STOCHASTIC MODELING OF TRANSIENT BOUNDARY LAYERS IN HIGH-RAYLEIGH-NUMBER THERMAL CONVECTION

Marten Klein*[1], Alan R. Kerstein[2], and Heiko Schmidt[1]
[1]*Department of Numerical Fluid and Gas Dynamics, Brandenburg University of Technology (BTU) Cottbus-Senftenberg, Cottbus, Germany*
[2]*Consultant, Danville, California, U.S.A.*

Summary One-dimensional turbulence (ODT) modeling is used to investigate the boundary layer in high-Rayleigh-number thermal convection for a notionally infinite horizontal layer of fluid. The model formulation distinguishes between turbulent advection, which is modeled by a stochastic process, and deterministic molecular diffusion to capture relevant vertical transport processes (including counter-gradient fluxes). For this study, statistical homogenization is applied to the two horizontal dimensions so that we use ODT as stand-alone tool. We show that the model yields mean and fluctuation temperature profiles that are in several respects consistent with available reference data. Furthermore, the profile of a surrogate for the fluctuation velocity is reminiscent of canonical wall turbulence.

INTRODUCTION

Turbulent thermal convection manifests itself by irregular fluid motions on a range of scales, which are the results of a nonlinear interplay of buoyancy, inertial, and viscous forces as well as thermal diffusion. The fluid-surface coupling is achieved by shallow boundary layers so that it is mandatory to accurately capture their dynamical properties in numerical simulations if one wishes to quantitatively predict, for instance, the heat transfer or the induced flow velocities. For typical control parameters found in applications, the resolution requirements are far too high as direct numerical simulation (DNS) would be possible in the foreseeable future (e.g. [1]). Unfortunately, widely used and economical gradient-diffusion closures for modeling the sub-filter scale dynamics will not be sufficient to be predictive (e.g. [2]). We address this dilemma by utilizing the stochastic one-dimensional turbulence (ODT) model [3] for the simulation of nonstationary boundary layers in thermal convection problems. The objective is to investigate the turbulent boundary layer properties up to very high Rayleigh numbers with a physics-based but feasible numerical model.

In the following we, first, describe the flow configuration and model application. After that, we present some key results for the temperature and velocity statistics including a surrogate analysis. Finally, we close with our main conclusions and an outlook to what will be presented at the conference.

OVERVIEW OF THE FLOW CONFIGURATION AND MODEL FORMULATION

Figure 1(a) shows a sketch of the canonical Rayleigh–Bénard set-up investigated together with the ODT computational domain (ODT line). A Boussinesq fluid is confined between two smooth horizontal isothermal no-slip walls that are located at $z = 0$ and $z = L$. The heated and cooled walls have prescribed temperatures T_hw and T_cw, respectively, with constant difference $\Delta T = T_\mathrm{hw} - T_\mathrm{cw}$. The flow properties are governed by the Rayleigh number, $Ra = g\,\beta\,\Delta T\,L^3/(\nu\kappa)$, and Prandtl number $Pr = \nu/\kappa$, where g is the background gravity and β, κ, ν denote the fluid's thermal expansion coefficient, thermal diffusivity, and kinematic viscosity, respectively.

Figure 1(b) shows statistically representative vertical profiles of the temperature, $T(z,t)$, and a horizontal velocity component, $u(z,t)$ (inset). Deterministic molecular diffusion is directly resolved along the ODT line whereas the effects of turbulent advection are modeled by a stochastic process. The computational domain is thus a single vertical line along which flow profiles are evolved in time as described in [3, 4, 5, 6]. In contrast to the original model formulation [3, 4], we incorporate the vector velocity, $\boldsymbol{u} = (u, v, w)^\mathrm{T}$ as in [5, 6].

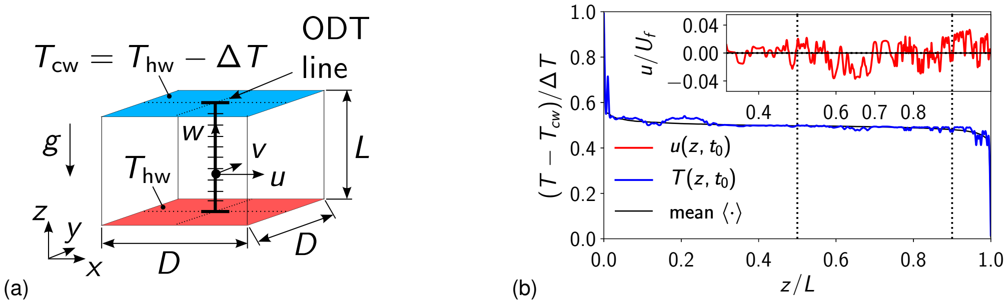

Figure 1: (a) Sketch of the Rayleigh–Bénard configuration investigated. Here, the vertical spacing, L, is fixed, whereas the horizontal dimension, D, is taken to infinity ($D \to \infty$). A stochastic ODT simulation aims to capture the nonstationary vertical transport and is carried out on a one-dimensional computational domain (ODT line). (b) Representative instantaneous and time-averaged vertical profiles of the temperature, T, and a horizontal velocity component, u, for $Ra = 10^{10}$, $Pr = 0.7$ (air). The velocity reference scale is the free-fall velocity, $U_f = \sqrt{g\,\beta\,\Delta T\,L}$.

*Corresponding author. E-mail: marten.klein@b-tu.de.

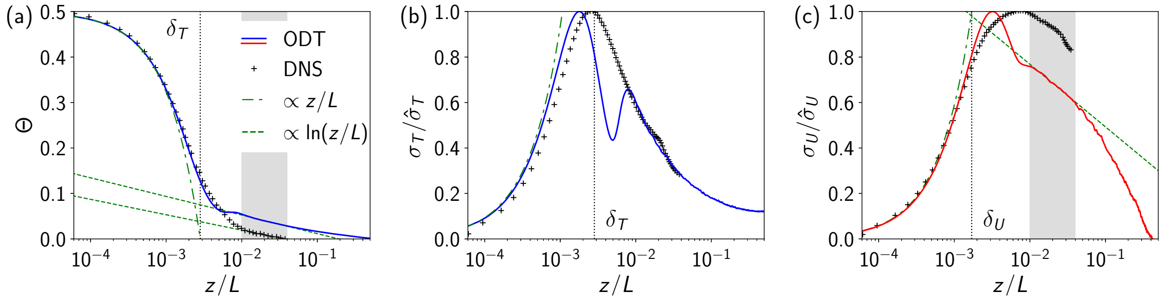

Figure 2: Boundary layer profiles over the heated wall for $Ra = 3 \times 10^{10}$ and $Pr = 0.7$ from ODT together with reference DNS data from [7] for the axis of a cylindrical set-up of aspect ratio one. (a) Normalized mean temperature, Θ; (b) normalized standard deviation of the fluctuation temperature, $\sigma_T/\hat{\sigma}_T$; and (c) normalized standard deviation of the fluctuation horizontal velocity, $\sigma_U/\hat{\sigma}_U$. The cap $(\hat{\cdot})$ denotes the maximum value. In ODT, a 'mean boundary-layer downstream velocity', $\langle u_\mathrm{d} \rangle$, is used as a surrogate for the fluctuation velocity, σ_U, that is, we take $\langle u_\mathrm{d} \rangle \sim \sigma_U$. Linear and logarithmic regions, as well as the thermal, δ_T, and viscous, δ_U, boundary-layer thicknesses (slope method) are indicated. Logarithmic profiles have been fitted across $10^{-2} \le z/L \le 4 \times 10^{-2}$ (shaded region).

KEY RESULTS

Figure 2(a) shows the normalized mean temperature, $\Theta = (\langle T \rangle - T_\mathrm{b})/\Delta T$, where $T_\mathrm{b} = (T_\mathrm{hw} + T_\mathrm{cw})/2$ is the bulk temperature and $\langle \cdot \rangle$ a conventional temporal average. ODT reproduces the near-wall structure, but Θ departs from the reference data towards the bulk. It is remarkable that the present ODT results suggest the onset of a logarithmic region, $\Theta(z) = A \ln(z/L) + B$, which is in general consistent with [8]. The ODT solution exhibits approximately the same prefactor, A, as the reference data but a larger additive constant, B, due to the earlier departure from the sublayer.

Next, figure 2(b) shows profiles of the standard deviation of the fluctuation temperature, $\sigma_T = \sqrt{\langle T^2 \rangle - \langle T \rangle^2}$. Reasonable agreement between ODT and the reference data is observed close to and far from the wall. However, an unphysical bimodal structure can be discerned around $z \approx \delta_T$, which is precisely where Θ departs from the linear sublayer. This structure is presumably of the same origin as a similar one in the ODT fluctuation velocity observed in turbulent channels [5]. Here, the responsible modeling error manifests itself primarily in the driving Boussinesq temperature.

At last, we consider a surrogate analysis for the fluctuation velocity since ODT exhibits zero-mean velocity (see figure 1(b)). This differs from available reference data, which exhibits a large-scale circulation (e.g. [7]). We therefore consider a synthetic 'boundary-layer downstream velocity', $u_\mathrm{d}(z,t) = \boldsymbol{u}(z,t) \cdot \boldsymbol{e}_\mathrm{d}(t) \ge 0$, which is a projection of the vector velocity, $\boldsymbol{u}$, on an instantaneous 'boundary-layer downstream direction', $\boldsymbol{e}_\mathrm{d} = [\boldsymbol{e}_x(\partial_z u)_\mathrm{hw} + \boldsymbol{e}_y(\partial_z v)_\mathrm{hw}]/[(\partial_z u)_\mathrm{hw}^2 + (\partial_z v)_\mathrm{hw}^2]^{1/2}$. The temporal mean, $\langle u_\mathrm{d} \rangle$, of instantaneous positive semi-definite $u_\mathrm{d}(z,t)$ profiles is used as surrogate for the positive semi-definite horizontal fluctuation velocity, $\sigma_U = \sqrt{\langle U^2 \rangle - \langle U \rangle^2}$, where $U = \sqrt{u^2 + v^2}|_\mathrm{axis}$ is evaluated at the axis of a cylindrical set-up [7]. Figure 2(c) shows profiles of the reference horizontal fluctuation velocity, σ_U, in comparison to the ODT surrogate, $\langle u_\mathrm{d} \rangle$. There is good qualitative agreement between both quantities and there is no bimodal structure in $\langle u_\mathrm{d} \rangle$ obtained by ODT. Furthermore, the profile of $\langle u_\mathrm{d} \rangle$ is reminiscent of the streamwise fluctuation velocity in canonical wall turbulence (e.g. [9]). Note that the profile seems to exhibit the onset of a logarithmic region but the vertical extend is small which suggests further analysis at higher Ra numbers.

CONCLUDING REMARKS AND OUTLOOK

The capabilities for economical but also reasonably accurate and robust modeling of transient boundary layers in turbulent thermal convection have been addressed by utilizing the stochastic one-dimensional turbulence (ODT) model. We have shown that point-wise temperature and velocity statistics (at least up to second order) exhibit reasonable agreement with available reference data. We have furthermore suggested a 'boundary-layer downstream velocity' surrogate for the fluctuation velocity. Vertical profiles of this surrogate are reminiscent of the streamwise fluctuation velocity in wall turbulence which is currently further investigated.

In the talk, we will outline the model formulation. After that, we will discuss the boundary-layer properties. At last, we will address the Ra number dependence and scaling properties by making use of the model's predictive capabilities.

References

[1] Schumacher J., Bandaru V., Pandey A., Scheel J. D. *Phys. Rev. Fluids* **1**: 084402, 2016.
[2] Deardorff J. W. *J. Atmos. Sci.* **23**: 503-506, 1966.
[3] Kerstein A. R. *J. Fluid Mech.* **392**: 277-334, 1999.
[4] Wunsch S., Kerstein A. R. *J. Fluid Mech.* **528**: 173-205, 2005.
[5] Lignell D. O., Kerstein A. R., Sun G., Monson E. I. *Theor. Comp. Fluid Dyn.* **27**(3): 273-295, 2013.
[6] Gonzalez-Juez E. D., Kerstein A. R., Lignell D. O. *Geophys. Astro. Fluid Dyn.* **107**(5): 506-525, 2013.
[7] Li L., Shi N., du Puits R., Resagk C., Schumacher J., Thess A. *Phys. Rev. E* **86**: 026315, 2012.
[8] Ahlers G., Bodenschatz E., Funfschilling D., Grossmann S., He X., Lohse D., Stevens R. J. A. M., Verzicco R. *Phys. Rev. Lett.* **109**: 114501, 2012.
[9] Marusic I., Kunkel G. J. *Phys. Fluids* **15**: 2461-2464, 2003.