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Mathematical model for the prediction of strength degradation of composites subjected to constant amplitude fatigue

Nenad Stojković [a,*], Radomir Folić [b], Hartmut Pasternak [c]

[a] College of Applied Technical Sciences, Department of Civil Engineering, Aleksandra Medvedeva 20, 18000 Niš, Serbia
[b] University of Novi Sad, Faculty of Technical Sciences, Department of Civil Engineering, Trg Dositeja Obradovića 6, 21000 Novi Sad, Serbia
[c] Brandenburg University of Technology, Chair of Steel and Timber Structures, Konrad-Wachsmann-Allee 2, 03046 Cottbus, Germany

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ABSTRACT

Residual strength models are widely used for the fatigue analysis of composites subjected to both constant and variable amplitude fatigue. However, they often require large experimental efforts to determine the model parameters. The aim of this research is to develop a simplified mathematical model which uses a single set of parameter values to predict the strength degradation of the elements subjected to different load levels, and thus, to reduce the required experimental effort for the determination of model parameters for each load level separately. The new two-parameter model is developed here on the basis of normalization of the difference between the residual strength and maximal applied load in the constant amplitude cyclic loading, herein referred to as the strength reserve, with respect to initial conditions. The model is validated using different experimental data sets from the literature. High correlation with the experimental data is observed. Moreover, the curves obtained by using the single set values of parameters matched the curves of some of the most accurate residual strength models from the literature.

1. Introduction

Due to their high strength-to-weight and stiffness-to-weight ratios, composite materials offer great possibilities for a variety of engineering applications, such as wind turbine blades [1] or bridge decks [2] etc. On the other hand, while reducing the weight of the structures, such composite materials provide shorter construction time and increased durability [1]. However, the lack of unified methods for assessing composites durability represents one of the main setbacks for their larger scale usage in practice. In recent years, many researchers have devoted considerable attention to the variety of influences on the long-term performance of composites, such as environmental conditions [2] and fatigue loading [3]. Although, during their service life, the real structures undergo extreme cases of static loading, they are also subjected to irregular type of cyclic loading, known as variable amplitude fatigue. In practical applications, fatigue is considered to be one of the most important forms of loading with respect to the long-term service life of composites [4]. This type of loading causes deterioration of material properties, such as strength or stiffness. Thus, the design considerations of extreme static loads should not be restricted to the original material properties. In recent years, much research has been conducted in the field of fatigue of composites, at both theoretical and practical levels. Different theories which could be successfully applied to the prediction of fatigue lives of existing products, such as composite pipes [5,6], high pressure vessels [7] or compressor and wind turbine blades [8,9] have been developed. The practical application of those theories is of great importance for the prediction of fatigue life of existing products as well as for the design considerations of new ones.

Following the traditional total life approach, in which the number of cycles to the failure is expressed as a function of the amplitude or maximum fatigue stress or load (S-N or L-N curves), fails to provide the insight into the level of deterioration. The solution for this problem could be found in two different approaches usually referred to in the literature as mechanistic and phenomenological [10,11]. The mechanistic approach accounts for the damage progression in the composites after a fraction of fatigue life by considering the micromechanical damage models. This approach fails to offer the convenience of the direct practical application, due to the fact that parameters needed for its implementation are often very difficult to obtain in structural engineering reality. On the other hand, phenomenological approaches consider deterioration of strength or stiffness, which are more useful material properties for engineering application. They are often characterized by the ease of practical implementation, although in many cases they require extensive experimental investigation.

* Corresponding author.
E-mail address: nenad.stojkovic@vtsnis.edu.rs (N. Stojković).

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There has been a large number of studies on the stiffness reduction with increasing number of loading cycles. Most of the models proposed in the literature considered specific stress state, stress ratio and layup, e.g. [12–16], which was the main setback for their further implementation. By coupling the *normalized residual strength* [17] and *normalized fatigue life* [18] models proposed by Harris and co-workers, Shokrieh and Lessard [19] developed a *general residual material property degradation model*, which was the first model capable of simulating the behaviour of unidirectional ply under multiaxial loading and arbitrary stress ratio. This model considered the gradual degradation of both strength and stiffness. Based on it, the same authors later developed technique called *progressive fatigue damage modeling* [20] which was a combination of failure criteria and degradation rules. A similar principle was later used in [21] for developing a residual stiffness model for cross-ply laminates in all material directions using a new fatigue life measure, proposed in [22].

Residual strength models are widely used for the fatigue life prediction of the fibre-reinforced polymer composites. They are originally developed for prediction of strength degradation under constant amplitude fatigue loading and later extended to variable amplitude fatigue. Furthermore, the advantage of these models is that residual strength could be used for the design in the case of extreme overloads that structures could experience in their service life, while they are undergoing regular cycles of loading during their normal service.

The philosophy of constant amplitude residual strength models is illustrated in Fig. 1, and will be explained here briefly. The damage accumulation in material occurs during the constant amplitude fatigue loading. The main parameter for assessing the damage level is the remaining static strength (residual strength), S_R. The damage accumulation leads to deterioration of the residual strength, proportional to the increasing number of cycles, n. When the residual strength becomes equal to the maximum load in the cycle, the structural element fails, and the number of cycles is equal to the fatigue life, N.

In the literature, residual strength models have been classified in different ways: probabilistic or deterministic, linear or nonlinear, sudden death or wear-out. Both deterministic and probabilistic models were introduced at approximately the same time, in early seventies. While investigating the fatigue of glass chopped strand mat/polyester resin specimens in [23], Owen and Howe proposed the residual strength model as a quadratic function of the ratio of endured number of cycles and the fatigue life. Halpin et al. [24], proposing the probabilistic approach, derived a three parameter Weibull function describing the residual strength. Broutman and Sahu [25] defined residual strength as a linear function of a number of cycles or fractional life. Hahn and Kim assumed the change of the residual strength to be described by the rate-type function, and proposed a new probabilistic approach [26]. The residual strength approach was subsequently accepted for the characterization of deterioration levels in composites, resulting in the proposal of various new models [17,19,27–34]. This approach served as a basis for predicting the fatigue life for variable amplitude loading. Schaff and Davidson, using a constant amplitude residual strength model, proposed a procedure for predicting the fatigue life of composites under two stress level and fatigue spectrum [35,36].

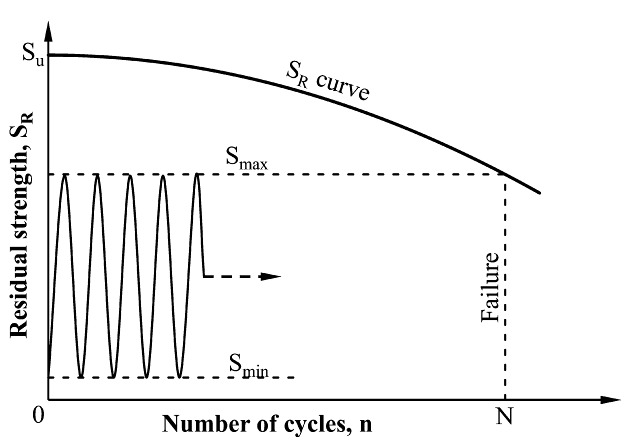

Fig. 1. Residual strength model philosophy.

With the exception of the Broutman and Sahu's model [25], all of the abovementioned residual strength models require experimental data for obtaining specific parameter values. The more model parameters there are, the larger experimental data set is required. In real life, structures are subjected to randomly ordered loading spectra, that consist of many different load levels. Utilization of the residual strength models for prediction of the fatigue life of structures subjected to such types of loading requires the knowledge of model parameters for each load level in the spectrum. Obtaining these parameters could be time and money consuming. In the past, some efforts were made to reduce the amount of experimental data required for determining the residual strength model parameters at different load levels of loading spectrum. The model and procedure that Schaff and Davidson proposed in [36] aimed to reduce the number of tests. Their one-parameter mathematical model correlated well with the experimental data used for validation, but failed to produce the curves for describing some of the different degradation trends reported later in the literature [37–40]. Starting from the same mathematical model, Shenoy et al. [10] utilized the normalization principle, in order to establish the same parameter for all load levels. The alternative for the fast determination of the fatigue life without performing residual strength tests, based solely on S-N curves, is the application of Palmgren-Miner cumulative damage rule [41]. Because of its simplicity and the fact that it does not require any other tests apart from obtaining S-N curves, this rule represents a powerful tool for practical engineering application, e.g. [42,43]. The main flaw of this model was the questionable conservatism of its predictions, which led to the development of other types of cumulative damage rules, such as isodamage lines [44]. However, despite the fact that it generates non-conservative results in some cases, even nowadays it serves as a baseline against which most of the approaches for the prediction of fatigue life of materials subjected to variable amplitude loading are compared [38].

The aim of this research is to develop the simplified mathematical model which could use single parameter values for predicting of the strength degradation of the elements subjected to different load levels; and thus reduce the required experimental effort for determination of the model parameters for each load level. Its derivation follows two main principles. The first applies to degradation curves which at different load levels should be modified so as to be represented with a single curve, achieved by following the normalization principle. The second principle applies to the new model which should be able to produce different shapes of degradation curves, in order to describe different strength degradation trends reported in the literature. The new two-parameter equation is developed based on the normalization of the difference between the residual strength and maximal applied load in the constant amplitude cyclic loading, herein referred to as the strength reserve, with respect to initial conditions. It considers only the constant amplitude fatigue, but offers the possibility of being expanded to the variable amplitude fatigue. The model is validated by using different experimental data sets from the literature, which are detailed in the next section.

2. Experimental data

The experimental data for validation of the model are taken from the Refs. [37–39]. In this section, the information about materials of the tested samples and the datasets will be summarized. More detailed information on the test procedure, manufacturing of the samples and test conditions can be found in those references.

Anderson and Korsgaard [37] investigated the fatigue behaviour of the glass fibre reinforced polyester typically used as a wind turbine blade material. It was produced by hand lay-up, and comprised chopped strand mat, unidirectionally reinforced. Fabric layers were in the following sequence: /CSM, fabric, (CSM, UD)$_2$/$_S$, with orthophthalic polyester (Norpol 410M910) resin. Constant amplitude fatigue tests were performed with the stress ratio R = 0.1 and the frequency of 17 Hz. The S-N curve was fitted to experimental data and the obtained equation was $logN = 10.9 - 0.019*S$. The residual strength dataset consists of 66 test results at six different levels, with different fractions of constant amplitude fatigue life before tensile tests for all stress levels, and under the same conditions as in fatigue tests. This dataset was chosen to represent the initial drop of static strength, as it contains six fractions of fatigue life between 6% and 17%. On the other hand, at the stress level of 284 MPa, the test results are available for four different life fractions distributed between 17% and 82%, which can serve to evaluate the model's prediction of degradation at a late period of fatigue life.

The experimental investigation published by Post et al. [38] was performed on pseudoquasi-isotropic laminate manufactured by Northrop Grumman, consisting of 10 layers of Vetrotex 324 woven roving, with the 5:4 bias in the warp direction and the [0/+45/90/45/0]s layup. The matrix used was Dow Derakane 510A vinyl-ester resin. The dataset consists of 20 tensile strength tests, 88 constant amplitude fatigue tests and 189 residual strength tests. Constant amplitude fatigue tests were performed at six stress levels with the obtained median number of cycles between 900 and 1.5 million. The residual strength tests were performed at five fractions of fatigue life at each stress level. The targeted fractions were 10%, 30%, 50%, 60% and 70% of the fatigue life. Actual numbers of cycles were rounded, which resulted in differences of life fractions at each stress level. Consequently, 15 different life fractions at 3 different load levels are available, rather evenly distributed between 9% and 93% of fatigue life. The data set is very convenient for examination of the model's accuracy in prediction of strength degradation trend in the middle part of the fatigue life. Both constant amplitude fatigue and residual strength tests were performed with the stress ratio R = 0.1 and frequency of 10 Hz. In some residual strength tests premature failure was reported and these samples were not taken into consideration either in Ref. [38] or in this study.

Philippidis and Passipoularidis [39] performed the tests on glass/epoxy laminate consisting of 4 unidirectional plies at a layup, used in the manufacturing of wind turbine rotor blades. Static strength dataset consists of 26 test values. Constant amplitude fatigue tests were conducted with the stress ratio R = 0.1, and the frequency was not kept constant for the various stress levels. The S-N curve fitted to the experimental data was given by the equation $logN = 24.64 - 11.06logS$. The residual strength tests dataset consists of 74 test results at three different stress levels. The life fractions of residual strength tests were approximately 20%, 50% and 80% for all stress levels. This set of data was chosen for investigation because the developed model could use a relatively small number of life fractions for evaluation of the strength reduction trend and gave good prediction for all load levels.

The mechanical properties of the composite specimens from Refs. [37–39] are given in Table 1. All the experimental data from fatigue life and residual strength tests were obtained under tensile cyclic loading with constant stress ratio (0 < R < 1 and R = const).

Table 1
Mechanical properties of composite specimens from Refs. [37–39].

Reference	Longitudinal modulus [GPa]	Tensile strength [MPa]
Andersons & Korsgaard [37]	30.0	672
Post et al. [38]	25.2	333
Philippidis & Passipoularidis [39]	14.1	113

3. Normalized Strength Reserve Model (NSRM)

The new mathematical model is developed in order to investigate the possibility of defining material parameters independent of load level. Following the standard principle of normalizing residual strength with respect to static strength and endured number of cycles with respect to fatigue life, it was found that normalized curves could not coincide, i.e. could not have the same parameter values. The explanation and solution of this problem is given in this section.

In Fig. 2, residual strength plots are given for arbitrary three load levels, where $S_{max1} > S_{max2} > S_{max3}$. The residual strength S_R (Eq. (1)) is considered to be a function of endured number of cycles, n, maximum cyclic stress or load, S_{max}, static strength, S_u, and fatigue life, N:

$$S_R = f(n, S_{\max}, S_u, N) \tag{1}$$

It is a monotonically decreasing function which is equal to S_u for the unloaded specimen (n = 0) and S_{max} at the moment of failure (n = N). Normalization of residual strength and endured number of cycles is given by the Eqs. (2) and (3)

$$S_{R,n,i} = \frac{S_{R,i}}{S_u}, \tag{2}$$

$$n_{n,i} = \frac{n_i}{N_i}, \tag{3}$$

where $S_{R,n,i}$ is normalized residual strength and $n_{n,i}$ is fractional life (normalized number of cycles with respect to fatigue life) of the arbitrary (i-th) stress level.

Normalized residual strength curves of the three arbitrary load levels are illustrated in Fig. 3. The three curves could theoretically coincide only if $S_{max1} = S_{max2} = S_{max3}$, which is not the case. This problem can be circumvented without derogation from the nor-

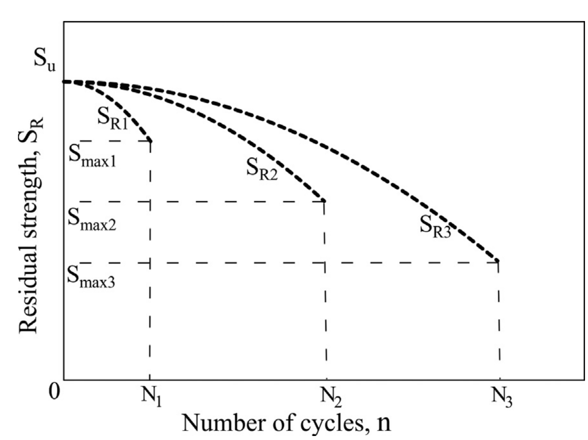

Fig. 2. Residual strength curves.

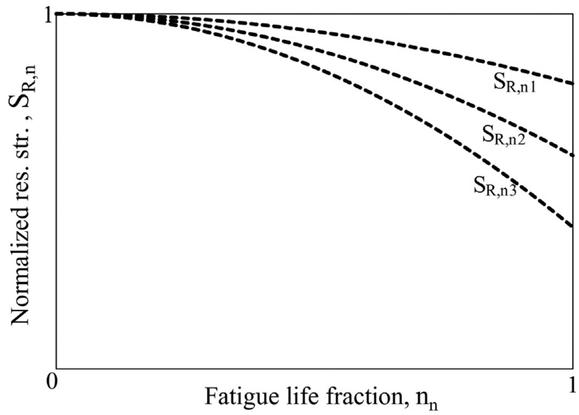

Fig. 3. Normalized residual strength curves.

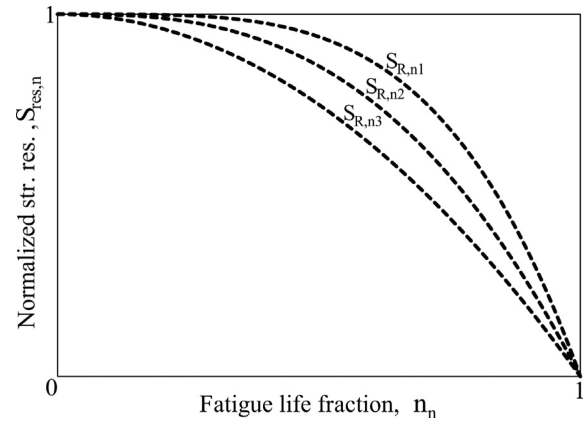

Fig. 5. Normalized strength reserve.

malization principle. Prerequisite for this assumption is that the curves must fulfill the same boundary conditions. In order to develop the model that could satisfy such boundary conditions, we start with residual strength equation that is proposed by Schaff and Davidson [35]:

$$S_R = S_u - (S_u - S_{\max})n_n^{\alpha}, \tag{4}$$

where α is the strength degradation parameter which can be determined by fitting the curve to experimental data. We observe the difference between residual strength and maximum stress, called the ***strength reserve*** (***S_{res}***). Starting from Eq. (4), the strength reserve can be expressed by the Eq. (5), which is represented in Fig. 4.

$$S_{res} = S_R - S_{\max} = [S_u - (S_u - S_{\max})n_n^{\alpha}] - S_{\max} \tag{5}$$

By applying the basic mathematical transformation, Eq. (5) can be rewritten in the form of:

$$S_{res} = (S_u - S_{\max})(1 - n_n^{\alpha}) \tag{6}$$

Eq. (6) satisfies the conditions that at the beginning ($n = 0$) strength reserve has the value of S_u-S_{max} and at the end ($n = N$) it is equal to 0 (see Fig. 4), since there is no reserve of strength left and the sample fails. Here the principle of normalization can be applied. If the strength reserve curves for different load levels $S_{res,i}$ are normalized with respect to the initial strength reserve, given by the Eq. (7), the obtained curves, herein referred to as "***normalized strength reserve***" curves, $S_{res,n,i}$ (Fig. 5), could be formulated by the Eq. (8):

$$S_{res,i}^{ini} = S_u - S_{\max,i}, \tag{7}$$

$$S_{res,n,i} = 1 - n_{n,i}^{\alpha,i}, \tag{8}$$

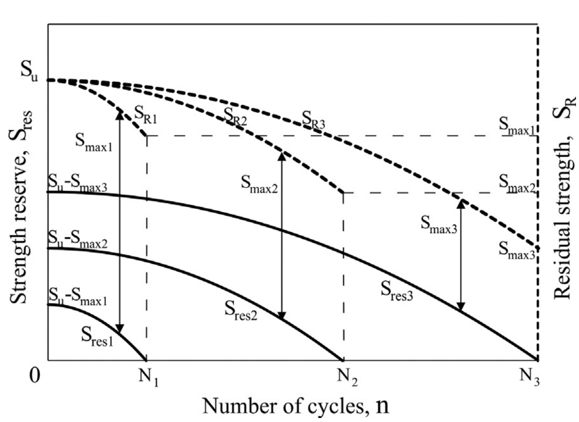

Fig. 4. Strength reserve and residual strength curves.

where $S_{res,n,i}$ is initial normalized strength reserve, $n_{n,i}$ is the fractional life and α_i is model parameter for the i-th load level.

These curves can coincide since they fulfill the same boundary conditions, i.e. the value at the beginning of the cyclic loading ($n_n = 0$) is 1 and the value at the moment of failure ($n_n = 1$) is 0. In this case, the value of parameter α must be the same for all load levels.

Using Eq. (8), the same shapes of strength degradation curves could be represented as in the case of Schaff and Davidson's model [35] (decreasing and concave up or concave down). Philippidis and Passipoularidis [39] analyzed the data from different sources and reported that in many cases the tests showed an initial decrease of strength followed by slow degradation in the middle part and steep degradation at the end of the fatigue life. In order to represent this type of degradation trend, a new parameter β is introduced into the Eq. (8). Finally, the new model, termed normalized strength reserve model (NSRM), is given by the following expression:

$$S_{res,n} = (1 - n_n^{\alpha})^{\beta} \tag{9}$$

The significance of the additional parameter is illustrated in Fig. 6, where the shape of the strength degradation is shown for different values of parameters α and β. It is shown that the Eq. (9), depending on the parameters, can describe most of the strength degradation behaviour that is reported in the literature: linear degradation; sudden drop at the end of life; and initial drop followed by slow degradation and sudden drop at the end. When parameter β is equal to 1, the model corresponds to the residual strength model given by Schaff and Davidson [35], and if both parameters are equal to 1 it corresponds to the model of Broutman and Sahu [25].

The parameters α and β can be obtained in two ways. The first is to transform the experimental residual strength data to normalized strength reserve data (NSRM data) using Eq. (10), and then fit the Eq. (9) to the transformed data. The second way is to fit the Eq. (11), which gives the residual strength curve that corresponds to the NSRM model, to the residual strength data.

$$S_{res,n} = \frac{S_R - S_{\max}}{S_u - S_{\max}} \tag{10}$$

$$S_R = S_{\max} + (S_u - S_{\max})(1 - n_n^{\alpha})^{\beta} \tag{11}$$

4. Results and discussion

In this section, the model's ability to accurately predict a monotonic strength degradation according to the experimental data is evaluated. Validating the model addresses two issues. The first

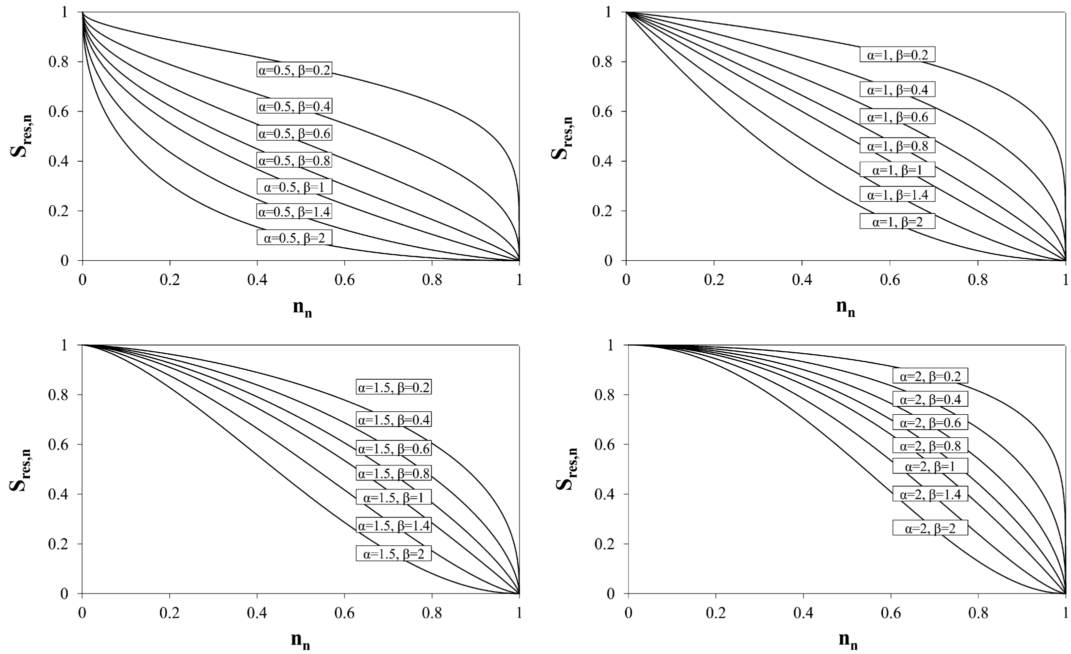

Fig. 6. The influence of the parameters α and β on the shape of the NSRM curve.

issue reassesses the assumption that the NSRM data tend to be at the same curve, i.e. the same parameters can describe strength degradation process regardless of stress level. The second issue considers the suitability of the developed model for prediction of general strength degradation trend in composites investigated in selected references. Therefore, the model is applied to the experimental data described in the Section 2. The results are presented in two different diagram types. The first one, the NSRM diagram, shows the normalized strength reserve vs. normalized number of cycles. The second one, residual strength diagram type, shows residual strength vs. endured number of cycles (logarithmic scale). The NSRM diagram is used to investigate the coincidence of experimental NSRM data obtained at different load levels, and both of them are used to evaluate the success in describing degradation trends.

The mean values of residual strength tests from [37] at different stress levels are transformed into normalized strength reserve data using Eq. (9). The NRSM curve is fitted to the generated data using regression analysis and the resulting model parameters are $\alpha = 0.489$ and $\beta = 0.290$, with $R^2 = 0.92$. The NSRM data together with the prediction curve are shown in Fig. 7. Using the resulting model parameters, the corresponding residual strength curves are derived and shown in Fig. 8. The residual strength curves are plotted for each stress level separately because of the low visibility of a large agglomerate of data points in the first part of fatigue life.

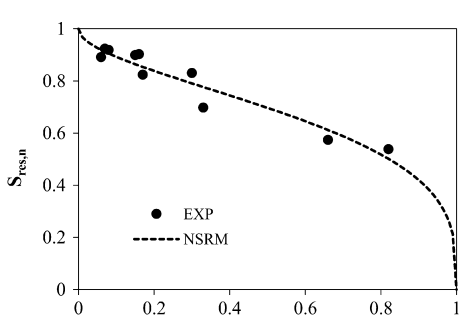

Fig. 7. NSRM diagram – prediction vs. experimental data from [37].

The NSRM data are obtained for all residual strength data from [38]. The mean values of each set of data of the same life fraction and load level are plotted in the Fig. 9, together with the NSRM prediction curve. The parameters are obtained in the same way as in the case of the first data set and are as follows: $\alpha = 0.217$ and $\beta = 0.280$, with $R^2 = 0.80$. The corresponding residual strength curves for each load level are obtained and plotted together with the experimental data in the Fig. 10.

As in the previous two cases, the NSRM curve is fitted to the experimental data from [39]. The obtained parameters are: $\alpha = 0.615$ and $\beta = 0.375$, with $R^2 = 0.88$. The plots of the NSRM curve and the corresponding residual strength curve are given in Figs. 11 and 12 respectively.

In Figs. 7, 9 and 11, where the comparison is given of the experimental NSRM data with the curves fitted using developed model; it is shown that the experimental data points lie in the vicinity of fitted curves. This is also supported with the high value of R^2, which is used as a quantitative measure of the goodness of fit. For the regression analysis and determination of R^2 values, the mean values of the residual strength are used, since the residual strength data are very scattered. This actually raises the question of whether probabilistic models should be preferred over deterministic ones. For all three datasets calculated R^2 is between 0.8 and 0.9. Additionally, after obtaining parameters α and β, the goodness of fit of corresponding residual strength curves are evaluated for each stress level separately (Figs. 8, 10 and 12). The curves are plotted using the same parameters obtained by fitting NSRM curves to the experimental data. In the case of the Ref. [38] dataset, the calculated values of R^2 are 0.88, 0.86 and 0.87 for 174 MPa, 147 MPa and 120 MPa respectively. For the experimental dataset taken from [39] R^2 is 0.91, 0.83 and 0.82 for 78.3 MPa, 63.6 MPa and 48.5 MPa respectively. The high values of R^2, as well as visual observation suggest that the curves obtained by applying developed mathematical model correlate very well with the experimental data, and it could be used for predicting the strength degradation after partial fatigue. Hence, the unique model parameters values, obtained by fitting the NSRM curve to the entire data-

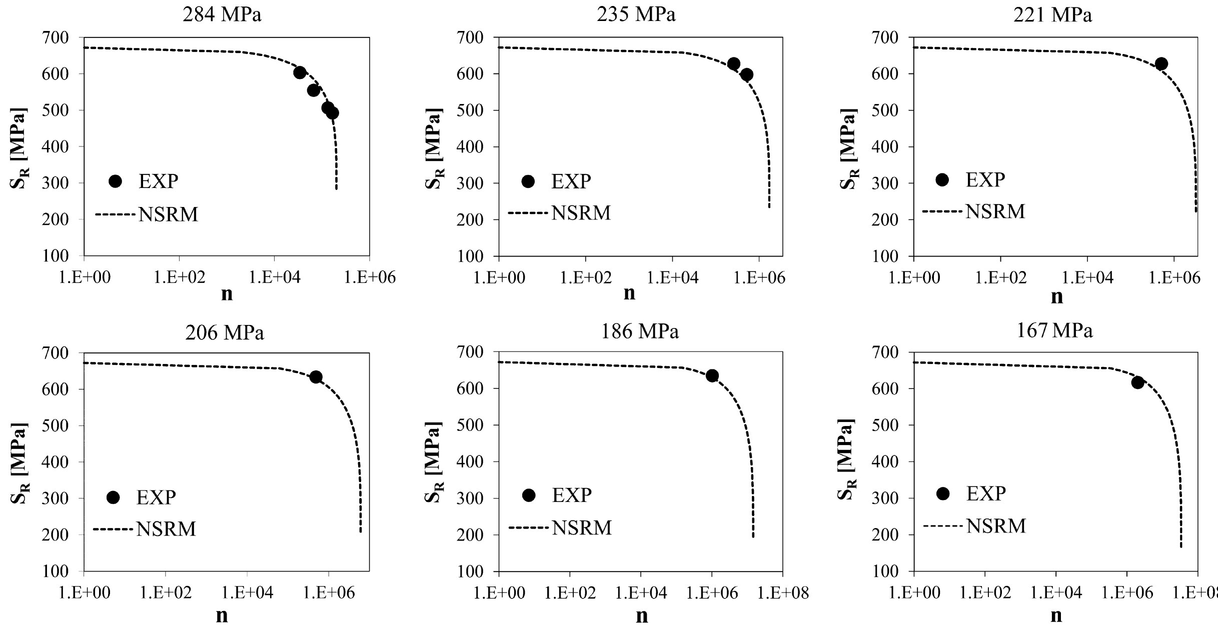

Fig. 8. The residual strength diagram – prediction vs. experimental data from [37].

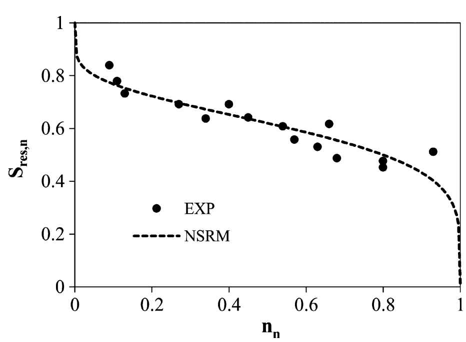

Fig. 9. NSRM diagram – prediction vs. experimental data from [38].

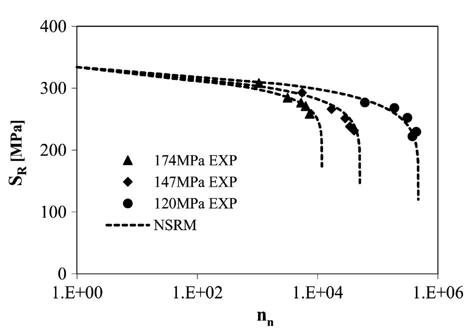

Fig. 10. The residual strength diagram – prediction vs. experimental data from [38].

set (transformed residual strength data of all load levels), could successfully predict strength degradation at each load level separately. This fact is very significant, as it allows the assumption that the single set of parameters could be used to predict the strength degradation of samples subjected to the arbitrary load level at the specific range, supposing that the S-N curve was already formulated and the stress ratio remains the same. Thus, the experimental effort for the analysis of a relatively large number of load levels of certain load spectrum could be reduced to only a few load levels.

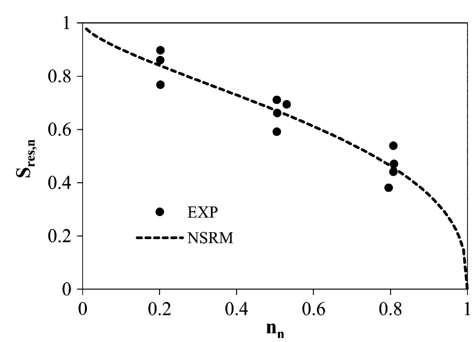

Fig. 11. NSRM diagram – prediction vs. experimental data from [39].

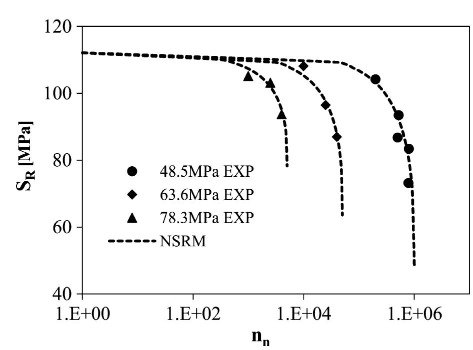

Fig. 12. The residual strength diagram – prediction vs. experimental data from [39].

Observing the trend of strength degradation, it might be noticed that the fitted curves exhibit some differences for all three datasets. In Fig. 7 the curve shows some initial loss of strength reserve, i.e. the residual strength, at the beginning of the fatigue life (in the first 5%–10%). This is followed by the slower degradation rate until 80%–90% of the fatigue life is reached, when the degradation rate increases with the very rapid loss of the last 40% of strength reserve in the last 5% of the fatigue life. In the case of the experimental results from Ref. [38] (Fig. 9), the prediction curve shows substantial loss of strength reserve at the very beginning of the fatigue life, after a small number of loading cycles. The initial drop is followed by much slower degradation and a much shallower slope of the curve. In the last 5%–10% of the fatigue life degradation accelerates considerably and, as in the previous case, a very rapid loss of strength occurs at the end of the fatigue life. Unlike the first two cases, the NSRM curve in Fig. 11 shows no significant drop of strength reserve at the beginning of the fatigue life. The degradation is almost linear until the last 10–15% of the fatigue life, when it significantly increases. Based on the experimental results from [37] and [38] and corresponding prediction curves it could be concluded that the findings of Philippidis and Passipoularidis [39] are correct: that certain composites subjected to the cyclic loading suffer initial loss of strength after which the degradation rate becomes lower and then suddenly increases in the last part of the fatigue life. This justifies the introduction of parameter β, because combining the initial loss and sudden decrease of the strength (or strength reserve) could not be described within a single parameter. The sudden failure could be represented using a high value of single parameter α (Eq. (8)), but it implies very slow preceding degradation. D'Amorre et al. have reported this kind of failure in their research [33], but because there were no experimental results at the beginning fractions of the fatigue life, the initial drop of strength could not be taken into consideration. One problem of the two parameter models is that they need data of the fractions at the beginning and the end of the fatigue life in order to represent the right shape of strength degradation. On the other hand, sometimes at the higher fractions of fatigue life the number of cycles could fall in the scatterband of the fatigue life for tested load level and premature failure could occur. This was the case in the experimental results from Ref. [37].

5. Comparison with the models from literature

The model developed here is compared with different models available in the literature, which are given in Table 2. This is done in order to evaluate the accuracy of the developed model in predicting strength degradation with the same parameters for all load levels. The available models are applied to the experimental data from [38] and [39]. In the case of the NSRM model the parameters are obtained using the whole experimental data set. They are calculated by fitting Eq. (9) to the NSRM data transformed from residual strength data using Eq. (10). These parameters are later used for calculating the residual strength curve applying the Eq. (11). For all other models, the parameters are obtained for each load level separately, in order to achieve the best possible fit, although it is seen that in some cases they do not differ much. All parameter values are obtained by applying regression analysis, and given in Table 2. The comparison of the developed model, more precisely the corresponding residual strength curves, with the most widely used models from the literature, is given in Fig. 13.

Comparing the curves with experimental data, makes it clear that not all of the models correctly describe degradation trends of the selected data. When the strength as monotonically decreasing function of the number of cycles that starts from static strength and ends with the strength equal to maximum load of the applied load level is considered, it becomes evident that the strength degradation of the observed materials is nonlinear. In [39], the authors argued for the use of nonlinear models with parameters to be determined by experimental investigation. They proposed the linear BS model to be used to predict the residual strength, since it does not require experimental results and was always on the safe side for the experimental data used in their research. However, in the case of the experimental results from [38], up to one third of the fatigue life, it is shown in Fig. 13 that this model is not on the safe side. SD and HA models show a similar kind of strength degradation with the increasing degradation rate from the beginning to the end of the fatigue life. In the case of the results from [38] they do not differ much from the BS model prediction, but in the case of the results from [39] they predict a higher residual strength than the linear model. However, in both cases, all of these three models fail to represent the right shape of the curve, i.e. the degradation trend. On the other hand, comparison with

Table 2
NSRM and residual strength models and obtained parameters for the experimental data from [38] and [39].

Reference	Denotation	Equation	[38]		[39]	
			Smax	α,β	Smax	α,β
Eq. (11)	NSRM	$S_R = S_{\max} + (S_u - S_{\max})\left[1 - \left(\frac{n}{N}\right)^{\alpha}\right]^{\beta}$	α = 0.217	β = 0.280	α = 0.615	β = 0.375
Hashin Ref. [31]	HA	$S_R^{\alpha} = S_u^{\alpha} - (S_u^{\alpha} - S_{\max}^{\alpha})\left(\frac{n}{N}\right)$	120 MPa	α = 2.57	48.5 MPa	α = 3.72
			147 MPa	α = 1.67	63.6 MPa	α = 5.09
			174 MPa	α = 2.06	78.3 MPa	α = 6.85
Schaff & Davidson Ref. [35]	SD	$S_R = S_u - (S_u - S_{\max})\left(\frac{n}{N}\right)^{\alpha}$	120 MPa	α = 1.43	48.5 MPa	α = 1.77
			147 MPa	α = 1.10	63.6 MPa	α = 2.03
			174 MPa	α = 1.19	78.3 MPa	α = 1.84
Phillipidis & Passipoularidis Ref. [39]	PP	$S_R = S_u - (S_u - S_{\max})\left(\frac{n}{N}\right)^{\alpha \cdot \exp(\beta \frac{n}{N})}$	120 MPa	α = 0.488	48.5 MPa	α = 0.715
				β = 2.13		β = 1.74
			147 MPa	α = 0.546	63.6 MPa	α = 0.94
				β = 1.53		β = 1.33
			174 MPa	α = 0.647	78.3 MPa	α = 0.645
				β = 1.42		β = 1.90
Adam et al. Ref. [19]	AD	$S_R = S_{\max} + (S_u - S_{\max})\cdot \cdot\left[1 - \left(\frac{\log n - 0.5}{\log N - 0.5}\right)^{\alpha}\right]^{1/\beta}$	120 MPa	α = 1.40	48.5 MPa	α = 5.87
				β = 5.28		β = 3.44
			147 MPa	α = 2.68	63.6 MPa	α = 8.41
				β = 3.12		β = 2.48
			174 MPa	α = 2.99	78.3 MPa	α = 3.01
				β = 317		β = 3.70
Broutman & Sahu Ref. [25]	BS	$S_R = S_u - (S_u - S_{\max})\left(\frac{n}{N}\right)$	no parameters (linear model)			
Sarkani et al. Ref. [34]	SA	$S_R^{\alpha} = S_u^{\alpha} - (S_u^{\alpha} - S_{\max}^{\alpha})\left(\frac{n}{N}\right)^{\beta}$	120 MPa	α = 16.90	48.5 MPa	α = 7.64
				β = 0.022		β = 0.333
			147 MPa	α = 8.96	63.6 MPa	α = 7.19
				β = 0.148		β = 0.640
			174 MPa	α = 8.25	78.3 MPa	α = 17.37
				β = 0.285		β = 0.232

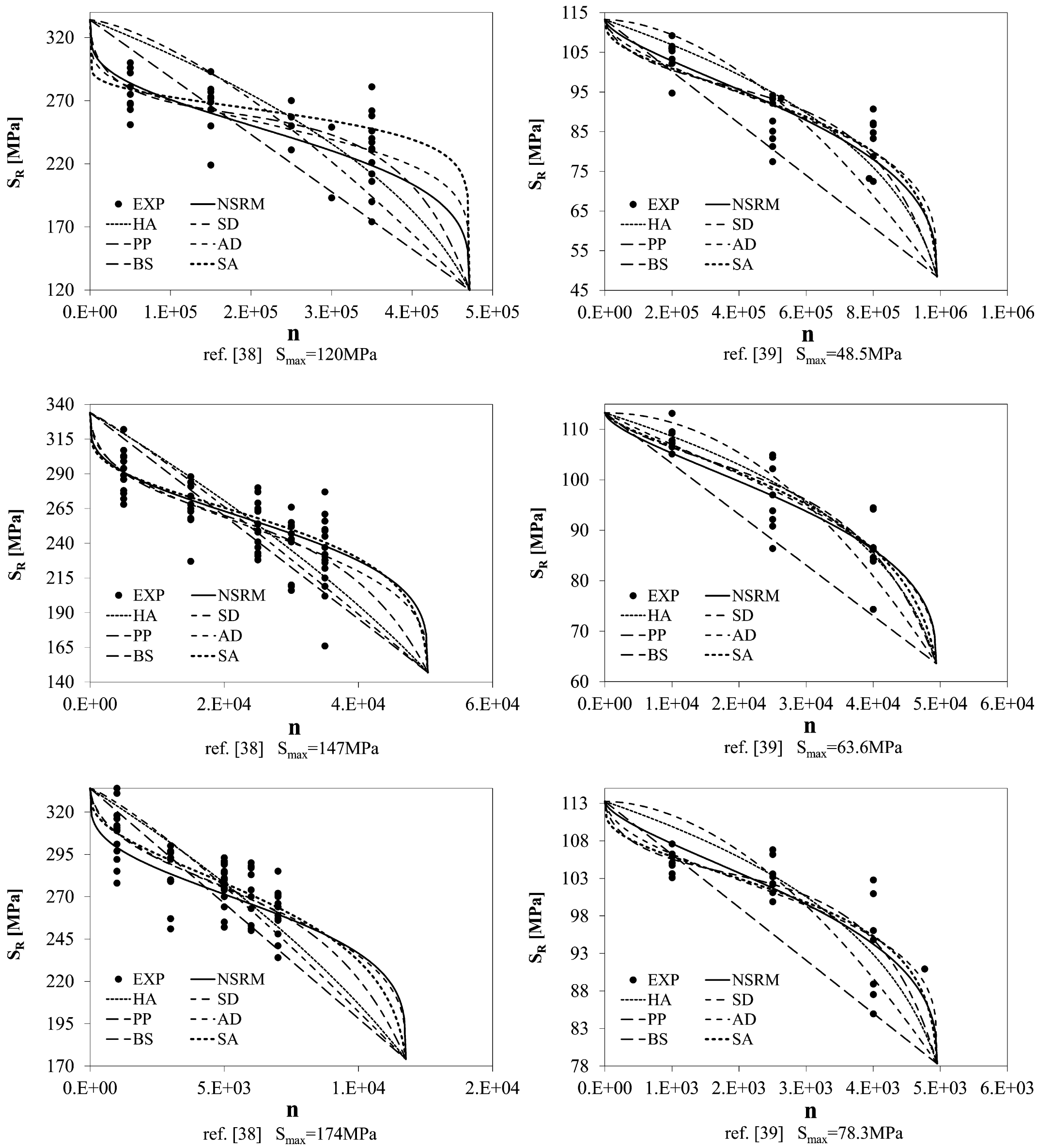

Fig. 13. Comparison of the residual strength curves obtained by application of the developed model and standard models from literature.

the experimental data shows that the NSRM model follows degradation trends much more accurately. The curve passes mainly through the middle of the observed data points and, unlike the HA, SA and BS model, never goes outside the scatterband of the observed residual strengths. The NSRM model represents one of the most conservative models in the first part of the fatigue life and a non-conservative one in the last part. Its degradation trend matches SA, AD and, to a slightly lesser extent, the PP model. These models were rated by Philippidis and Passipoularidis [39] as some of the most accurate residual strength models. All of them have two parameters to be determined experimentally and can take into consideration the initial drop of strength. The difference between the NSRM model and the models from literature can be found in Table 2. In the case of the models from literature, the values of their parameters in this analysis show that there is no linear connection between the load level and parameter values. Moreover, the parameter α of the SA model is almost identical for two stress levels, but significantly different at the third level. This is found for both experimental data sets. On the other hand, the NSRM model shows a very good shape of the curve for all stress levels using a single set of parameter values. This could be attributed to the application of the strength reserve approach.

It must be pointed out that all the data used to validate the proposed model were obtained by testing under single stress ratio and layup configuration, which implies an extensive experimental effort is needed for full characterization of the material. However, the normalized nature of the model proposed here offers the possibility, like the model proposed by Harris [17], to be coupled with the normalized fatigue life models and used in techniques such as progressive fatigue damage modeling. The major advantage of the model proposed here is that, by using a single set of parameters for all stress levels, it matches the strength degradation trend observed in analyzed experimental data better than the best fits of other models, obtained for each stress level separately. Thus, obtained parameters could be used for stress levels that were not tested experimentally, which reduces the required experimental effort. This is of great importance for accurate analysis and failure prediction of composites under fatigue loading, especially of those subjected to occasional overloads during the service life. On the other hand, it was shown in Fig. 6 that the proposed equation could represent other types of degradation trends seen in composites, such as sudden death and wear-out.

6. Conclusions

The mathematical NSRM model was developed to predict strength degradation of the composites subjected to cyclic loading with constant amplitude. The model is phenomenological, and based on the difference between the residual strength and the maximum load in the loading cycle. The two-parameter equation was proposed, which could account for the initial drop of strength at the beginning of the fatigue life, as well as sudden drop in the latter part of it. The mathematical nature of the developed model allows for various shapes of degradation curve, depending on the values of the parameters. It requires a single set of parameters values to predict residual strength regardless of load level.

It was validated using experimental data from three different sources, and compared to the standard models from the literature. A high correlation with the experimental data from the literature was observed. The accuracy of the model is emphasized by matching the shape of the curve of the models that are described in the literature as some of the most accurate. The main advantage of the NSRM model is that it allows parameters to be determined using experimental results obtained from testing at only a few levels of cyclic loading and subsequent prediction of strength degradation for load levels that are not tested experimentally. Therefore, the developed model could significantly reduce the required experimental effort for determining strength degradation curves in cases where the loading spectrum consists of many loading levels. This makes the model suitable for use as a basis to predict degradation of composites at variable amplitude loadings.

Further research will focus on the suitability of this model for the cases of compressive or changing sign cyclic loading as well as for investigating the influence of load ratio on the values of model parameters. Finally, the model could be used to predict strength degradation and, subsequently, the fatigue life of composites subjected to variable amplitude loading.

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