Investigating thermal convection at low Prandtl numbers using one-dimensional turbulence

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Introduction

Turbulent thermal convection denotes the chaotic flow driven by buoyancy forces due to an unstable temperature stratification imposed to a layer of fluid. Convecting flows are encountered in numerous engineering and geophysical applications (see [1] references therein). When buoyancy forces are much larger than viscous forces, the flow may reach the **ultimate regime** of thermal convection [2, 3, 4]. This regime, however, has remained inaccessible to 3-D direct numerical simulations (DNSs) as it imposes very high resolution requirements [5, 6]. Recently 2-D DNSs [7] have reached the ultimate regime but at the price of an inverse energy cascade in contrast to 3-D turbulence.

Accurate and **efficient numerical models** are needed if one wishes to study convection under strong forcing conditions. Here we utilize the **one-dimensional turbulence (ODT)** model [8, 9, 10] in which a **stochastic process** is used to model 3-D turbulence within a dimensionally reduced setting. This model is able to yield, for example, a direct energy cascade [8]. Using ODT as **forward model** we investigate what might happen in turbulent convection at very high Rayleigh numbers.

ODT model formulation

ODT aims to **resolve all relevant scales** of a turbulent flow but only for a representative 1-D domain (ODT line) [8]. Flow variables are resolved along this notional line-of-sight on a dynamically adaptive grid [11]. Instantaneous flow profiles are evolved by **deterministic diffusion**, which is resolved along the ODT line, and a **stochastic process** that models the effects of 3-D turbulence. These are turbulent advection [8], but also fluctuating pressure gradient [9] and buoyancy forces [10].

- The **ODT governing equations** for the velocity vector u_i and a scalar T read [8, 9, 10],

$$\frac{\partial u_i}{\partial t} + \mathcal{E}_i(\alpha) = \nu \frac{\partial^2 u_i}{\partial z^2}, \qquad \frac{\partial T}{\partial t} + \mathcal{E}_T = \kappa \frac{\partial^2 T}{\partial z^2}.$$

- The **stochastic terms** $\mathcal{E}_i$ and $\mathcal{E}_T$ are formulated with discrete **mapping (eddy) events** (Fig. 1).
- The **eddy rate** $\tau^{-1}(\ell, z_0; t)$ of a size ℓ eddy event at location z_0 depends on the **locally available specific energy**, $E_{kin} + E_{pot} - Z\,E_{vp}$, for the **momentary flow state** at time t [8, 9, 10],

$$\tau^{-1} = C\sqrt{2\,\ell^{-2}\left(E_{kin} + E_{pot} - Z\,E_{vp}\right)}.$$

E_{kin} and E_{pot} are the map-induced changes of the kinetic and the potential energy, respectively, and E_{vp} is a viscous penalty energy on the scale ℓ.

- The main **ODT model parameters** are C and Z [8, 10], together with $\alpha = 2/3$ [9].

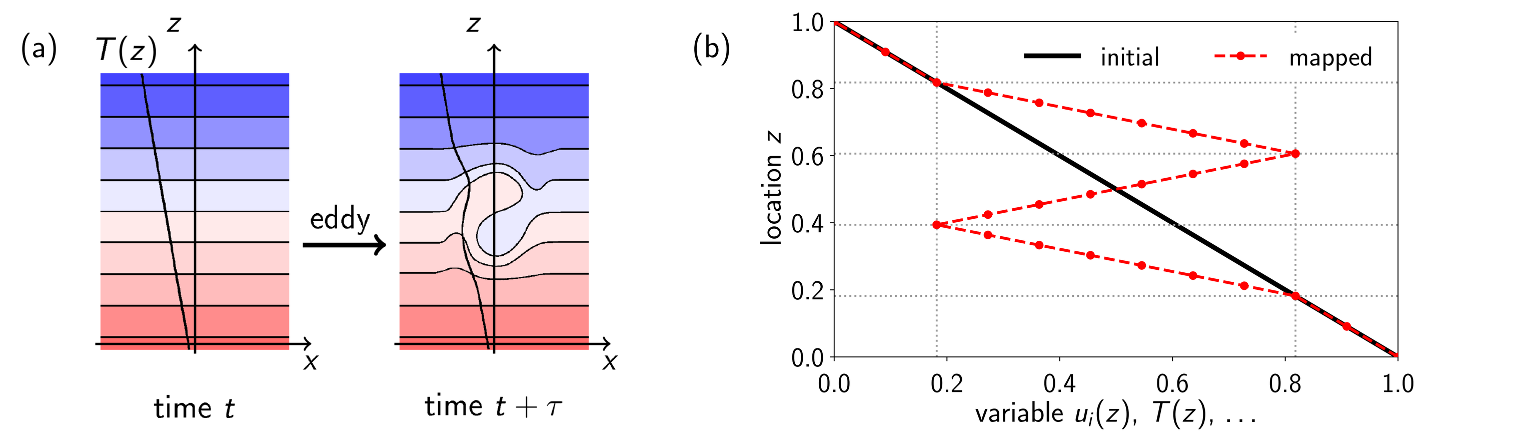

Figure 1: (a) Schematic of an eddy turnover. (b) Triplet map as 1-D model for turbulent advection. Effects of an eddy turnover are modeled by a permutation of fluid parcels along a stochastically selected ODT line interval (here $0.2 \leqslant z \leqslant 0.8$).

ODT application to Rayleigh–Bénard convection

- **Thin layer of fluid** (aspect ratio $\Gamma = 2R/L \simeq D/L \to \infty$) subject to const. gravity g (see Fig. 2).
- **Oberbeck–Boussinesq approximation**, $\rho(T) = \rho_0[1 - \beta(T - T_0)]$, where β is the thermal expansion coefficient, ρ the density, and T the temperature; the subscript 0 denotes reference values.
- Smooth isothermal no-slip walls with constant temperature difference ΔT.
- The flow is characterized by the **Rayleigh** (Ra), **Prandtl** (Pr) and **Nusselt** (Nu) **number**,

$$Ra = \frac{g\,\beta\,\Delta T\,L^3}{\nu\kappa}, \qquad Pr = \frac{\nu}{\kappa}, \qquad Nu = 1 + \frac{\langle w'T'\rangle_{V,t}}{\kappa\,\Delta T/L}.$$

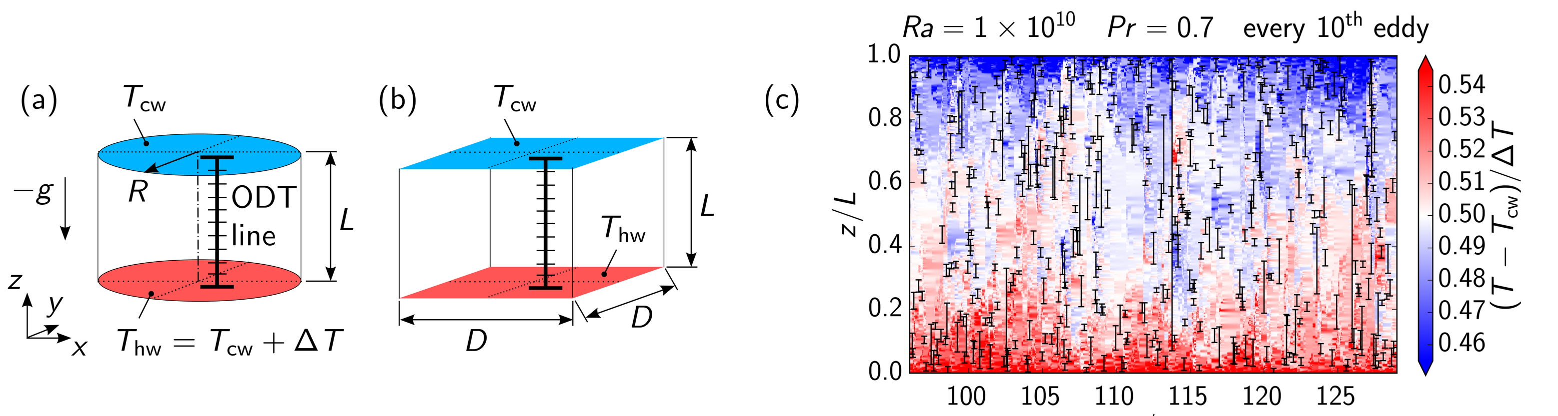

Figure 2: (a, b) Sketches of the setups considered. (c) Space-time diagram of the instantaneous vertical temperature profile $T(z, t)$ of an ODT solution. Black lines explicitly mark the line interval of every 10[th] eddy event. Reference scales are given by the free-fall velocity, $U_f = \sqrt{g\,\beta\,\Delta T\,L}$, and the free-fall time, $t_f = L/U_f$.

ODT model parameter estimation

The **ODT model parameters** are **estimated** by matching Nu (see Fig. 3 and [12]) and the mean temperature profile (see [13]) for a **single** Ra **number** but **both** Pr **numbers** to available reference DNS data. Note that these reference data are *not* particularly well-suited for the ODT model calibration as they are for $\Gamma \simeq 1$ and limited to low Nu numbers whereas ODT is built for the ultimate regime.

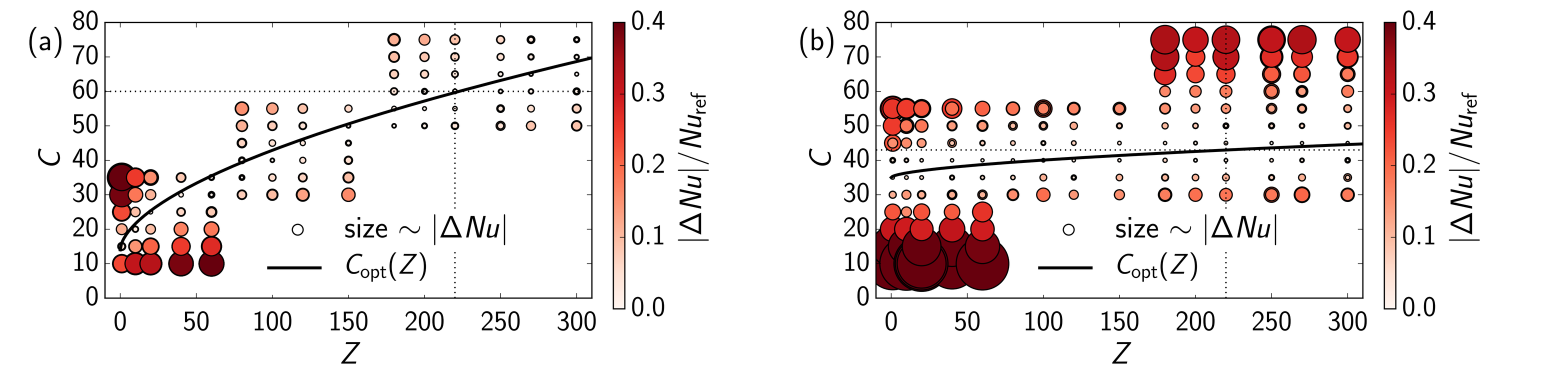

Figure 3: Model validation for the Nusselt number Nu. (a) $Pr = 0.7$, $Ra = (2.5 \pm 0.5) \times 10^{10}$, $Nu_{ref} = 176 \pm 5$ [5, 14]; (b) $Pr = 0.021$, $Ra = 4 \times 10^8$, $Nu_{ref} = 27.5 \pm 2.5$ [6]. Reference values are from cylindrical samples with $0.5 \leqslant \Gamma \leqslant 1$. Near-optimal values for C and Z are given by $C_{opt}(Z)$; the selected values are given by dotted lines (see [12]).

Rayleigh-number dependence of the Nusselt number

The Nusselt number exhibits **effective scalings** $Nu \propto Ra^{\gamma}$ as shown in Fig. 4. For moderately high Ra numbers, the present ODT results exhibit approximately the **classical (Malkus) scaling** $\gamma = 1/3$ [15] to a degree that is comparable with the reference data. For high Ra numbers, a **transition to the ultimate regime** can be discerned. The effective scaling exponent is only slightly lower than the asymptotic value $\gamma = 1/2$, which is due to a logarithmic correction, that is, $Nu \propto Ra^{1/2}\,[\ln(Ra)]^{-3/2}$ according to Kraichnan [2]. For very high Ra numbers, the few available ODT data points are quite well described by the Kraichnan prediction up to a Pr-dependent prefactor [12, 16].

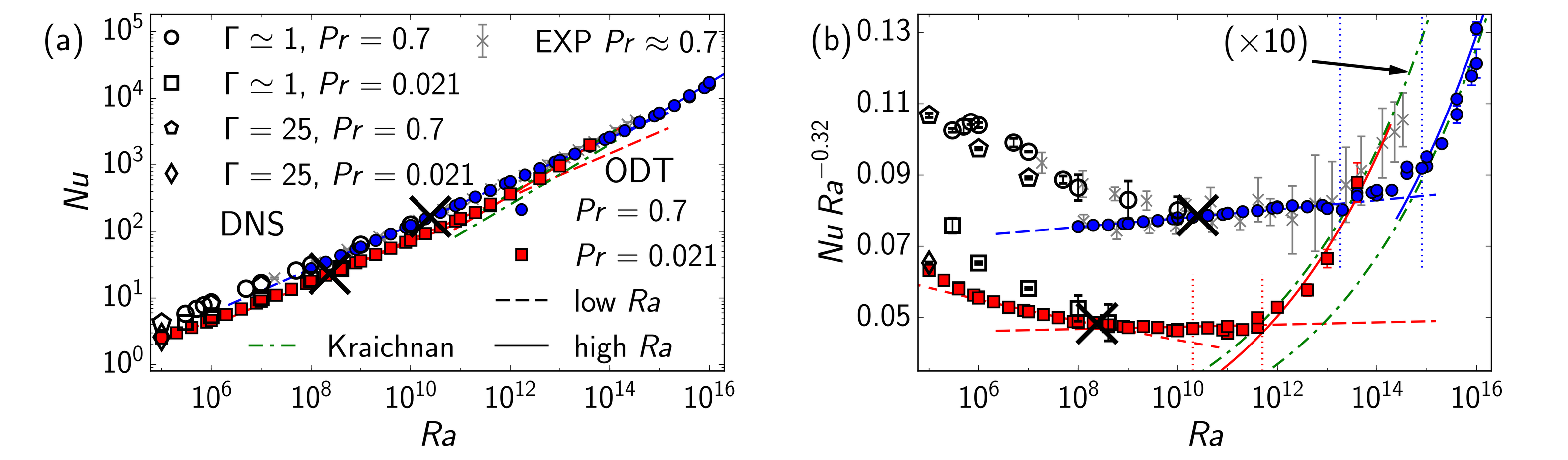

Figure 4: (a) Scaling of the Nusselt number Nu versus Rayleigh number Ra for the Prandtl numbers $Pr = 0.7$ and 0.021. (b) Same data but compensated with $Ra^{0.32}$. ODT results and the corresponding scaling laws are given in blue and red. Reference DNS data (black) is given for $1 \leqslant \Gamma \leqslant 3$, $Pr = 0.7$ [17]; $\Gamma = 1$, $Pr = 0.021$ [6, 18]; $\Gamma = 25$, $Pr = 0.7$ and 0.021 [19]. Reference measurement data (EXP, gray) encompasses $0.23 \leqslant \Gamma \leqslant 20$, $0.5 \leqslant Pr \leqslant 10$ as compiled in [1]. The Kraichnan [2] prediction is given by dash-dotted lines with a magnified prefactor ($\times 10$) for $Pr = 0.021$. Transitional Ra numbers expected from the literature are marked by dotted lines [4, 6]. Thick crosses mark the calibration cases from Fig. 3.

Vertical profiles of the turbulent temperature flux

Vertical profiles of the ODT turbulent temperature flux per unit area, $\langle w'T'\rangle$, are shown for the lower half, $0 \leqslant z/L \leqslant 0.5$, of the domain in Fig. 5. For moderately large Ra numbers, in the **classical regime**, $\langle w'T'\rangle$ increases rapidly across the thermal boundary layer with thickness $z \lesssim \delta/L = (2\,Nu)^{-1}$ but levels out towards the bulk. By contrast, for large Ra numbers after transition to the **ultimate regime**, $\langle w'T'\rangle$ **increases again towards the bulk** where it attains its maximum at $z/L \approx 0.18$ for both Pr numbers investigated [12, 16]. This is consistent with an assumption of Kraichnan [2].

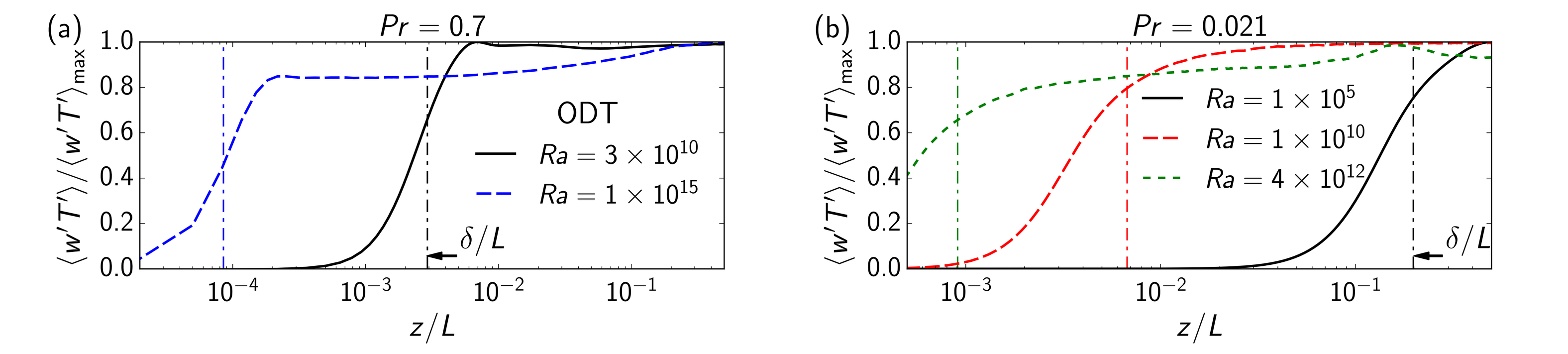

Figure 5: Time-averaged turbulent temperature flux per unit area $\langle w'T'\rangle$ for (a) $Pr = 0.7$ and (b) $Pr = 0.021$ corresponding to the regimes identified in Fig. 4(b). Normalization with the maximum value emphasizes profile shape variations.

Conclusion

- The **ODT model parameters** for thermal convection have been **estimated** for $Pr = 0.021$ and $Pr = 0.7$ using available DNS reference data for $\Gamma \simeq 1$ in the **classical regime**.
- ODT has **good predictive capabilities**. ODT exhibits the **transition to the ultimate regime** close to the expected critical Ra values. $Nu(Ra)$ is described well by the Kraichnan [2] theory.
- The ODT results obtained support Kraichnan's [2] assumption of a **relative increase** of the **turbulent temperature flux in the bulk** of the fluid for large Ra numbers.

Forthcoming Research

- Further analysis of the ODT scaling for the dependencies $Nu(Ra, Pr)$ and $Re(Ra, Pr)$.
- Variation of the boundary conditions (e.g. sheared convection).
- Model application to spherical-shell, non-Oberbeck–Boussinesq, and rotating convection.

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