Energie-Innovationszentrum Cottbus

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Simulating Homogenous Isotropic Turbulence with Deterministic and Stochastic Forcings Using a One-Dimensional Turbulence Model

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Introduction

To understand the intermittency present in scalar fields we need to address the expense of current start-of-art DNS to probe the higher-order structure functions. These higher-order moments become increasingly sensitive to increasing Re_λ and much more prone to extreme events. Here, in this work, we investigate using a Reduced order model(ODT) to simulate Homogenous Isotropic turbulence as an initial step towards that goal by employing a linear forcing [1] proposed by Lundgren that is proportional to local and instantaneous velocity.

ODT Model

The **ODT model** aims to **resolve all relevant scales** of a turbulent flow along a notional line-of-sight ('ODT line'). Flow variables are resolved along this line on a uniform grid.Instantaneous flow profiles are evolved by **deterministic diffusion** along the ODT domain, and a **stochastic process** that models the effects of turbulent advection and pressure fluctuations [2, 3].

- The **ODT governing equations** for Homogenous Isotropic Turbulence read

$$\frac{\partial u_i}{\partial t} + \mathcal{E}_i(u, v, w, \alpha) = \nu \frac{\partial^2 u_i}{\partial x_1^2} - f_i$$

  where x_1 denotes the spatial coordinate of the high-resultion ODT domain, t the time, $(u_i) = (u, v, w)^\mathrm{T}$ with $i \in \{1, 2, 3\}$ the Cartesian components of the velocity vector, and f_i a physical forcing that is detailed below assuring convergence to a statistically stationary flow solution.
- The **stochastic term** $\mathcal{E}_i$ represents the effects of turbulent **eddy events**. For each stochastically sampled event, the turnover of a notional turbulent eddy (**Fig. 1(a)**) is modeled by the instantaneous application of a spatial mapping, the **triplet map** [2] (**Fig. 1(b)**). In addition, a pressure redistribution model [?] is applied instantaneously in order to induce a tendency to isotropic turbulence efen if f_i is unidirectional. The strength is controlled by the model parameter α.
- Herere, we limit our attention to a scalar velocity representation setting $u_1 \neq 0$ and $f_1 \neq 0$, but $u_2 = u_3 = f_2 = f_3 = 0$. Correspondingly, pressure-redistribution effects are neglected by letting $\alpha = 0$. This assumption will be relaxed in forthcoming research.
- The turbulent **eddy rate** $\tau^{-1}(\ell, z_0; t)$ of a size-ℓ eddy event at location z_0 at time t depends on the **total available eddy specific energy** for the momentary flow state.

$$\tau^{-1} = C\sqrt{2\,\ell^{-2}\left(E_\mathrm{kin} - Z\,E_\mathrm{vp}\right)}\ ,$$

  where E_kin and E_vp denote eddy specific kinetic energy and viscous penalty energy, respectively. The latter effectively suppresses eddies below a viscous (Kolmogorov) length scale.
- The **ODT model parameters** $C = 1.8$ (eddy-rate parameter), $Z = 10$ (viscous-suppression parameter) have been fixed after calibration with reference direct numerical simulation (DNS) [4].

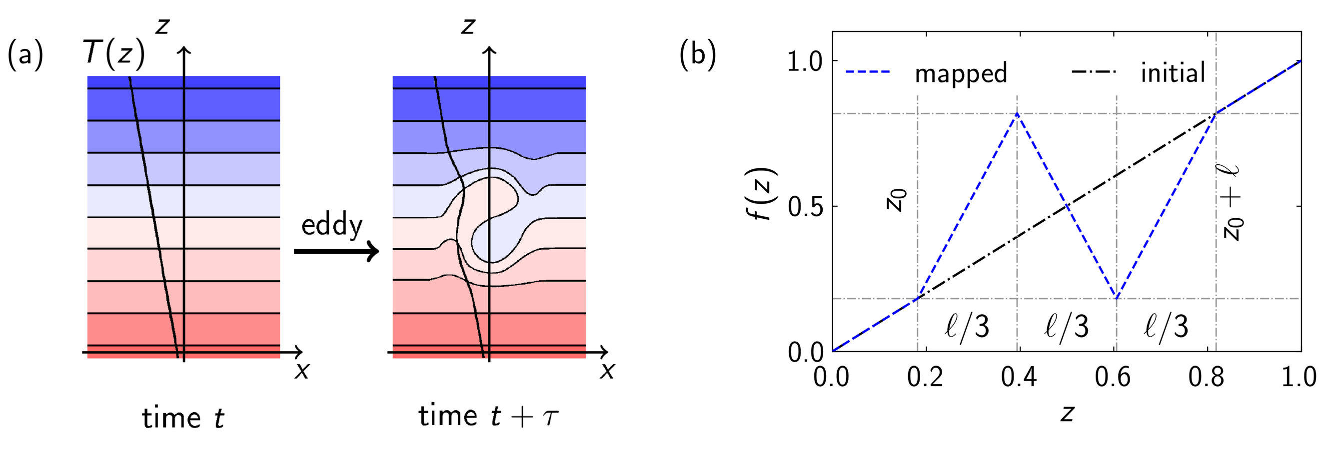


Figure 1: (a) Schematic of an eddy turnover. (b) Triplet map for an eddy event that covers the interval $0.18 \leqslant z \leqslant 0.82$.

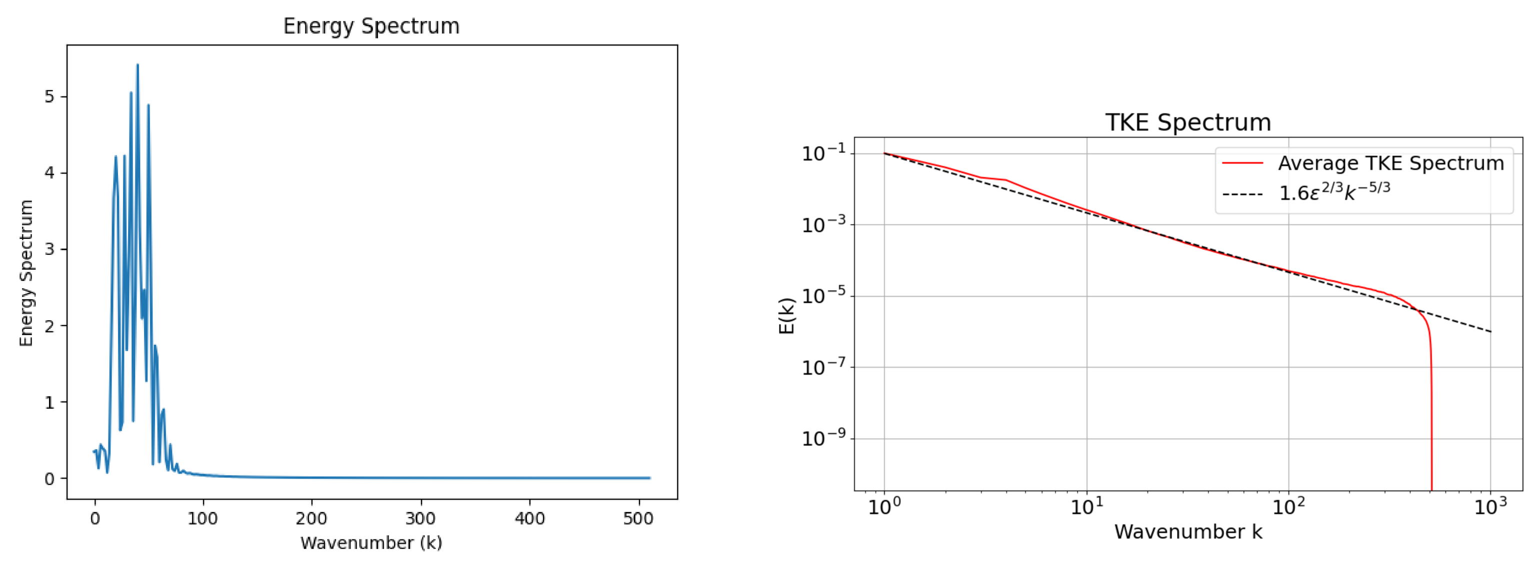
Linear Forcing

In this study, we investigate the application of **linear forcing** within dimensionally reduced flow models, specifically One-Dimensional Turbulence (ODT) [2] [3]. We build upon recent work by Giddey *et al.* (2018) [5], utilizing a deterministic forcing that incorporates a time-dependent forcing coefficient as proposed by Basenne *et al.* [4]. The forcing term for achieving constant turbulent kinetic energy can be written as

$$f_1 = Au_1$$
$$A(t) = \frac{\epsilon(t) - G[k(t) - k_0]/t_{\ell,\infty}}{2k(t)}$$

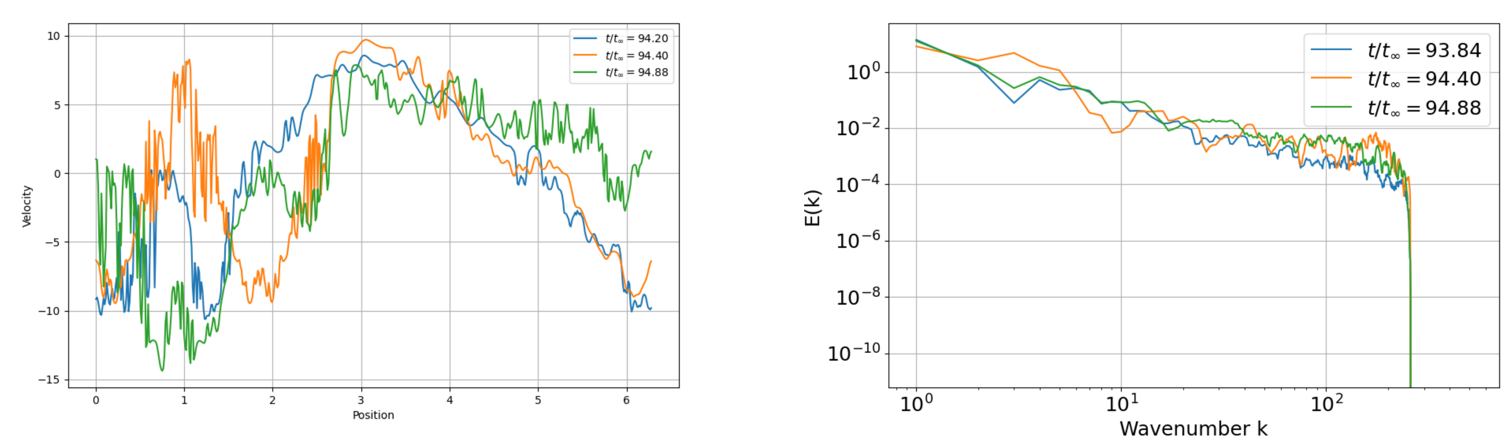
where $k(t) = \frac{1}{2}\langle u_i u_i \rangle$, $\epsilon_{1\mathrm{D}} = \nu \left\langle \frac{\partial u_i}{\partial x_2}\frac{\partial u_i}{\partial x_2} \right\rangle$ are volume averaged kinetic energy and dissipation. k_0 is a the target kinetic energy (initial kinetic energy) and t_l, ∞ is integral time. G is an amplification parameter. We reproduce the results from [5] and recover an inertial scaling in the kinetic energy cascade with the spectral exponent $-5/3$. We investigate the spectra and temporal evolution of spatially averaged kinetic energy and other derived quantities. The initial conditions typically involve a synthetic, solenoidal isotropic velocity field generated from an energy spectrum [6](**Fig. 2(a)**). A 1-D projection of such a synthetic flow field is given by

$$E_0(\kappa) = \frac{32\kappa_0}{3\kappa_0}\sqrt{\frac{2}{\pi}}\left(\frac{\kappa}{\kappa_0}\right)^4 \exp\left[-2\left(\frac{\kappa}{\kappa_0}\right)^2\right],$$



(a) (b)

- **Fig. 2(b)** shows the statistically stationary **kinetic energy spectrum** averaged over 5000 time steps.

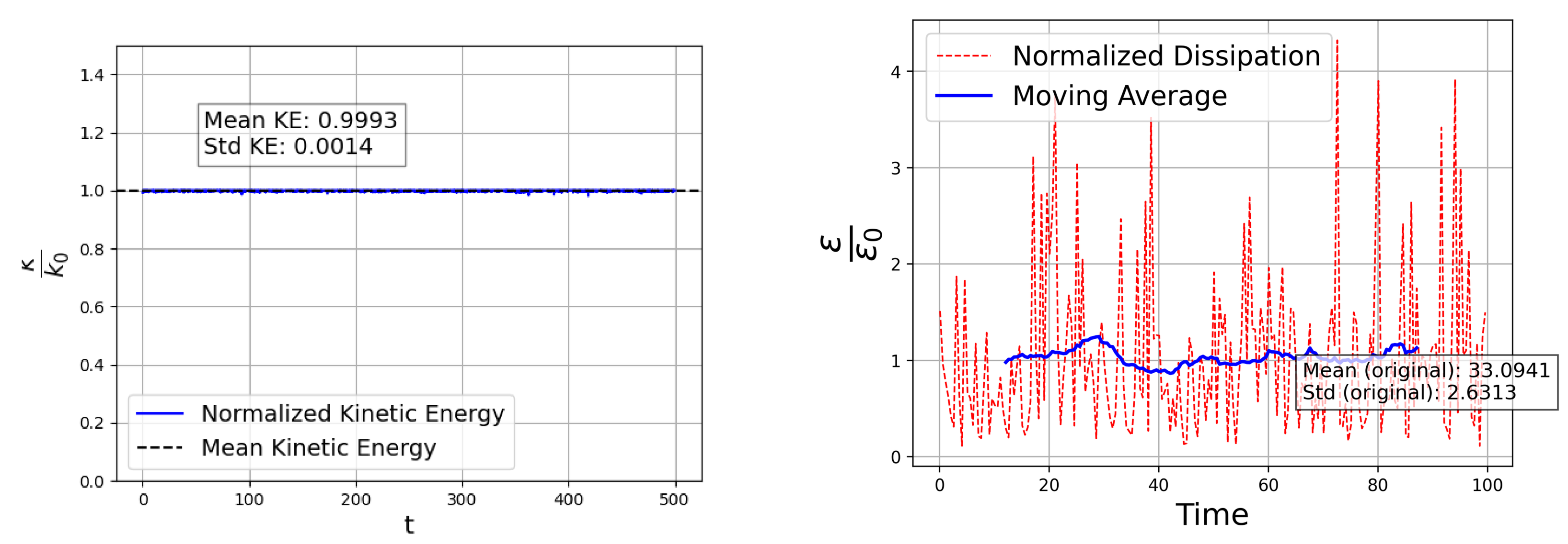


(a) (b)

Figure 2: (a) Initial Energy spectrum (b) Inertial Scaling with exponent -5/3 (c) Instantaneous velocity profiles at different eddy turnover times (d) Corresponding instantaneous energy profiles

Kinetic energy and Dissipation

We also look at the $\mathbf{k(t)}$ and $\epsilon_{\mathbf{1D}}$. We see that the kinetic energy has relatively fewer fluctuations and is around $k_0 = 17.1$ which we have set as our target kinetic energy. The dissipation ϵ_{1D} has extreme events but the moving average is around $\epsilon_0 = 32.3$ which is per the $Re_\lambda = 110$ that we have investigated.



(a) (b)

Figure 3: (a) Normalised Kinetic Energy with time (b) Normalised Dissipation with time

Results and Outlook

With this work, we were able to able to achieve Homogenous isotropic turbulence using ODT in a significantly less computational time when compared to its DNS counterpart. The next step is to investigate passive scalar intermittency and the higher structure functions to comment on the anomalous scaling behaviour observed in the model.

References

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