

A Globally Convergent State Observer for Multimachine Power Systems with Lossy Lines

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Abstract: We present the first solution to the problem of estimation of the state of multimachine power systems with lossy transmission lines. We consider the classical three-dimensional “flux-decay” model of the power system and assume that the active and reactive power as well as the rotor angle and excitation voltage at each generator is available for measurement—a scenario that is feasible with current technology. The design of the observer relies on two recent developments proposed by the authors: a parameter estimation based approach to the problem of state estimation and the use of the dynamic regressor extension and mixing technique to estimate these parameters. Thanks to the combination of these techniques it is possible to overcome the problem of lack of persistent excitation that stymies the application of standard observer designs. Simulation results illustrate the performance of the proposed observer.

Keywords: Electric power systems, Observers, Nonlinear systems, Dynamic state estimation

1. INTRODUCTION

Power systems are experiencing major changes and challenges, such as an increasing amount of power-electronics-interfaced equipment, growing transit power flows and fluctuating (renewable) generation, see (Winter et al., 2015). Therefore power systems are operated under more and more stressed conditions and, thus, closer to their stability limits as ever before, see (Winter et al., 2015). Their dynamics become faster, more uncertain and also more volatile. Fast and accurate monitoring of the system states is crucial in order to ensure a stable and reliable system operation, see (Zhao et al., 2019). This, however, implies that the conventional monitoring approaches based on steady-state assumptions are no longer appropriate and instead novel *dynamic state estimation (DSE)* tools have to be developed.

By recognizing this need, DSE has become a very active research area in the past years, see (Zhao et al., 2019; Singh and Pal, 2018). As of today, the prevalent algorithms in the literature for DSE are Kalman Filter techniques, including Extended Kalman Filters, see (Ghahremani and Kamwa, 2011; Paul et al., 2018) and Unscented Kalman Filters,

see (Valverde and Terzija, 2011; Wang et al., 2011; Anagnostou and Pal, 2017). The main reasoning behind using these techniques is that they are in principle applicable to nonlinear systems, see (Julier and Uhlmann, 1997; Wan and Van Der Merwe, 2000), such as power system models. However, a major drawback of the aforementioned results is that the estimator convergence is only shown empirically, *e.g.*, via simulations, but no rigorous convergence guarantees are provided.

Recently, power system state observers have been proposed in (Rinaldi et al., 2017, 2018, 2019) based on sliding mode techniques and their convergence has been established under certain assumptions. Yet, all these results neglect the voltage dynamics and assume purely inductive transmission lines. In (Anagnostou et al., 2018) an observer-based anomaly scheme has been derived using a detailed linearized power system model in combination with a linear time-varying observer.

In this paper we consider a large-scale power system consisting of n synchronous generators represented by the standard three-dimensional “flux-decay” model, see (Kundur, 1994; Sauer et al., 2017), and interconnected through a transmission network, which we assume to be lossy. That is, we explicitly take into account the presence of transfer conductances. It is well known that

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the dynamics of power systems with lossy transmission lines is considerably more complicated than the lossless one—see (Anderson and Fouad, 2003; Ortega et al., 2005) for a discussion on this issue.

The main contribution of the present work is the proof that using the *measurements* of active and reactive power as well as rotor angle and excitation voltage at each generator—a reasonable assumption given the latest technology, see (Yang et al., 2007)—it is possible to recover *the full state* of a multimachine power system, even in the presence of lossy lines. An important observation that is made in the paper is that, under the assumed measurement scenario, the problem of estimation of the generator voltages can be recast as estimation of the state of a *linear time-varying* (LTV) system, to which classical state estimation techniques, *e.g.*, Kalman-Bucy filters (Anderson, 1971), can be applied. Unfortunately the associated LTV system is not uniformly completely observable (UCO), a necessary condition to ensure convergence of the estimated states, see (Rueda-Escobedo et al., 2019).

To overcome this problem we invoke two recent results reported by the authors.

- (1) Following the parameter estimation based observer (PEBO) design proposed in (Ortega et al., 2015), we reformulate the problem of state observation into one of *parameter* estimation.
- (2) The unknown parameters are reconstructed using the dynamic regressor extension and mixing (DREM) procedure (Aranovskiy et al., 2017).

See (Ortega et al., 2019; Pyrkin et al., 2019) for two recent applications of these combined techniques. Two outstanding features of DREM estimators that are exploited in the paper are that, on one hand, no appeal is made to persistent excitation (PE) arguments to prove its convergence¹. Instead, a condition of non-square integrability of a scalar signal is imposed in DREM, see (Aranovskiy et al., 2017) for further details. On the other hand, as shown in (Gerasimov et al., 2018), a simple modification to DREM allows to achieve the very important feature of *finite-time convergence* (FTC) under the weakest sufficient excitation assumption (Kreisselmeier and Rietze-Augst, 1990). The convergence conditions mentioned above are, of course, necessary since as is well-known some excitation assumptions are needed to ensure the success of state observers (Besançon, 2007; Bernard, 2019) and parameter estimators (Ljung, 1987; Sastry and Bodson, 1989).

2. SYSTEM MODEL AND STATE OBSERVER PROBLEM FORMULATION

As standard in centralized DSE, see (Zhao et al., 2019), we consider a Kron-reduced power system consisting of $n > 1$ interconnected machines and with the dynamics of the i -th generator described by the classical third order model² (Anderson and Fouad, 2003; Kundur, 1994; Sauer et al., 2017)

¹ We recall that PE of the classical linear regression model is equivalent to non UCO of the associated LTV system (Sastry and Bodson, 1989).

² To simplify the notation, whenever clear from the context, the qualifier “ $i \in \bar{n}$ ” will be omitted in the sequel.

$$\begin{aligned} \dot{\delta}_i &= \omega_i, \\ M_i \dot{\omega}_i &= -D_{mi} \omega_i + \omega_0 (P_{mi} - P_{ei}), \\ \tau_i \dot{E}_i &= -E_i - (x_{di} - x'_{di}) I_{di} + E_{fi} + \nu_i, \quad i \in \bar{n} := \{1, \dots, n\}, \end{aligned} \quad (1)$$

where the state variables are the rotor angle $\delta_i \in \mathbb{R}$, rad, the speed deviation $\omega_i \in \mathbb{R}$ in rad/sec and the generator quadrature internal voltage $E_i \in \mathbb{R}_+$, I_{di} is the d axis current, P_{ei} is the electromagnetic power, the voltages E_{fi} and ν_i are the constant voltage component applied to the field winding, and the control voltage input. Furthermore, D_{mi} , M_i , P_{mi} , τ_i , ω_0 , x_{di} and x'_{di} are positive parameters.

The active P_{ei} and reactive Q_{ei} power are defined as

$$P_{ei} = E_i I_{qi}; \quad Q_{ei} = E_i I_{di}, \quad (2)$$

where I_{qi} is the q axis current.

These currents establish the connections between the machines and are given by

$$\begin{aligned} I_{qi} &= G_{mii} E_i + \sum_{j=1, j \neq i}^n E_j Y_{ij} \sin(\delta_{ij} + \alpha_{ij}) \\ I_{di} &= -B_{mii} E_i - \sum_{j=1, j \neq i}^n E_j Y_{ij} \cos(\delta_{ij} + \alpha_{ij}), \end{aligned} \quad (3)$$

where we defined $\delta_{ij} := \delta_i - \delta_j$ and the constants $Y_{ij} = Y_{ji}$ and $\alpha_{ij} = \alpha_{ji}$ are the admittance magnitude and admittance angle of the power line connecting nodes i and j , respectively. Furthermore, G_{mii} is the shunt conductance and B_{mii} the shunt susceptance at node i . Finally, combining (1), (2) and (3) results in the well-known compact form (Langarica et al., 2015)

$$\begin{aligned} \dot{\delta}_i &= \omega_i \\ \dot{\omega}_i &= -D_i \omega_i + P_i - d_i \left[G_{mii} E_i^2 - E_i \sum_{j=1, j \neq i}^n E_j Y_{ij} \sin(\delta_{ij} + \alpha_{ij}) \right] \\ \dot{E}_i &= -a_i E_i + b_i \sum_{j=1, j \neq i}^n E_j Y_{ij} \cos(\delta_{ij} + \alpha_{ij}) + u_i, \end{aligned} \quad (4)$$

where we have defined the signals

$$u_i := \frac{1}{\tau_i} (E_{fi} + \nu_i)$$

and the positive constants

$$\begin{aligned} D_i &:= \frac{D_{mi}}{M_i}, \quad P_i := d_i P_{mi}, \quad d_i := \frac{\omega_0}{M_i} \\ a_i &:= \frac{1}{\tau_i} [1 - (x_{di} - x'_{di}) B_{mii}], \quad b_i := \frac{1}{\tau_i} (x_{di} - x'_{di}). \end{aligned}$$

To formulate the observer problem we consider that all parameters are known, and make the following assumption on the available *measurements*.

Assumption 1. The signals δ_i , u_i , P_{ei} and Q_{ei} of all generating units are measurable.

As usual in observer problems (Besançon, 2007; Bernard, 2019) we assume that u_i is bounded and such that all state trajectories are *bounded*.

Problem Formulation: Consider the multimachine power system (4), verifying Assumption 1. Design an observer³

³ For a set of scalar quantities x_i we define the vector $x := \text{col}(x_1, x_2, \dots, x_n)$.

$$\dot{\chi} = F(\chi, \delta, P_e, Q_e); \begin{bmatrix} \hat{E} \\ \hat{\omega} \end{bmatrix} := H(\chi, \delta, P_e, Q_e, \hat{E}, \hat{\omega}),$$

such that

$$\lim_{t \rightarrow \infty} \begin{bmatrix} \tilde{E}(t) \\ \tilde{\omega}(t) \end{bmatrix} = 0, \quad (5)$$

where we define the estimation errors $(\tilde{\cdot}) := (\hat{\cdot}) - (\cdot)$.

Although the centralized measurement of the rotor angles δ_i is unlikely in applications, the observer problem without this assumption is a daunting task. Even if this transfer of information is possible, another practical consideration that is neglected in the problem formulation is the difficulty of synchronizing these measurements (Sauer et al., 2017).

3. MAIN RESULT

Before presenting the proposed observer we first give in this section an LTV systems perspective of the voltage dynamics and show its fundamental lack of observability. To overcome the latter, we then adopt a PEBO perspective for the state observation problem and apply the DREM procedure to estimate the unknown parameters.

3.1 An LTV description of the voltage dynamics

To simplify the notation we find convenient to introduce the following notation

$$\Delta_{ij}(t) := \delta_i(t) - \delta_j(t) + \alpha_{ij}$$

$$A(t) := \begin{bmatrix} -a_1 & b_1 Y_{12} \cos \Delta_{12}(t) & \dots & b_1 Y_{1n} \cos \Delta_{1n}(t) \\ b_2 Y_{21} \cos \Delta_{21}(t) & -a_2 & \dots & b_2 Y_{2n} \cos \Delta_{2n}(t) \\ \vdots & \vdots & \ddots & \vdots \\ b_n Y_{n1} \cos \Delta_{n1}(t) & b_n Y_{n2} \cos \Delta_{n2}(t) & \dots & -a_n \end{bmatrix} \quad (6)$$

Given this notation we observe that we can write the voltage equations of (4) as an LTV system

$$\dot{E} = A(t)E + u. \quad (7)$$

We underscore that, under the standing assumptions, $A(t)$ is *measurable* and obviously, bounded.

The following simple lemma is instrumental for our further developments.

Lemma 2. There exists a *measurable* matrix $C(t) := C(P_e(t), Q_e(t), \delta(t)) \in \mathbb{R}^{n \times n}$ such that

$$C(t)E = 0 \quad (8)$$

Proof. From (2) we have that

$$P_e I_d - Q_e I_q = 0.$$

Clearly, the equations (3)—which are linearly dependent on E —may be written in the compact form

$$I_q = S(\delta)E, \quad I_d = T(\delta)E, \quad (9)$$

for some suitably defined $n \times n$ matrices $S(\delta)$ and $D(\delta)$. The proof is completed replacing (9) in the identity above and defining

$$C(P_e, Q_e, \delta) := \begin{bmatrix} P_{e1} T_1^\top(\delta) - Q_{e1} S_1^\top(\delta) \\ \vdots \\ P_{en} T_n^\top(\delta) - Q_{en} S_n^\top(\delta) \end{bmatrix}, \quad (10)$$

where $T_i^\top(\delta)$, $S_i^\top(\delta)$ are the rows of the matrices $D(\delta)$ and $S(\delta)$, respectively. $\square\square\square$

Now, we recall that the state trajectories of the system $\dot{x} = A(t)x$ with initial condition $x(t_0) \in \mathbb{R}^n$ satisfy (Demidovich, 1967; Rugh, 1996)

$$x(t) = \Phi(t, t_0)x(t_0), \quad \forall t \geq t_0. \quad (11)$$

3.2 A PEBO approach to the estimation of the voltage

The lemma below illustrates how, applying the PEBO approach proposed in (Ortega et al., 2015), we can generate a linear regression equation (LRE) where estimation of the unknown parameters leads to estimation of the unknown state E .

Lemma 3. Consider the dynamic equations of the voltage (6)-(8) and the dynamic extension

$$\dot{\xi}_E = A(t)\xi_E + u; \quad \dot{\Phi} = A(t)\Phi, \quad \Phi(0) = \mathcal{I}_n. \quad (12)$$

There exists a *constant* vector $\theta \in \mathbb{R}^n$, and *measurable* signals $y \in \mathbb{R}^n$ and $\psi \in \mathbb{R}^{n \times n}$ such that

$$E = \xi_E - \Phi\theta \quad (13)$$

$$y = \psi\theta. \quad (14)$$

Proof. Define the signal $e := \xi_E - E$. From (7) and (12) we see that e satisfies

$$\dot{e} = A(t)e.$$

From the equation above and the properties of the state transition matrix Φ of $A(t)$ given in (11), we conclude that there exists a constant vector⁴ $\theta \in \mathbb{R}^n$ such that $e = \Phi\theta$ and, consequently, (13) holds.

To establish (14) we multiply (13) by $C(t)$ and, recalling (8), obtain the equation

$$C(t)\xi_E = C(t)\Phi\theta.$$

The proof is completed defining

$$y := C(t)\xi_E; \quad \psi := C(t)\Phi. \quad (15)$$

$\square\square\square$

3.3 Generation via DREM of n scalar LRE for the unknown parameters θ_i

In view of Lemma 3 the only remaining step to construct the observer for E is to, proceeding from the linear regression (14), propose an estimator for θ . Standard gradient or least squares estimators will not ensure (exponential) convergence (Sasthy and Bodson, 1989). To bypass this difficulty we invoke the DREM parameter estimation procedure proposed in (Aranovskiy et al., 2017).

Although the construction of DREM allows for the use of general, LTV, \mathcal{L}_∞ -stable operators, for the sake of simplicity we consider here the use of simple LTI filters. Towards this end, we propose a stable transfer matrix $F(s) \in \mathbb{R}^{n \times n}(s)$, and define the signals

$$\mathbf{Y} := F(p)y \in \mathbb{R}^n; \quad \Psi := F(p)\psi \in \mathbb{R}^{n \times n}$$

$$\mathcal{Y} := \text{adj}\{\Psi\}\mathbf{Y} \in \mathbb{R}^n; \quad \Delta := \det\{\Psi\} \in \mathbb{R}, \quad (16)$$

where $p := \frac{d}{dt}$ and $\text{adj}\{\cdot\}$ is the adjugate operator.

Lemma 4. Consider the LRE (14) and the signals (16). Then, the following *scalar* LREs hold

$$\mathcal{Y}_i = \Delta\theta_i. \quad (17)$$

⁴ Clearly, we have that $\theta := e(0)$.

Proof. Applying the filter $F(p)$ to the LRE (14) we obtain

$$\mathbf{Y} = \Psi\theta + \boldsymbol{\eta},$$

where $\boldsymbol{\eta}$ is an exponentially decaying term that, following the standard procedure, is neglected in the sequel. The proof is completed by multiplying the equation above by $\text{adj}\{\Psi\}$ and recalling that for any, possibly singular, $n \times n$ matrix B we have $\text{adj}\{B\}B = \det\{B\}\mathcal{I}_n$. $\square\square\square$

3.4 Proposed state observers

We are now in position to present our main result: two globally convergent observers for the state of the multimachine power system (4). The first one ensures *asymptotic* convergence while the latter guarantees FTC. In both cases, the required excitation conditions are rather weak.

Proposition 5. Consider the multimachine power system (4), verifying Assumption 1. Fix an $n \times n$ stable transfer matrix $F(s)$ and $2n$ positive numbers γ_i and k_{ω_i} . The voltage state observer defined by (6), (10), (12), (15), (16), together with

$$\dot{\hat{\theta}}_i = -\gamma_i \Delta(\Delta\hat{\theta}_i - \mathcal{Y}_i); \hat{E} = \xi_E - \Phi\hat{\theta}, \quad (18)$$

and the speed observer given by

$$\begin{aligned} \dot{\xi}_{\omega_i} &= -D_i\hat{\omega}_i + P_i - d_i P_{ei} - k_{\omega_i}\hat{\omega}_i \\ \hat{\omega}_i &= \xi_{\omega_i} + k_{\omega_i}\delta_i, \end{aligned} \quad (19)$$

ensure (5), with all signals bounded, provided $\Delta \notin \mathcal{L}_2$.

Proof. Replacing (14) in the parameter estimator equation yields

$$\dot{\tilde{\theta}}_i = -\gamma_i \Delta^2 \tilde{\theta}_i. \quad (20)$$

Given the standing assumption on Δ we have that $\tilde{\theta}(t) \rightarrow 0$. Hence, invoking (13) and (18), this implies that $\tilde{E}(t) \rightarrow 0$.

To prove the convergence of the speed estimator notice that, using (2) and (3), the rotor speed dynamics (4) may be written as

$$\dot{\omega}_i = -D_i\omega_i + P_i - d_i P_{ei}.$$

Then, compute from (4) and (19) the error dynamics

$$\dot{\tilde{\omega}}_i = -(D_i + k_{\omega_i})\tilde{\omega}_i.$$

This completes the proof of the proposition. $\square\square\square$

As shown in (Gerasimov et al., 2018), a simple modification to DREM allows to achieve FTC for the voltage observer, under the weakest *sufficient excitation* assumption (Kreisselmeier and Rietze-Augst, 1990), which is articulated in the assumption below. For ease of presentation, and without loss of generality, we assume that all the adaptation gains γ_i in the parameter estimator (18) are equal to $\gamma > 0$. The proof is omitted for space reasons.

Assumption 6. Fix a small constant $\mu \in (0, 1)$. There exists a time $t_c > 0$ such that

$$\int_0^{t_c} \Delta^2(\tau) d\tau \geq -\frac{1}{\gamma} \ln(1 - \mu). \quad (21)$$

Proposition 7. Consider the multimachine power system (4), verifying Assumption 1. Fix an $n \times n$ stable transfer matrix $F(s)$, a positive numbers γ and $\mu \in (0, 1)$. The

voltage state observer defined by (6), (10), (12), (15), (16), together with

$$\begin{aligned} \dot{\hat{\theta}}_i &= -\gamma \Delta(\Delta\hat{\theta}_i - \mathcal{Y}_i); \dot{w} = -\gamma \Delta^2 w, w(0) = 1 \\ \hat{E} &= \xi_E - \Phi \frac{1}{1 - w_c} [\hat{\theta} - w_c \hat{\theta}(0)], \end{aligned} \quad (22)$$

where w_c is defined via the clipping function

$$w_c = \begin{cases} w & \text{if } w < 1 - \mu \\ 1 - \mu & \text{if } w \geq 1 - \mu, \end{cases}$$

ensures

$$\tilde{E}(t) = 0, \forall t \geq t_c$$

with all signals bounded, provided $\Delta(t)$ verifies Assumption 6.

The following remarks are in order.

- (R1) The stability properties established in Propositions 5 and 7 are “trajectory-dependent”, in the sense that they pertain only to the trajectory generated for the given initial conditions. This means that the flow of the observer dynamics may contain unbounded trajectories, and the appearance of a perturbation may drive our “good” trajectory towards a “bad” one. This is, of course, a robustness problem that needs to be further investigated.
- (R2) It is interesting to note that the “excitation” condition imposed by Assumption 6 can be weakened increasing the adaptation gain γ . However, as is well-known, the use of fast adaptation entails a series of undesirable phenomena. On the other hand, if the assumption of $\Delta \notin \mathcal{L}_2$ is strengthened to Δ being PE, then the convergence is *exponential*.

4. SIMULATION RESULTS

For simulation we consider a two-machine system, see for instance (Ortega et al., 2005). The dynamics of the system result in the sixth-order model

$$\begin{cases} \dot{\delta}_1 &= \omega_1, \\ \dot{\omega}_1 &= -D_1\omega_1 + P_1 - G_{11}E_1^2 - Y_{12}E_1E_2 \sin(\delta_{12} + \alpha_{12}) \\ \dot{E}_1 &= -a_1E_1 + b_1E_2 \cos(\delta_{12} + \alpha_{12}) + E_{f1} + \nu_1; \\ \dot{\delta}_2 &= \omega_2, \\ \dot{\omega}_2 &= -D_2\omega_2 + P_2 - G_{22}E_2^2 + Y_{21}E_1E_2 \sin(\delta_{12} + \alpha_{12}) \\ \dot{E}_2 &= -a_2E_2 + b_2E_1 \cos(\delta_{21} + \alpha_{21}) + E_{f2} + \nu_2, \end{cases} \quad (23)$$

with the current equations defined as

$$I_{q1} = G_{11}E_1 + E_2Y_{12} \sin(\delta_{12} + \alpha_{12}) \quad (24)$$

$$I_{d1} = -B_{11}E_1 - E_2Y_{12} \cos(\delta_{12} + \alpha_{12}) \quad (25)$$

$$I_{q2} = G_{22}E_2 + E_1Y_{21} \sin(\delta_{21} + \alpha_{21}) \quad (26)$$

$$I_{d2} = -B_{22}E_2 - E_1Y_{21} \cos(\delta_{21} + \alpha_{21}). \quad (27)$$

In this case we have that

$$\begin{aligned} A(t) &= \begin{bmatrix} -a_1 & b_1 \cos(\delta_{12}(t) + \alpha_{12}) \\ b_2 \cos(\delta_{21}(t) + \alpha_{21}) & -a_2 \end{bmatrix} \\ S(\delta) &= \begin{bmatrix} G_{11} & Y_{12} \sin(\delta_{12} + \alpha_{12}) \\ Y_{21} \sin(\delta_{21} + \alpha_{21}) & G_{22} \end{bmatrix} \\ T(\delta) &= \begin{bmatrix} -B_{11} & -Y_{12} \cos(\delta_{12} + \alpha_{12}) \\ -Y_{21} \cos(\delta_{21} + \alpha_{21}) & -B_{22} \end{bmatrix}. \end{aligned}$$

For the observer design we selected the simplest filter

$$F(p) = \begin{bmatrix} 1 & 0 \\ \frac{k}{p+k} & 0 \end{bmatrix},$$

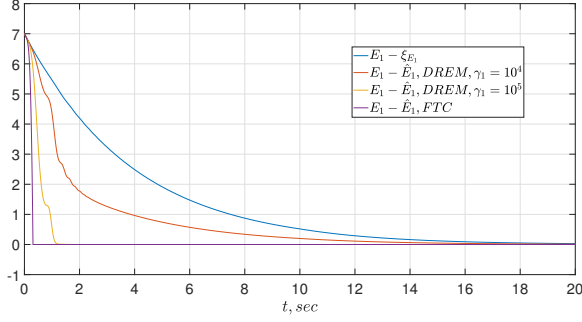


Fig. 1. Transients of the first voltage observation error $E_1 - \hat{E}_1$ for DREM and FTC observers with a 30% load change at $t = 10$ sec

with $k > 0$. The parameters of the model (23) are given in Table 1.

Table 1. System parameters

Parameter	Initial values	After load change
Y_{12}	0.1032	0.1032
Y_{21}	0.1032	0.1032
b_1	0.0223	0.02236
b_2	0.0265	0.0265
D_1	1	1
D_2	0.2	0.2
ν_1	1	1
ν_2	1	1
B_{11}	-0.4373	-0.5685
B_{22}	-0.4294	-0.5582
G_{11}	0.0966	0.1256
G_{22}	0.0926	0.1204
a_1	0.2614	0.2898
a_2	0.2532	0.2864
P_1	28.22	28.22
P_2	28.22	28.22
E_{f1}	0.2405	0.2405
E_{f2}	0.2405	0.2405

Simulations were carried out for the observers proposed in Proposition 5 (DREM) and Proposition 7 (FTC). We consider a scenario of a 30% load change, that happens at $t = 10$ sec. It is clear from the proof of Proposition 5 that this load change does not affect the speed observation. We used the following initial conditions for the system $E(0) = \text{col}(7, 6)$, and zero for all remaining states. All initial conditions for the observers were also set equal to zero. For the DREM observer two different adaptation gains γ_i were tried. For the FTC observer we set $\mu = 0.1$ of Assumption 6. For the speed observer we selected different values of the coefficient k_{ω_i} . For the sake of comparison we also present the behavior of the signals ξ_{E_i} that—since the matrix $A(t)$ corresponds to an asymptotically stable system—provides also an estimate of the voltage \hat{E} . The results of the simulations are shown in Fig. 1-Fig. 4. As expected by the theory, the speed of convergence of the DREM estimator for the voltage increases for large values of γ_i , with a similar behavior of the speed error with respect to k_{ω_i} . Also, it is seen that the FTC estimator has the fastest convergence, and this happens in finite-time. It is interesting to remark that the behavior of the observers is highly insensitive to the load change, as it is hardly visible in \hat{E}_1 and \hat{E}_2 .

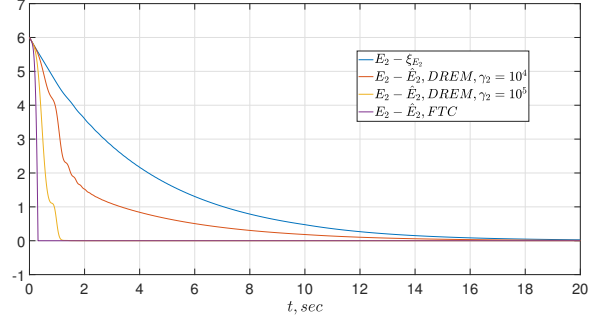


Fig. 2. Transients of the second voltage observation error $E_2 - \hat{E}_2$ for DREM and FTC observers with a 30% load change at $t = 10$ sec

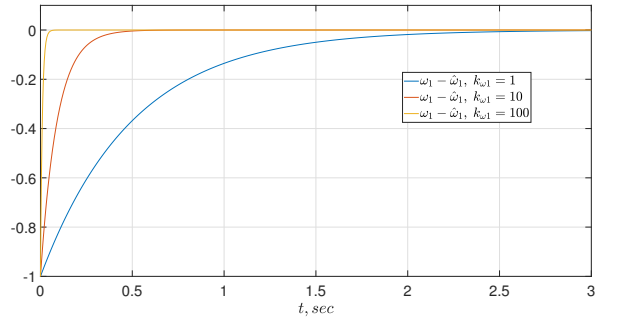


Fig. 3. Transients of the first speed observation error $\omega_1 - \hat{\omega}_1$

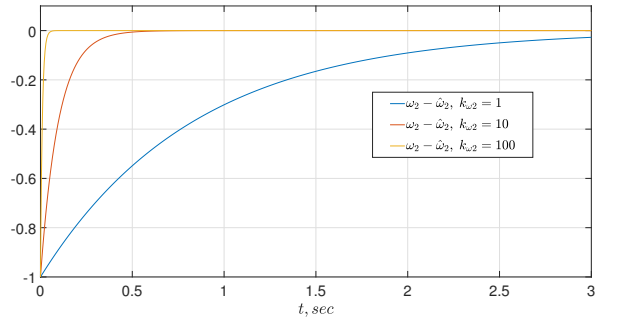


Fig. 4. Transients of the second speed observation error $\omega_2 - \hat{\omega}_2$

5. CONCLUDING REMARKS

We have given in this paper a solution to the problem of state observation of multimachine power systems with lossy lines assuming measurable the rotor angles, the excitation voltage and the active and reactive power of all generators. To the best of the authors' knowledge this is the first time a solution like this is presented. It is shown that, under a suitable observability assumption, global convergence of the voltage estimator is achieved. By imposing the classical PE condition, the convergence is exponential ensuring good robustness properties. The speed observation is ensured without imposing any excitation condition, and it is exponential with tunable and, possibly, arbitrarily fast rate of convergence.

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