Map-based Modeling of Turbulent Boundary Layers Subject to Rotation and Stratification

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Introduction

Turbulent boundary layers are ubiquitous in geophysical and technical applications, and often affected by **stratification** and/or **rotation** (like in the atmosphere or ocean). The small-scale turbulence is *not* always just increasing the dissipation rate. On the contrary, it can be crucial for the developing flow (e.g. [1, 2]). Full resolution of the boundary layer flow is numerically challenging and only possible for a limited Reynolds number (e.g. [3, 4]). We address the issues of **small-scale resolution** and **numerical costs** with the aid of a map-based, stochastic, the so-called **one-dimensional turbulence (ODT)** model [5]. In the following, ODT is applied to Rayleigh–Bénard convection (i.e. unstable stratification), neutrally-stratified Ekman flow (i.e. effect of rotation), and stably-stratified Ekman flow (i.e. combination of stratification and rotation). The present work is fundamentally relevant for future applications of ODT as a stochastic closure model [6].

ODT Model Formulation

ODT is a **stochastic turbulence model** that aims to **resolve all scales** of a turbulent flow but for a lower-order computational domain, the so-called **ODT line**. 1-D Profiles of the flow variables are evolved in time by a deterministic and a stochastic part [5]. The **deterministic part** is due to molecular diffusion and external forces (like the Coriolis force [7]). The **stochastic part** aims to mimic the effect of 3-D turbulence for the variables on the ODT line. This includes turbulent advection, but also pressure fluctuations [8] and buoyancy forces [9] that are driving or suppressing turbulence.

The ODT formulation uses instantaneous **eddy events**, which involve a mapping operation given by the **triplet map** (TM). The TM mimics the local modification of flow profiles in analogy to an eddy turnover (Fig. 1). The eddy position, size and time of occurrence are random variables.

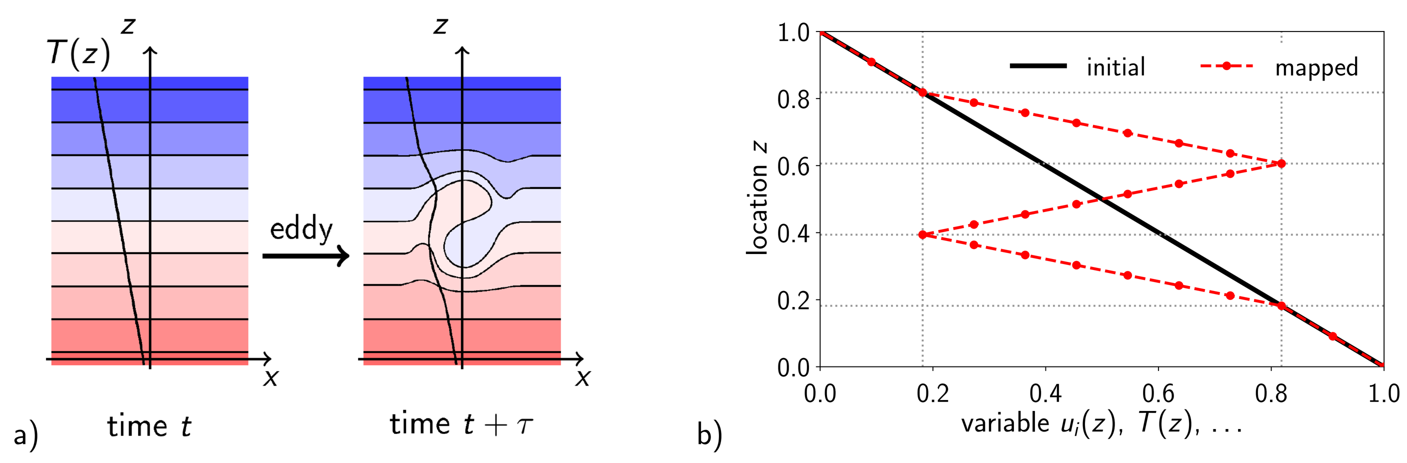

Figure 1: a) Schematic of an eddy turnover. b) Triplet map as a model for turbulent advection.

A **thinning-and-rejection method** is used to select eddy events [5]. A candidate eddy event is sampled from an empirical distribution so that its plausibility needs to be checked for the current flow state. This is done using the **eddy timescale** τ, which, following [10], is given by

$$1/\tau = C\sqrt{\left(\Delta E_{\mathrm{kin}}(\alpha) + \Delta E_{\mathrm{pot}} - Z\,E_{\mathrm{vp}}\right)/\left(\rho_0\,V\,l^2/2\right)} \qquad \text{so that} \qquad p_{\mathrm{a}} = \Delta t_{\mathrm{s}}/\tau.$$

Here, ΔE_{kin} and ΔE_{pot} denote the map-induced changes of the kinetic and the potential energy, respectively, E_{vp} is a viscous penalty energy, V the eddy volume, l the eddy size, ρ_0 an average density, Δt_{s} a time scale related to the sampling and p_{a} the acceptance probability. The **ODT model parameters** are C, Z and α [5, 8].

Rayleigh–Bénard Convection

Thermal convection in a **thin fluid layer** (thickness L, aspect ratio $R/L \to \infty$) is considered as shown in Fig. 2(a). It is heated from the bottom and cooled from the top. The gravity g is constant. The flow is continuously driven by the prescribed unstable temperature difference $\Delta T > 0$. Dirichlet boundary conditions are used for all variables. We utilize the **Oberbeck–Boussinesq approximation** due to which density variations manifest themselves only by **buoyancy forces**, which act **on all scales**. The fluid properties are otherwise constant (like the mean density ρ_0, the kinematic viscosity ν, the thermal diffusivity κ and the thermal expansion coefficient β). The flow is characterized by the Rayleigh (Ra), Prandtl (Pr) and Nusselt (Nu) number,

$$Ra = \frac{g\,\beta\,\Delta T\,L^3}{\nu\kappa}, \qquad Pr = \frac{\nu}{\kappa}, \qquad Nu = \frac{\left(\mathrm{d}\langle T\rangle/\mathrm{d}z\right)_{\mathrm{w}}}{\Delta T/L}.$$

Ra is a measure for the strength of the forcing and Pr is the time-scale ratio of the momentum and thermal molecular diffusion. Both numbers encompass several orders of magnitude, $10^6 \lesssim Ra \lesssim 10^{27}$ and $10^{-7} \lesssim Pr \lesssim 10^{23}$ [11]. Nu is the total heat flux measured in units of the purely conductive one. Here it is expressed by the temporally averaged temperature gradient $(\mathrm{d}\langle T\rangle/\mathrm{d}z)_{\mathrm{w}}$ at the wall.

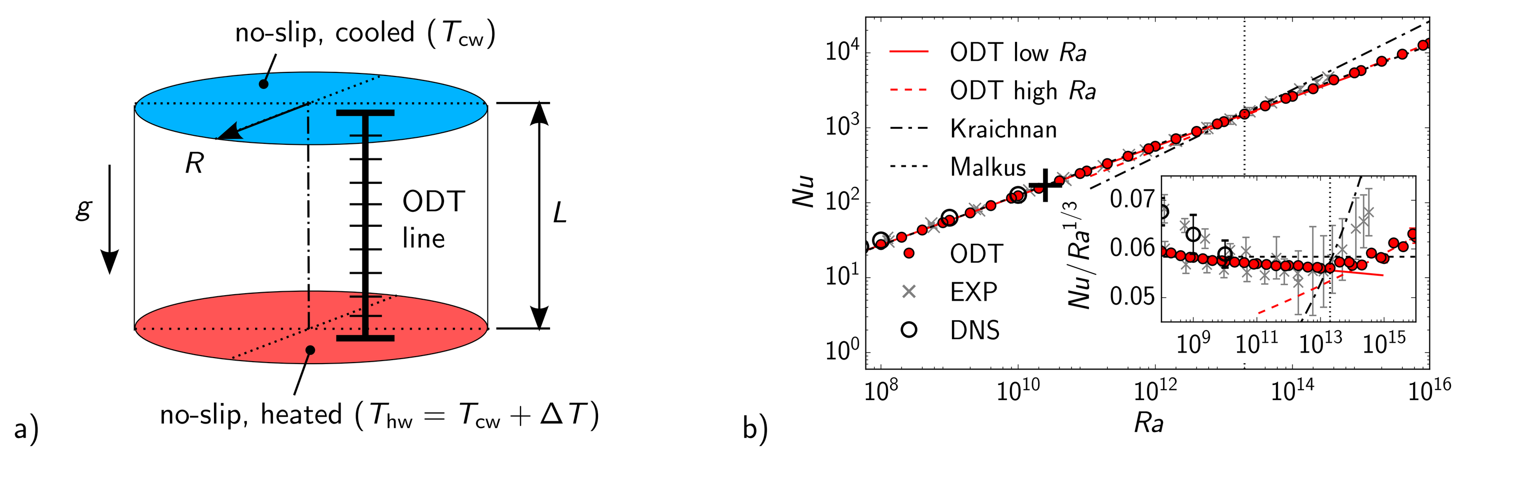

Figure 2: a) Sketch of the setup. b) Heat transfer (Nusselt number Nu) as function of the Rayleigh number Ra for air ($Pr = 0.7$) with reference data [11, 12] and scalings [13, 14]. ODT model parameters were estimated for one Ra (+).

The Nusselt number and effective scalings $Nu \propto Ra^{\gamma}$ are shown in Fig. 2(b). For low Ra, present ODT results exhibit approximately the **Malkus scaling** $\gamma = 1/3$ [14] to a degree that is comparable with the reference data. For high Ra, a **transition** can be discerned due to an increase of the scaling exponent to $\gamma \approx 0.36$. This is reminiscent of the transition to the ultimate state of convection [13, 15] but currently investigated further. Note that the ODT model parameters have been estimated only for a single point (+) [16] which hints at the **predictive capabilities** of the model.

Neutrally Stratified Ekman Flow

Turbulent Ekman boundary layers are investigated with ODT as sketched in Fig. 3(a). The setup consists of a geostrophically balanced bulk flow G over a wall fixed in the rotating frame of reference (rotation rate Ω). High resolution requirements arise from the near-wall dynamics, which posses typical length scales as small as the **Ekman layer thickness** $\delta = \sqrt{\nu/\Omega}$. For Earth's mid-latitude atmosphere and oceans, $\delta \sim O(10\,\mathrm{cm})$. A better understanding of idealized Ekman flows is needed in order to improve available models and parameterizations (e.g. [2, 3]). For this task, ODT can help to extend the range of accessible Reynolds numbers.

Fig. 3(b) shows mean velocity hodographs and wall-shear angles for various Reynolds numbers $Re = G\delta/\nu$ comparing ODT to reference DNS [3]. The ODT model parameters are those for turbulent channel flow [17]. A **hodograph** is a 2-D projection of the tip of the mean velocity vector (U, V) taken across the boundary layer. The "kink" in the ODT results is a modeling artifact that occurs at a finite distance from the wall. Nevertheless, the momentum transfer to the wall is well captured since the **wall-shear angles** $\gamma_\tau = \arctan\left[(\mathrm{d}V/\mathrm{d}z)_{\mathrm{w}}/(\mathrm{d}U/\mathrm{d}z)_{\mathrm{w}}\right]$ exhibit reasonable agreement with the reference data.

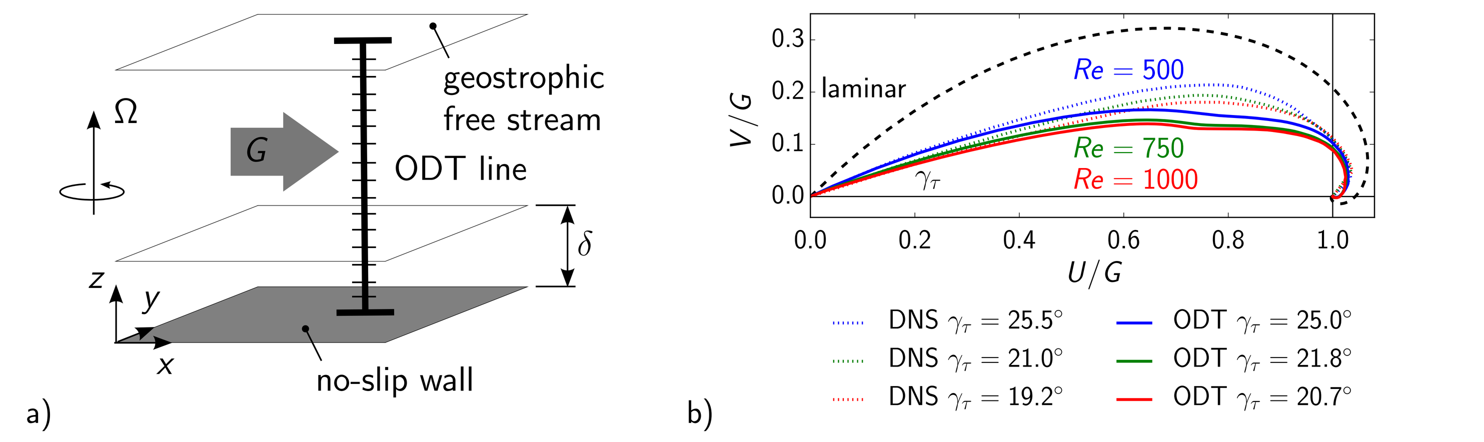

Figure 3: a) Sketch of the setup. b) Hodographs for various Reynolds numbers Re in comparison to reference DNS [3].

Towards Stably Stratified Ekman Flow

The **stable boundary layer** is strongly affected by its **density stratification**. When the stratification is strong, turbulence can be suppressed which is frequently encountered in the **night-time** boundary layer (e.g. [18, 19]). Under dry conditions, a strong near-wall stratification can arise from radiative cooling (e.g. [3]). The **idealized configuration** considered is shown in Fig. 4(a), which is a combination of those in Figs. 2 and 3. The **Oberbeck–Boussinesq approximation** is used. The ODT model parameters are the same as for the neutrally-stratified Ekman flow.

The **turbulent initial condition** is given by the ODT solution of the neutrally-stratified case, which exhibits the **turbulent boundary layer thickness** δ_*. A **sudden cooling** is modeled by prescribing a temperature profile with a strongly localized near-wall gradient [3]. For unity Prandtl number ($Pr = 1$), the flow is governed by the Reynolds (Re), Froude (Fr) and bulk Richardson (Ri) number,

$$Re = \frac{G\,\delta}{\nu}, \qquad Fr = \frac{G^2}{g\,\beta\,\Delta T\,\delta}, \qquad Ri = \frac{g\,\beta\,\Delta T\,\delta_*}{G^2}.$$

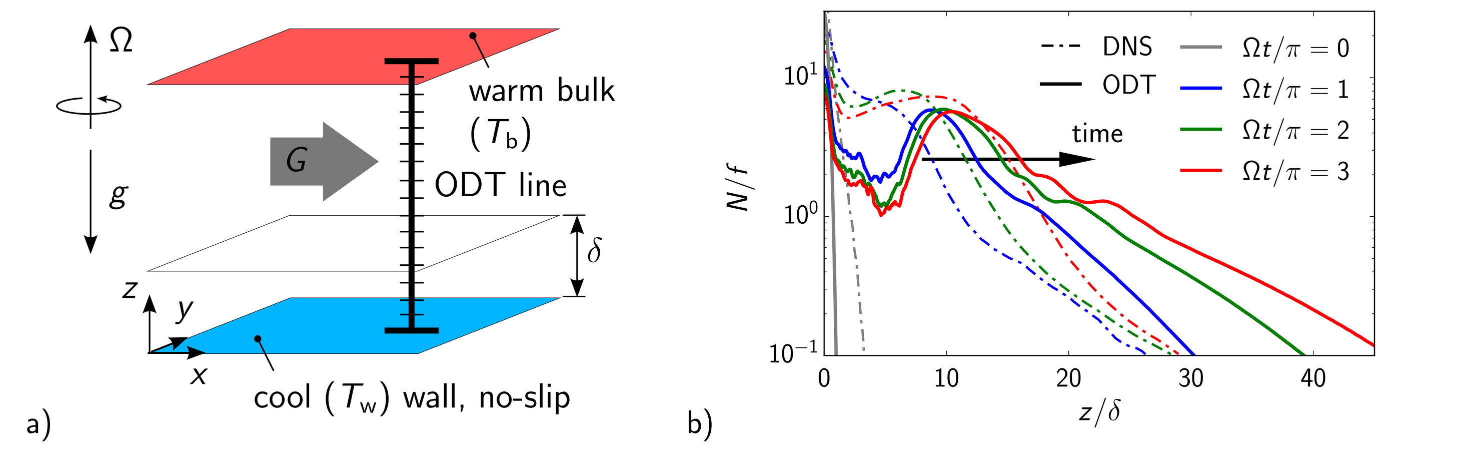

Figure 4: a) Sketch of the configuration. b) Vertical profiles of the mean buoyancy frequency N/f for various times using $Re = 500$, $Pr = 1$, $Fr = 250$, $Ri = 0.063$ (ODT) and $Ri = 0.15$ (DNS [3]). The initial condition of the preliminary ODT simulation differs from the reference DNS. At present, only qualitative comparisons are possible.

Fig 4(b) shows **vertical profiles** of the ensemble-averaged **Brunt–Väisälä (buoyancy) frequency** $N = \sqrt{g\,\beta\,(\mathrm{d}\langle T\rangle/\mathrm{d}z)}$ computed from 2000 members. N is a measure for the strength of the stratification and in the plot normalized by $f = 2\Omega$. Snapshots are shown for integer multiples of the inertial period $2\pi/f = \pi/\Omega$. The **preliminary ODT results** shown exhibit an **order-of-magnitude agreement** with the corresponding reference DNS [3]. Essential features, like the near-wall gradient and the drop at $z/\delta \approx 15$ (for $\Omega t/\pi = 3$), are captured but lesser for earlier times and further away from the wall. This hints at the **importance of the initial condition** that we currently investigate further.

Conclusions

- **ODT** can be used as **stand-alone tool** for **canonical flows**.
- **Lower-order formulation** of the model makes **small-scale resolution** feasible.
- ODT has **good predictive capabilities** as shown here for thermal convection and Ekman flows.
- Present results are encouraging for a **future** application as **small-scale closure model** in **LES**.

Forthcoming Research

- **Rayleigh–Bénard convection:** Pr-variations; non-Oberbeck–Boussinesq effects
- **Ekman flow:** influence of model parameters on structure of turbulent boundary layer
- **Stably-stratified Ekman flow:** initial conditions; model parameters; temporal evolution

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