

INFLUENCE OF STATISTICAL SIZE EFFECT IN STEEL ON STRUCTURAL SAFETY

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Abstract

This paper presents the investigation of the statistical size effect in steel structure and the corresponding reliability. This study describes the randomness of material properties through two different ways. These two proposed simulation methods, which are an extension and supplement to traditional simulation methods, can effectively simulate the statistical size effect for the tensile and flexural components in steel structures. The test results show that the variations of the yield and tensile strength increase with the decreasing specimen volume. Besides, the structural component strength is not only related to the specimen volume, but also the stress distribution. It is found by studying the influence of statistical size effect on structural reliability that the strength, which is obtained by small specimens through statistical analysis in the laboratory, is no more accurately applicable to large construction. The core idea is that the stochastic material properties are directly embedded in mechanical calculations to develop a more accurate and economical design method for steel structure.

Key Words

Statistical size effect, Steel structure, Stochastic material model, Stochastic FEM, Structural safety

1 INTRODUCTION

Since the conditions of the occurrence of an event are not sufficiently known, there is no direct causal relationship between the condition and the result of an event, and thus the event exhibiting uncertainty. Practical experience of engineering shows that the uncertainty is not only in the load assessment, but also involves the engineering system of materials and geometric characteristics.

In general, the computational model and the numerical software usually are based on the deterministic method, which means that all geometries, materials, and loads of the structure must be deterministic values. However, many parameters of the structural engineering exhibit complex randomness, which cannot be completely captured and characterized by a deterministic model. The traditional approach is used to rationalize these uncertainties through probabilistic and statistical methods, and then the required determined values for the calculation model are obtained based on the extreme, minimum, maximum or mean values of system parameters. This approach contains the assumption that the results obtained by the deterministic analysis represent all possible situations of load and resistance in the structural components. In some cases, this assumption may be correct. However, it is definitely that the deterministic method cannot get the best solutions or optimum design for the structure.

In general, the safety factor has been used as an evaluation criterion of civil engineering. However, the safety factor is only a determined value obtained from known information, and it fails to account for any variability of

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parameters for structural design. In other words, the safety factor does not precisely characterize the safety of the structure, because the statistical parameters of the design variables are also variable with the change of external conditions.

With the continuous development of computational science, the precision of the structural calculation is continually increasing. Hence, the benefits obtained by the precise analysis of the structure are submerged by the traditional safety factor, if the variability of the statistical parameters of the random variables such as (material properties, geometries, etc.) is not considered. Thus, it has a great sign that the uncertain factors are introduced indirectly or directly into the mechanical calculation to analyze the structural mechanics and to evaluate the reliability of practical engineering.

As is well known, the elastic and plastic design in steel structure is widely used [1], since the steel has outstanding elasto-plastic material properties. It is certain that the yield strength of steel has variability because the uncertainties and spatial variability is inherently present in nature. The influence of this uncertainty of strength on the structure and corresponding safety is unclear. The statistical size effect phenomenon, which is originally used to describe the effect of the characteristic size on structural strength in the concrete structure, can explain the relationship between the volume of the similar structure and the material strength. The current elastic and elasto-plastic theory don't involve the statistical size effect and its influence. Whether the uncertainty of the steel strength can produce statistical size effect, is a problem that needs to be discussed, because the statistical size effect will change the strength of structural components and the corresponding safety level.

The structural reliability analysis is typically based on random variables of load and resistance. In principle, the reliability can be evaluated once the probability distribution of the load and resistance (or response) becomes available. However, the rationality of the reliability calculation is based on the fact that the random variables distribution model and the relevant statistical parameters are correct. For yield strength of the material, some studies [2,3] suggest that a lognormal distribution is appropriate. But the reference [4] shows that the goodness-of-fit tests suggest that the lognormal, Weibull and extreme value distribution are all equally valid choice for describing the yield strength of steel. Through a series of experiments, it was found that there was a correlation between the steel strength and the sizes of the specimen [5]. The statistical parameters of the strength vary with size. Hence, the key problem for steel structure safety is to obtain the suitable distribution under the consideration of the uncertain-ties of the material properties.

This paper focus on the introduction of the influence of statistical size effect on the material strength and the corresponding reliability in steel structures. It mainly involves the latest research in the following aspects: developing a stochastic material model based on the known probability distribution model by mathematical methods, establishing a discrete random field and combining it with stochastic finite element method, showing some experimental and simulation results on statistical size effects, and evaluating the influence of statistical size effect on structural reliability.

2 Statistical size effect in steel structure

In the past few decades, the question of whether the strength of the material is affected by the absolute volume of the structure has been continuously raised. According to previous studies, the size effect phenomena can be divided into two different methods, namely deterministic and statistical explanations. The deterministic size effect based on the Energy release is widely studied by most researchers [6,7]. The statistical size effect of steel structures was mentioned decades ago [2]. However, the statistical size effect on the steel structure is currently still belong to a forgotten research field. As described in the reference [8], there are two main reasons why the statistical size effect is covered in steel structures: firstly, the statistical size effect in steel structures is not as prominent as in concrete since the strength variability of steel is relatively small. Another reason is that most of the components in the steel structure are plate-type and not bulk-type, and some studies have focused on the relationship between strength and material thickness.

From the perspective of the microstructure of the material, all materials are imperfect and have more or less some defects, which are caused by imperfections in crystal combination or slippage of crystal contact surfaces. Recently, some kinds of failure prediction of structural elements based on the microstructural analysis are widely mentioned and studied, for example, Gurson-Tvergaard-Needleman (GTN) material model [9]. However, these researches only describe macroscopic material models to describe phenomena caused by microscopic defects, and it does not link the randomness of microstructural defects with the probability distribution of material properties at macroscopic scales. Recent studies have shown that [10], material properties are affected by the size of the structure, because as the volume of the structure increases, the size, number and corresponding distribution of microstructure defects will change. From a statistical point of view, if the probability of failure is kept constant, the strength of the material will decrease. This is because the increased defects lead to higher failure probability

under the same stress conditions. The randomly distributed microscale defects determines the probability distribution of material strength at the macroscopic scale. Therefore, the size effects can be studied by statistical methods. Based on some simplifying assumptions of microscale structure, macroscopic material strength can be obtained using stochastic material models with probabilistic and statistical theories. Besides, this research method based on statistical theory can avoid the difficulty of studying the microstructure of materials.

The stochastic material model, which is employed to analyze the randomness of the material properties, can be roughly divided into three types: weakest link model, fiber bundle model and the combination of both models. The weakest link model was proposed by Weibull [11] to analyze the material strength, and the fiber bundle model was used to treat load-sharing among fibers by Daniel [12]. The combination of both classical models contains two different type, which means that the chain of bundle and bundle of chain models [13]. The chain of bundle model can provide a flexible selection of the generic probability distribution for different specimen size [14]. Recently, this model has been employed to analyze the statistical size effect in steel structure for the tensile and flexural members [8].

In the recently research, the stochastic material model for the elasto-plastic material was proposed. Firstly, the classical stochastic material model has been briefly described, as well as the modeling and application scope of the two most fundamental models are discussed in detail. According to the analysis of real material properties, it is confirmed that the two classical stochastic material models can not describe material properties very accurately. Then, the chain of bundle model based on two classical models was proposed and employed to describe the statistical size effect for steel. Besides, the proposed model was extended to analyze the steel structure with multi-axial stress state using the von Mises yield criterion. This model was also applied to structural components with non-uniform stress distribution. Furthermore, the stochastic material model was integrated into the commercial FEM software ABAQUS by user subroutine. The statistical size effect of steel can be verified by uniaxial tensile tests and bending tests, and the parameters of statistical size effect are normally determined by experimental results and simulations.

3 Uncertainty modeling and stochastic FEM

Because the material mechanical defects exist in the entire structure, the uncertainties of the material properties are necessarily presented in the whole structural component. The conventional method based on stochastic material model can only describe the phenomenon of statistical size effects through indirect mathematical models, and it is still difficult to directly obtain the statistical distribution of material strength for different structure sizes. Nowadays, it is widely recognized that the material properties are characterized by intrinsic randomness and uncertainty, as well as these properties exhibit stochastic variation in space. The random field is a useful mathematical model that can describe the properties of spatial variability accurately in the structural component. From the reference in the past times [15], it can be seen that the realization all random fields revolve around two main categories, which are converting Gaussian random field to non-Gaussian, and the discretization of high-dimensional random field.

Although the Gaussian random field is relatively simple and it has a wide range of applicability, several quantities arising in practical engineering exhibit non-Gaussian probabilistic characteristics. In general, there are two different approaches to implement the non-Gaussian random field. The first method is generating a sample function that matches the specified power spectral density function or the statistical moments of the target random field [16]. It is possible that the different marginal probability distributions have similar statistical moments even if the tails are dissimilar. The other approach is generating a sample function, which contains the probability information of the target function [17].

From the numerical calculation point of view, the random field must be discretized using a finite number of random variables, since the numerical method used in structure analysis cannot directly deal with continuous models. Besides, if the discretization of the random fields used to describe the performance of the structure is not accurate enough, the results of numerical simulations may produce erroneous conclusions. Therefore, obtaining an approximate random field with the correct statistical parameters is a prerequisite for the investigation of uncertainty modeling and stochastic FEM. Some different random field discretization methods have been published in many references, for example, midpoint method, interpolation method, Karhunen-Loève (K-L) expansion [18] and the spectral representation method [19], etc. At present, the main problem of random field discretization is still concentrated on the required amount of computation for the generation of high-dimensional random field, and the corresponding major difficulty is to treat the problem that the solution of the eigenvalue function is included in the high dimensional integral. To speed up the assembly of a matrix with high-order integral, a method which approximates matrixes as a hierarchical matrix proposed [20]. More recently in [21], the cost to solve the homogeneous Fredholm integral equation was reduced by decomposing the 3D eigenfunction problem

in orthogonal coordinate axes as well as com-posing the matrixes again with the coordinates of elements. Fig. 1 shows the different dimensional Random fields realized by the proposed approach based on the 1D random field.

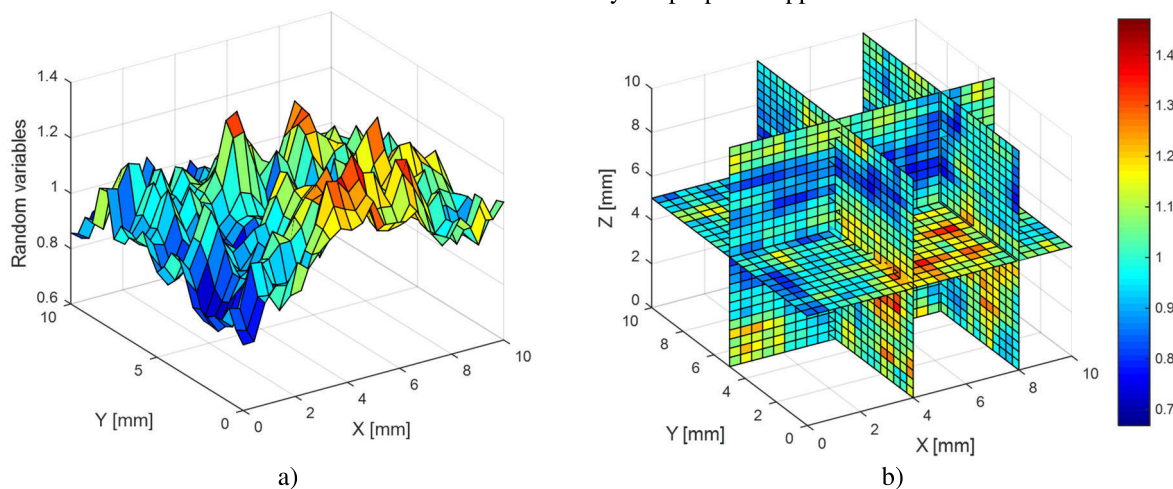


Fig. 1 Random field, a) Two-dimensional, b) Three-dimensional

After the material randomness can be defined with the mathematical model, i.e. random field, the statistical size effect can be analyzed with the FEM simulation. The stochastic FEM is a powerful tool to solve this kind of problem. In fact, this approach can be seen as an extension of the conventional deterministic FEM. From a mathematical point of view, the solution of the stochastic stiffness matrix is essentially to solve non-linear stochastic partial differential equations. Since the stochastic FEM with elasto-plastic materials involves nonlinear iterative calculation at integration points, the direct Monte Carlo Simulation is the simplest approach to calculate the structural response with random properties under the SFEM framework; especially it is one or only one general method for treating the elasto-plastic problems. The main idea in this method is that the samples of random properties are generated and discretized using K-L expansion, and then the samples of the response vector are obtained by repeating the deterministic finite element calculations. The probability distribution function of structural response can be calculated based on the obtained samples of the response with the first n -th moments of the statistics. The influence of statistical size effect on structural reliability can be also analyzed based on the accurate probabilistic descriptions of the response with stochastic FEM.

4 Reliability assessment with statistical size effect

Recently, the design of structures has focused on the probabilistic limit state design in civil engineering. The key problem in the structural design is to properly account for the uncertainties on the boundary conditions and the material properties. The finite element approaches in connection with stochastic and probabilistic methods, which can analyze the structure with the inputted uncertainty, has developed very fast in the last decades. The stochastic FEM can estimate the statistical description of response quantities. A more accurate and efficient calculation approach of the structural reliability based on the structural response from stochastic FEM has always been the focus in this research area. Usually, the statistical response characterization method uses an explicit limit state function in which the response of the structure is obtained by the stochastic FEM. However, this approach can be applied to analyze reliability problems involving linear problems. Because the stochastic FEM with direct Monte Carlo Simulation for the elasto-plastic material requires many iterative calculations, the sample size of the structural response is limited, and the accuracy of the reliability evaluation is often difficult to be satisfied. As an alternative, the probability density function of the structural response can be obtained using traditional fitting techniques, and the corresponding parameters of the theoretical probability distribution can be estimated. The maximum Entropy fitting method provides a possibility to obtain relatively accurate probability distribution with small available data.

To analyze the reliability and the ultimate bearing capacity of the bending members under considering the statistical size effect, the 3 and 4 points bending simulations of IPE beam with random fields of yield strength can be implemented by stochastic FEM. The simulations are performed with different sizes of IPE profile in reference [10] to study the effect of the component volume on structure safety. The random field with lognormal distribution is employed for the yield strength, as well as the isotropic exponential function is defined as autocorrelation function of random field. Besides, the correlation length and coefficient of variance of the random field are

assumed respectively as 40 mm and 10%. The length of the discrete random field is defined as half the correlation length. Fig. 2 a) show the corresponding distributed yield strength in the numerical model.

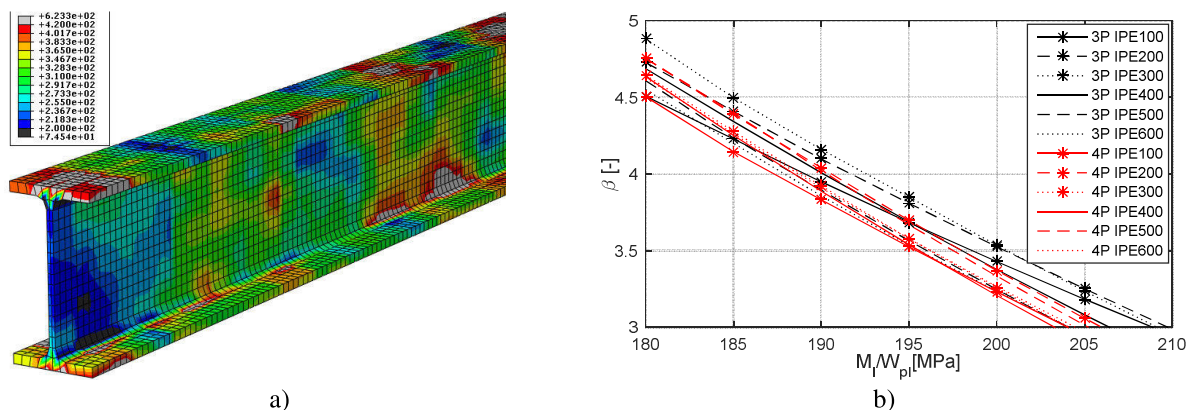


Fig. 2 a) Distribution of yield strength in IPE-section beam, b) influence of statistical size effect on the structural safety

Fehler! Verweisquelle konnte nicht gefunden werden. 2 b) shows the reliability indices β for 3P and 4P bending simulation with different specimen volume. The reliability indices of 3P and 4P bending beam have a relatively significant difference even if the structure is applied by the same bending moment. This is due to the statistical size effect caused by different yield volume under the ultimate bearing capacity. The M_l / W_{pl} in abscissa represents equivalent stress of the structure, where M_l is the mean value of the applied moment. For different sizes of beams, the reliability index of the 3P bending beam is always greater than the 4P bending beam with the same M_l / W_{pl} value. The relationship of the reliability index and the volume of structure cannot be simply summarized. For example, in the case of three-point bending the structural reliability index increases with the enlarged volume of the structure, while in the case of four-point bending it decreases. Therefore, the reliability analysis for flexural members considering the statistical size effect needs to be calculated separately for each structure, because the reliability index is significantly influenced by the structural volume and stress gradient of the structure.

5 Conclusion

This paper shows that the statistical size effect presents in steel structures due to the uncertain microscopic structure of the material properties. The numerical studies exhibit the great potential of the embedding uncertainty directly into the structural analysis. Furthermore, the influence of statistical size effect on the reliability of the structure is not negligible. The stochastic modeling framework has been used for derivation of the statistical size effect for the considered randomness of material properties. The results show that the changing rates and variation coefficients of the strengths decrease with increasing structural component sizes. The statistical distribution of the material strength will be varied due to the change of the coefficient of variation. Therefore, the influence of the statistical size effect on structural safety must be noted by structural engineers.

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