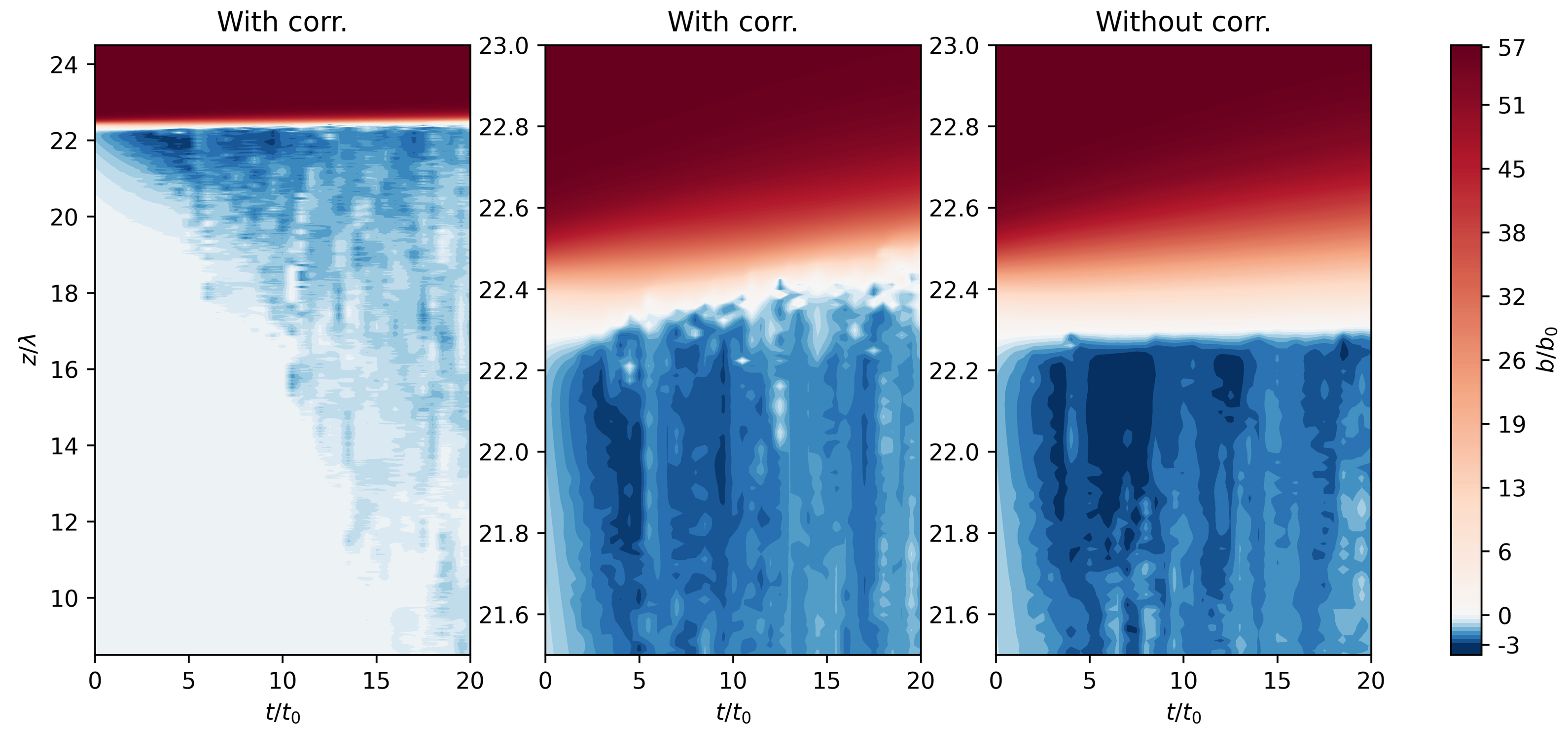


Figure 2. Buoyancy Hovmöller diagrams. The left and middle subplots are from a realisation with the pressure correction, the middle one being a zoomed-in view near the inversion height, while the right subplot is the same zoomed-in view from a realisation without the correction.

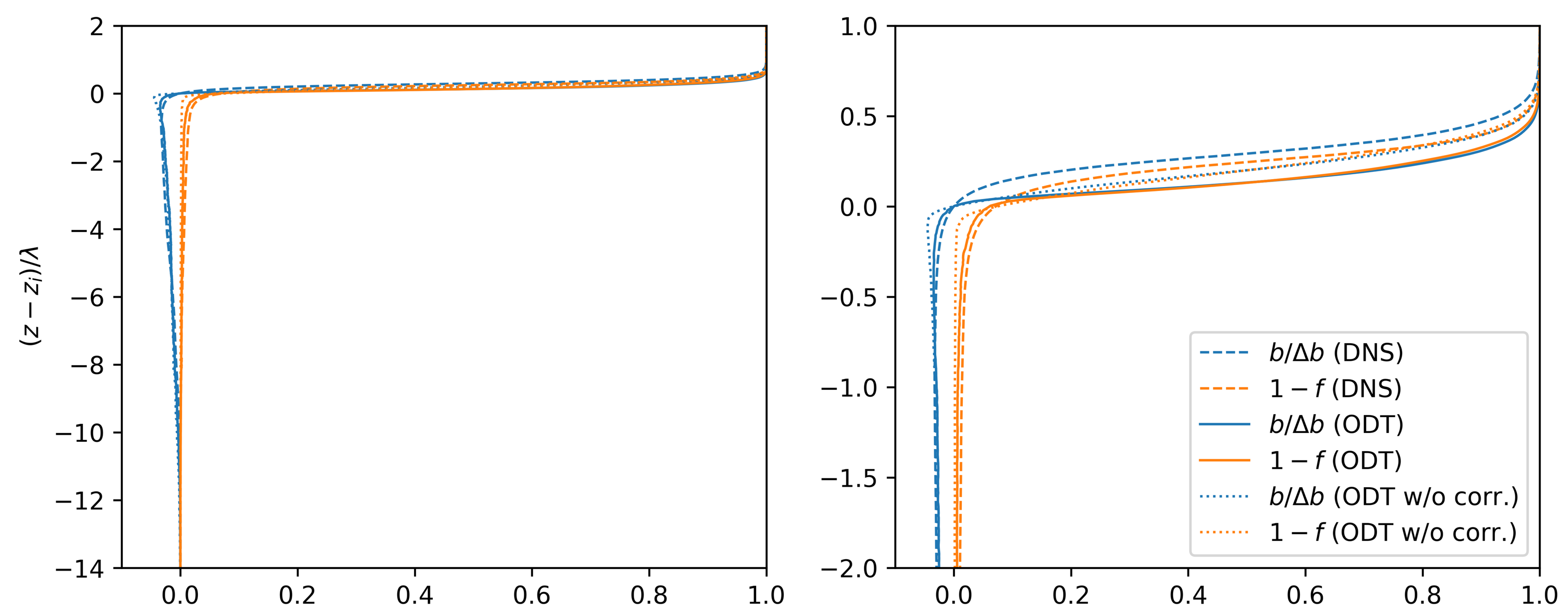


Figure 3. Mean profiles. The right subplot is a zoomed-in version of the left one near the inversion height.

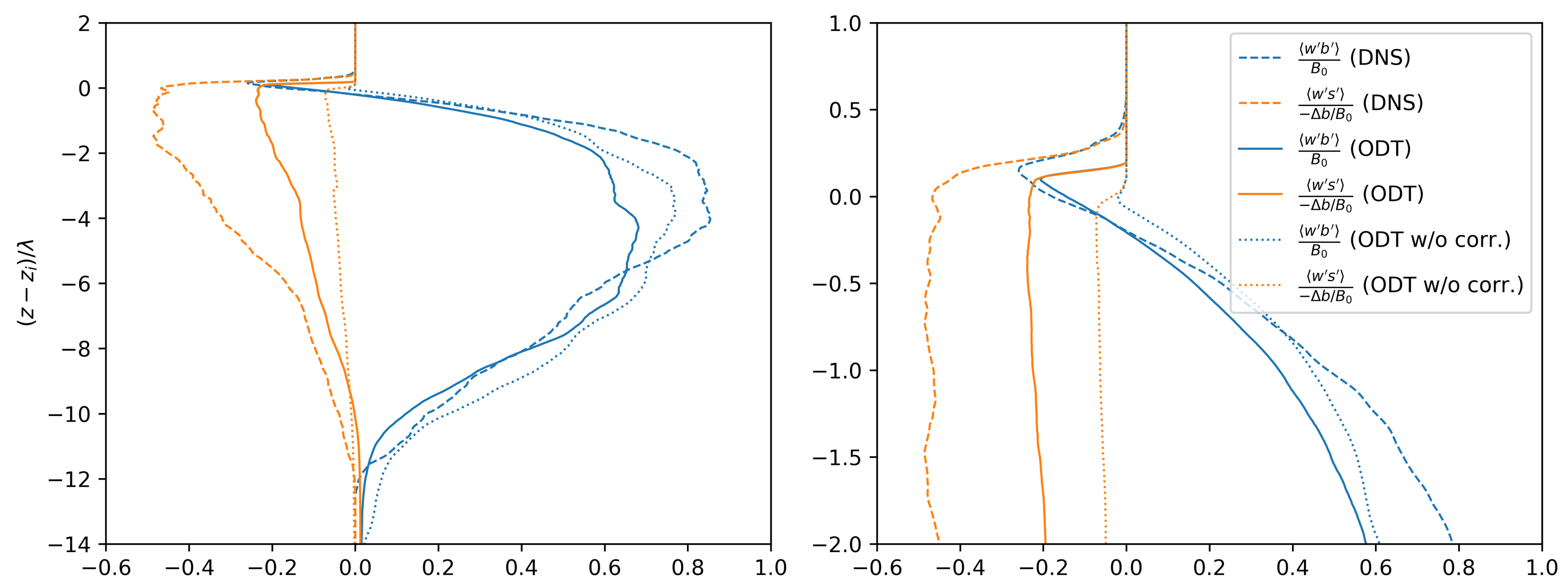


Figure 4. Turbulent flux profiles. The right subplot is a zoomed-in version of the left one near the inversion height.

## Scaling analysis

- **Fig. 5** The turbulent buoyancy flux at the inversion falls within the range $\langle w'b' \rangle_{z_i} = (-0.175 \pm 0.05)B_0$ from the DNS study [2] for $\mathrm{Re}_0 > 800$ and $\mathrm{Ri}_0 > 50$.
- **Fig. 6** The entrainment velocity approaches the $\mathrm{Ri}_0^{-1}$ scaling as $\mathrm{Re}_0$ increases.

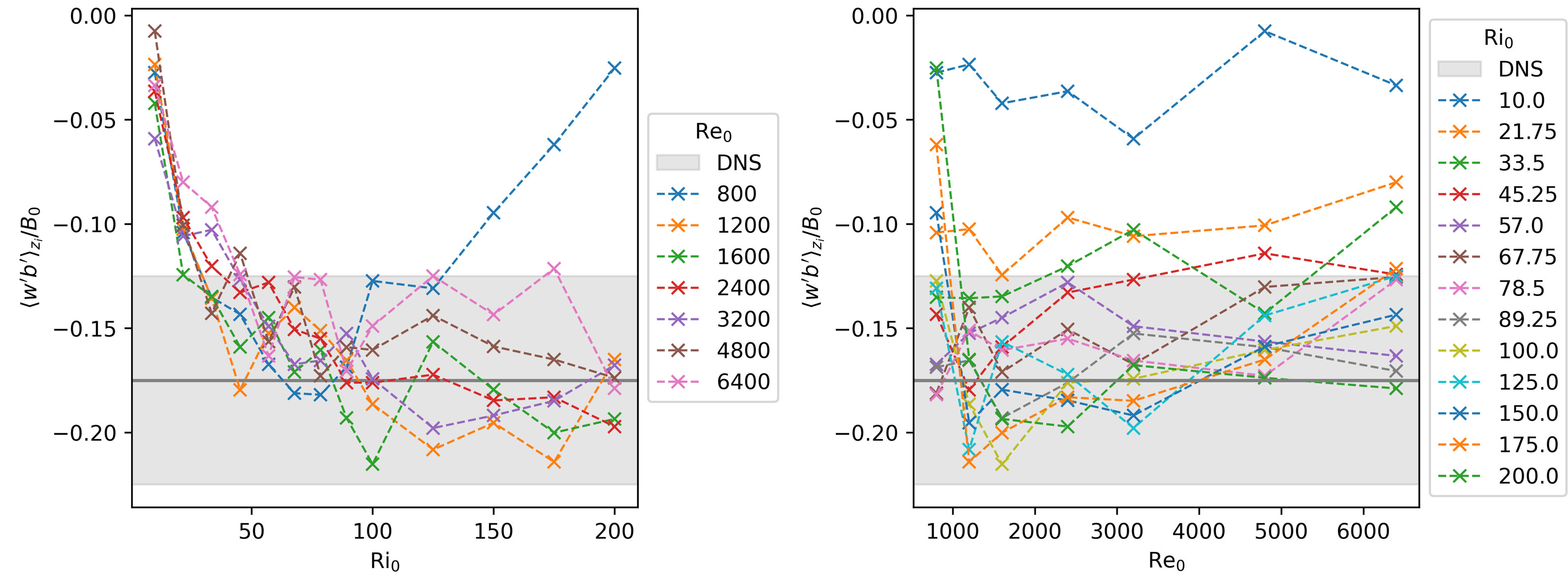


Figure 5. The turbulent buoyancy flux at inversion as functions of the Reynolds and Richardson number.

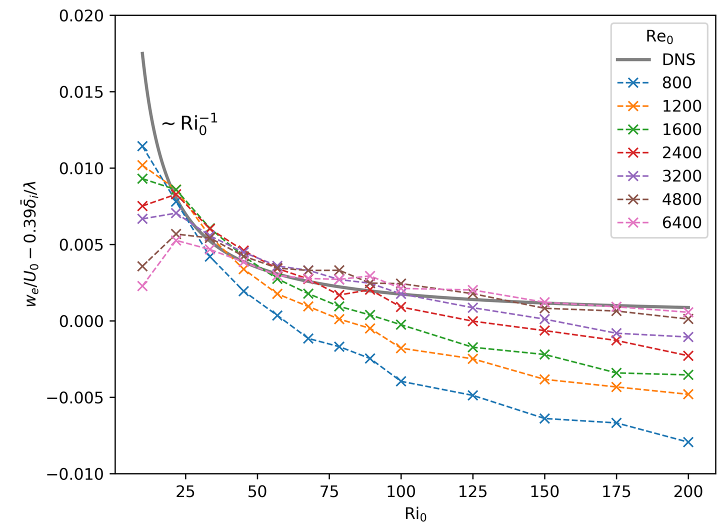


Figure 6. The entrainment velocity (adjusted for direct cooling) as a function of the Richardson number.

## Conclusions

- Cloud-top entrainment is triggered by an increase of the vertical TKE at the inversion, which must be included in ODT's advection modelling.
- ODT is capable of reproducing the scaling law for high Reynolds numbers.

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