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INVESTIGATING THE SKIN FRICTION DRAG ACROSS ELECTROLYTES AND ELECTRICAL FIELDS USING ONE-DIMENSIONAL TURBULENCE MODELING

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INTRODUCTION

We investigate the skin friction drag across various electrolytes and applied electrical fields using the stochastic one-dimensional turbulence (ODT) model. Turbulent Couette flows are studied as canonical example of engineering applications that may benefit from less friction and internal heating due to improved lubrication. The aim is to validate the lower order formulation for turbulent electro-hydrodynamic (EHD) flows. We limit our attention to weakly turbulent flows due to the available reference data, but it is planned to move on to vigorously turbulent flow regimes.

APPLICATION OF ODT TO ELECTROLYTE FLOWS

The one-dimensional turbulence model aims to resolve all scales of a turbulent flow by reducing the dimensionality [1]. The effects of Navier–Stokes turbulence are modeled by a stochastic process acting on a notional line-of-sight. The computational domain is, hence, a line (ODT line in figure 1).

Flows of electrolytes are described by coupling the Navier–Stokes equations to the Poisson–Nernst–Planck equations and an equation for the electrical potential (see e.g. [6]). The corresponding dimensionally-reduced, stochastic ODT equations for constant-density flows and univalent ions read

$$\frac{\partial u_i}{\partial t} + \mathcal{E}_i = \frac{\partial}{\partial y}\left(\nu \frac{\partial u_i}{\partial y}\right), \tag{1}$$

$$\frac{\partial c_\pm}{\partial t} + \mathcal{E}_\pm = \frac{\partial}{\partial y}\left(D\frac{\partial c_\pm}{\partial y} \pm \frac{Dc_\pm}{V_T}\frac{\partial \Phi}{\partial y}\right), \tag{2}$$

$$\frac{\partial}{\partial y}\left(-\varepsilon\frac{\partial \Phi}{\partial y}\right) = \rho e(c_+ - c_-), \tag{3}$$

where y denotes the wall-normal ODT line coordinate, t the time, $(u_i) = (u, v, w)$ the velocity vector, ν the kinematic viscosity, $c_\pm$ the positive/negative ion concentration, D the diffusion coefficient of the ions, Φ the electrical potential, $V_T = k_\mathrm{B}T/e$ the thermal voltage, k_B the Boltzmann constant, T the temperature, e the (positive) electrical charge of an electron, ε the dielectric permittivity of the electrolyte, ρ the constant fluid density, and $\mathcal{E}$ the stochastic eddy events. $\mathcal{E}_i$ models turbulent advection, fluctuating pressure-gradient and electrical forces, whereas $\mathcal{E}_\pm$ models only turbulent advection of the ion species.

Eddy events are realized as instantaneous, measure-preserving mapping events for which time, location, and size are sampled from an unknown probability density function (PDF) that depends on the current state of the flow. A construction of this PDF is avoided in practice by using a more efficient thinning-and-rejection algorithm [1], in which the inverse eddy turnover time,

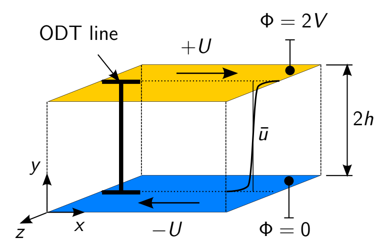

Figure 1: Sketch of the EHD Couette flow configuration.

$$\tau^{-1} = C\sqrt{2\big(E_\mathrm{kin}(\alpha) + E_\mathrm{pot} - ZE_\mathrm{vp}\big)/(\rho l^3)}, \tag{4}$$

is used to accept or reject candidate events [1, 2]. For a given eddy size l, E_kin is the shear-extractable kinetic energy, E_pot is the extractable potential (electrical) energy, and E_vp is the viscous suppression energy. E_kin is redistributed among the velocity components modeling pressure-velocity couplings (parameter α) [2]. E_pot is only released to the wall-normal (v) velocity component. The selected ODT model parameters $C = 10$, $Z = 600$ and $\alpha = 2/3$ [4] are kept fixed.

The stochastic terms $\mathcal{E}_i$ and $\mathcal{E}_\pm$ are zero between two subsequent eddy events. The deterministic parts of (1) and (2) are integrated here with an explicit time-marching scheme. The spatial discretization is done with finite volumes on an adaptive grid [4]. The Thomas algorithm is used to solve (3).

Figure 1 shows a sketch of the flow configurations investigated. The ODT line spans the height $2h$ meaning that the lateral directions (x, z) are taken to infinity. No-slip boundary conditions are prescribed for the velocity, where only $u(0) = -U$ and $u(2h) = U$ are nonzero. Dirichlet boundary conditions $\Phi(0) = 0$ and $\Phi(2h) = 2V$ are prescribed for the electrical potential. The ion concentrations obey a zero-flux boundary condition (right-hand side of (2) vanishes) so that the charges being redistributed are prescribed by the initial ion concentrations $c_\pm = c_0$.

ELECTROKINETICS WITHOUT FLOW

Figure 2 shows the stationary ion concentrations over the lower (negative) plate in the case of quiescent flow due to the boundary condition $U = 0$. The ODT results exhibit excellent agreement with the reference direct numerical simulation (DNS) results [6]. The ion concentrations at the wall differ slightly due to the details of the spatial discretization. Note that c_+ is larger than c_- due to the development of a space-charge region, which is roughly confined to the Debye layer thickness $\lambda_\mathrm{D} = \sqrt{\varepsilon V_T/(2\rho c_0 e)}$.

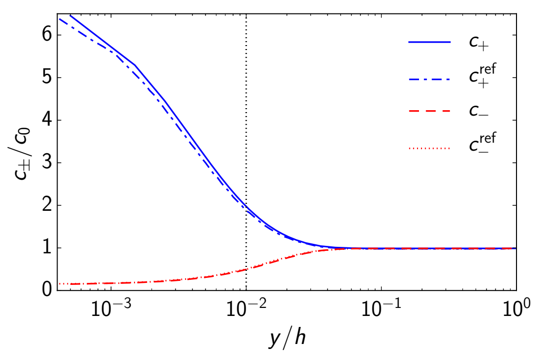

Figure 2: Ion concentrations $c_\pm$ over the negative plate for quiescent flow, dimensionless voltage $\hat{V} = 4$, and Debye length $\lambda_{\mathrm{D}}/h = 0.01$ (dotted). The reference data is from [6].

VELOCITY STATISTICS

Figure 3 shows the dimensionless velocity deficit $u^+ = |U - \bar{u}|/u_\tau$ over the boundary layer coordinate $y^+ = u_\tau y/\nu$ for various values of the Debye length λ_{D} and the coupling constant $\beta = \varepsilon V_T^2/(\rho\nu D)$. The friction velocity reads $u_\tau = \sqrt{\nu\,|\mathrm{d}\bar{u}/\mathrm{d}y|_{\mathrm{wall}}}$, where the overbar denotes a temporal average in the statistically stationary state. The dimensionless voltage $\hat{V} = 4$, the Reynolds number $Re = Uh/\nu = 3000$, and the Schmidt number $Sc = \nu/D = 3$ are kept fixed.

A small value of β means weak electrical forces so that $\beta = 1.5 \times 10^{-4}$ yields hydrodynamically dominated flow conditions. The channel centerline is located at $u_\tau h/\nu = Re_\tau \approx 170$, which corresponds well with reference data ($Re_\tau^{\mathrm{ref}} = 172$ [6]. Very good agreement is observed for the sublayer ($y^+ < 5$) and the log layer ($y^+ > 30$) with respect to the empirical solution [5]. The largest discrepancy between ODT and DNS is observed for the buffer layer ($5 < y^+ < 30$), which is a known ODT modeling artifact [3]. Strikingly, the much larger coupling coefficient $\beta = 1/2$ and the larger Debye length $\lambda_{\mathrm{D}}/h = 0.03$ do not notably change the results.

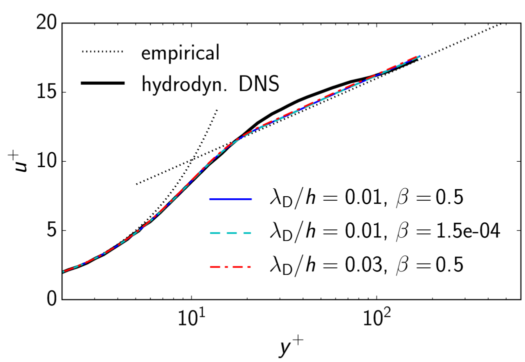

Figure 3: Mean streamwise velocity deficit u^+ across the momentum boundary layer for various coupling constants β and Debye lengths λ_{D} using the dimensionless voltage $\hat{V} = 4$. The ODT results collapse on the empirical law-of-the-wall [5]. A hydrodynamic DNS [6] is given for comparison.

SKIN FRICTION DRAG

Figure 4 shows the skin friction drag $c_{\mathrm{f}} = 2\tau_{\mathrm{wall}}/(\rho U^2)$ for various $\hat{V}$, β, and λ_{D} keeping $Re = 3000$ and $Sc = 3$ fixed. Without electrical fields or vanishing β, the skin friction drag is slightly larger in ODT ($c_{\mathrm{f},0} \approx 6.0 \times 10^{-3}$) than in the reference DNS ($c_{\mathrm{f},0}^{\mathrm{ref}} \approx 5.6 \times 10^{-3}$) [6]. Furthermore, the ODT results suggest a weak but systematic drag enhancement when the applied voltage increases. The magnitude of the effect is so tiny that it remains in the confidence interval. This interval has been estimated by computing the wall shear stress $\tau_{\mathrm{wall}}/\rho$ by $\nu\,|\mathrm{d}\bar{u}/\mathrm{d}y|_{\mathrm{wall}}$ and $\nu\,|\overline{\partial u/\partial y|_{\mathrm{wall}}}|$. The minimum drag is obtained for $\lambda_{\mathrm{D}}/h = 0.03$ in agreement with the reference DNS [6], but in ODT the applied voltage is much lower.

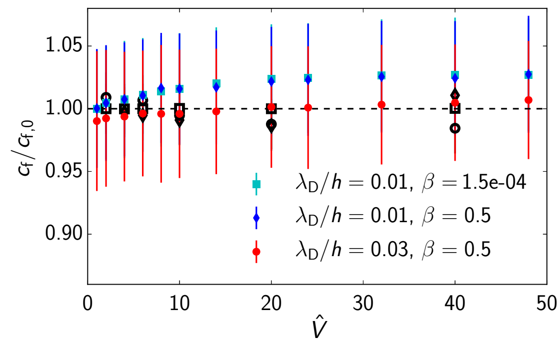

Figure 4: Normalized skin friction drag $c_{\mathrm{f}}/c_{\mathrm{f},0}$ for various voltages $\hat{V}$, coupling coefficients β, and Debye lengths λ_{D}. Reference data (black symbols of matching shape) is from [6].

CONCLUSION

Stochastic ODT simulations of turbulent Couette flows of electrolytes have been conducted and compared to available reference data. The skin friction drag is found to vary by only 1–3% in the investigated range of parameters which suggests only a very weak modification of the flow. The magnitude of the skin friction modification is comparable to the reference data [6] although it has remained in the confidence interval. The weak trends in the available data suggest a slight reduction of the skin friction drag when the Debye length λ_{D} is of approximately the same size as the viscous sublayer, that is, $\lambda_{\mathrm{D}}^+ \approx 5$, which is the case for $\lambda_{\mathrm{D}}/h = 0.03$ and $Re_\tau \approx 170$. A much smaller dimensionless voltage $\hat{V}$ is needed in ODT ($\hat{V} = 1$) than in the reference DNS ($20 \leq \hat{V} \leq 40$) to achieve the same level of drag reduction for otherwise the same parameters. We are currently extending the parameter range to identify a regime in which the coupling between electrokinetics and hydrodynamics increases.

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