Towards a stochastic model for electrohydrodynamic turbulence with application to electrolytes

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We investigate turbulent Couette flows of dilute, weakly-conducting electrolytes by utilizing the stochastic one-dimensional turbulence (ODT) model. The flow is driven by relative motion of the top and bottom wall and affected by an electric field between these walls that is prescribed by a voltage difference. The electrolytes considered have zero bulk charge and consist of two ion species with the same mobility, valence, and initial concentration. The stochastic model predicts a decrease of the mean streamwise velocity when an external voltage is applied provided that both Schmidt (Sc) and Reynolds (Re) numbers are sufficiently large, that is, $Sc \geqslant 30$ for $Re = 12000$ investigated. The effect observed is relevant for flow control, but the mechanism awaits clarification. Present ODT results may help to develop this understanding or design laboratory experiments.

1 Introduction

Turbulent electrohydrodynamic (EHD) flows occur in various applications that range from technical (e.g. [1–3]) to geophysical (e.g. [4]) scales. EHD flows may be divided into two categories. In the first category EHD effects do *not* affect the flow although they may enhance specific properties of an application [2]. In the second category EHD effects interact with the flow [1,3,4]. Electrokinetics and hydrodynamics are coupled by transient processes on different scales so that robust, accurate, but also economical modeling strategies are required. We address this by utilizing the stochastic one-dimensional turbulence (ODT) model [5] that we have extended to incorporate some EHD effects that are relevant for wall-bounded turbulent flows.

2 Model formulation and application to EHD Couette flow

The ODT model aims to resolve all relevant scales of a turbulent flow by reducing the dimensionality of the problem [5]. Here we apply ODT to turbulent Couette flows of electrolytes as sketched in figure 1. The ODT computational domain ('ODT line') is oriented vertically in order to resolve and statistically represent the leading-order transport across the channel configuration.

In general, the flow is described by the Navier–Stokes equations coupled to the Poisson–Nernst–Planck equations, and the Gauss law. The corresponding lower-order, stochastic equations for a constant-density liquid with univalent ion species read

$$\frac{\partial u_i}{\partial t} + \mathcal{E}_i = \frac{\partial}{\partial y}\left(\nu \frac{\partial u_i}{\partial y}\right), \qquad \frac{\partial c_\pm}{\partial t} + \mathcal{E}_\pm = \frac{\partial}{\partial y}\left(D\frac{\partial c_\pm}{\partial y} \pm \frac{Dc_\pm}{V_T}\frac{\partial \Phi}{\partial y}\right), \qquad \frac{\partial}{\partial y}\left(-\varepsilon\frac{\partial \Phi}{\partial y}\right) = \rho e(c_+ - c_-), \tag{1a, b, c}$$

where y denotes the wall-normal coordinate, t the time, $(u_i) = (u, v, w)^{\mathrm{T}}$ the Cartesian velocity components, ν the kinematic viscosity, $c_\pm$ the concentration and D the diffusion coefficient of the positive and negative ion species, Φ the electrical potential, $V_T = k_{\mathrm{B}}T/e$ the thermal voltage, k_{B} the Boltzmann constant, T the prescribed uniform temperature, $e > 0$ the electrical charge on an electron, ε the dielectric permittivity of the electrolyte, ρ the constant fluid density, and $\mathcal{E}$ the discrete stochastic eddy events that are discussed below. Equations (1a–c) are closed by prescribing the no-slip boundary condition for u_i, fixed-value boundary condition for Φ, and zero-total-flux boundary condition for $c_\pm$ at the channel wall.

The stochastic terms denoted by $\mathcal{E}$ in (1a, b) mimic the effects of Navier–Stokes turbulence by a stochastic sequence of discrete mapping (eddy) events. $\mathcal{E}_i$ models turbulent advection, fluctuating pressure-gradient and electric forces, but $\mathcal{E}_\pm$ only turbulent advection. Eddy events are sampled from an unknown probability density function (PDF) that depends on the flow state. In practice, the construction of this PDF is avoided and a more economical thinning-and-rejection algorithm [5] is used instead. In the latter, eddy events of size l are probabilistically accepted with the rate $\tau^{-1} = C\sqrt{2E/(\rho V_{\mathrm{e}} l^2)}$, where V_{e} denotes the volume and $E = E_{\mathrm{kin}}(\alpha) + E_{\mathrm{pot}} - ZE_{\mathrm{visc}}$ the extractable energy of an eddy event that possesses kinetic, potential (electric), and viscous contributions [5,6]. Model parameters are from [7] and fixed at $C = 10$, $Z = 600$, and $\alpha = 2/3$.

The deterministic terms in (1a–c) are spatially discretized with a finite-volume method on an adaptive grid [7]. The parabolic equations (1a, b) are integrated with an explicit time-marching scheme, whereas the elliptic equation (1c) is solved with the Thomas algorithm whenever the electric potential has to be renewed. This is usually the case when the ion concentrations have changed either due to a deterministic or a discrete stochastic advancement step.

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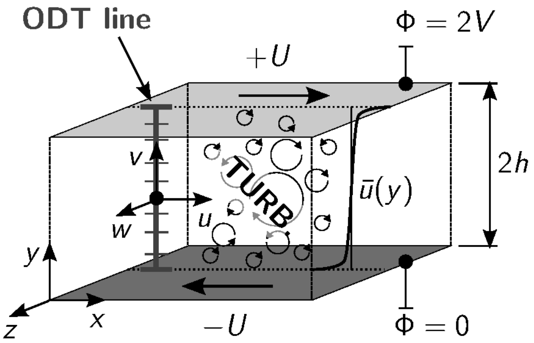

Fig. 1: Sketch of the flow configuration investigated.

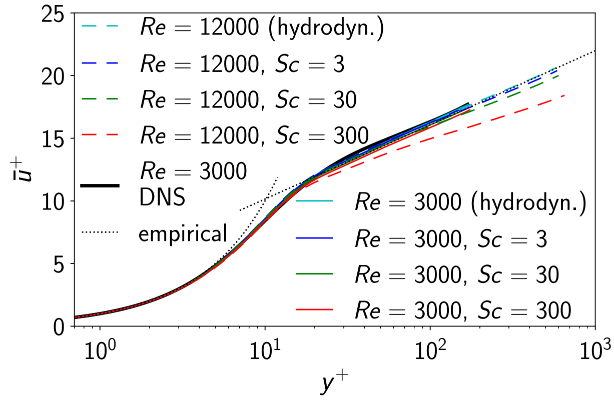

Fig. 2: Mean velocity deficit, $\bar{u}^+$, for various Re and Sc numbers using $\beta = 0.5$, $\lambda = 0.01$, $\hat{V} = 4$. Reference DNS data is from [3] and the empirical fit from [8].

3 Mean velocity deficit in the boundary layer with and without EHD effects

Figure 2 shows the dimensionless mean streamwise velocity deficit, $\bar{u}^+ = |U - \bar{u}|/u_\tau$, over the dimensionless boundary layer coordinate, $y^+ = u_\tau y/\nu$, where $u_\tau = \sqrt{\nu\,|\mathrm{d}\bar{u}/\mathrm{d}y|_{\mathrm{wall}}}$ denotes the friction velocity and the overbar $\bar{(\cdot)}$ the temporal average in the statistically stationary state. For $Re = 3000$ and 12000 in the hydrodynamic (HD) regime, ODT yields the friction Reynolds number $Re_\tau = u_\tau h/\nu = 170$ and 581, respectively. Former is in quantitative agreement with available direct numerical simulations (DNS) that predict $Re_\tau^{\mathrm{ref}} = 170$ (e.g. [3]). HD ODT simulation results (cyan lines in figure 2) exhibit good agreement with DNS [3] and the empirical law of the wall for the von-Kármán constant $\kappa \simeq 0.39$ and additive constant $B \simeq 4.2$ [8]. ODT captures the viscous sublayer ($y^+ \lesssim 10$) and the logarithmic layer ($y^+ \gtrsim 80$), but it does not capture the relative 'overshoot' in the buffer layer (across $10 \lesssim y^+ \lesssim 80$), which is a similar as in turbulent Poiseuille flow [9].

We now turn to the EHD regime. Similarity solutions to (1a–c) are obtained for the following five dimensionless control parameters: The bulk Reynolds number, $Re = Uh/\nu$; the Schmidt number, $Sc = \nu/D$, of the ions; the coupling constant, $\beta = \varepsilon V_T^2/(\rho\nu D)$; the dimensionless voltage, $\hat{V} = 2V/V_T$; and the normalized Debye length, $\lambda = \sqrt{\varepsilon V_T/(2\rho c_0 e)}/h$, where $c_0 = c_{\pm,0}$ is the initial ion concentration. The velocity profiles given in blue, green, and red in figure 2 address the Sc-number dependence for two Re numbers and otherwise fixed parameters as specified in the caption. For $Re = 3000$, the flow is not notably affected by EHD effects. This is consistent with the reference DNS [3], which, however, was restricted to $Sc = 3$ for otherwise similar parameters. For $Re = 12000$, ODT predicts a reduction of $\bar{u}^+$ with increasing Sc number for $y^+ \gtrsim 100$. The effect is weak for $Sc = 30$, but notable for $Sc = 300$ investigated. Correspondingly, the friction Reynolds number, Re_τ, increases from $Re_\tau^{\mathrm{HD}} = 581$ to $Re_\tau^{\mathrm{EHD}} = 600$ (by $\approx 3\%$) for $Sc = 30$, and to $Re_\tau^{\mathrm{EHD}} = 650$ (by $\approx 12\%$) for $Sc = 300$.

4 Conclusion

We have performed stochastic ODT simulations of turbulent Couette flows in hydrodynamic (HD) and electrohydrodynamic (EHD) flow regimes. For the HD regime, we have shown that the model captures the law of the wall in turbulent Couette flow. For the EHD regime, and using the same model parameters as in the HD regime, ODT predicts a decrease of the boundary-layer mean velocity (deficit) and a corresponding increase of the friction Reynolds number with increasing Sc number for $Sc \geqslant 30$ and $Re = 12000$ investigated. A heuristic estimate suggests that $Sc \lesssim \beta\hat{V}^2$ for strong interactions of electrokinetics and turbulent boundary layer dynamics [3]. For the control parameters selected, we expect strong EHD effects for $Sc \lesssim O(10)$. Present ODT results indeed hint at a change of the flow regime at $Sc \simeq O(10)$, but this change seems to manifest itself somewhat differently than expected from the literature. Further investigation is required in order to better understand this EHD-related mechanism of flow control; a task for which ODT predictions may stimulate dedicated laboratory experiments.

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