A passivity-inspired design of power-voltage droop controllers for DC microgrids with electrical network dynamics

Juan E. Machado[1] and Johannes Schiffer[2]

Abstract— **We propose a design procedure for a power-voltage droop controller in structure-preserving DC microgrids under explicit consideration of the electrical network dynamics. Differently from most related literature, the system's controlled output is taken as the power—not the current—injection at each generation unit, yielding a nonlinear closed-loop system. This makes the output regulation problem non-trivial, yet far more appealing in a practical setting than the usual linear current-voltage droop control. Our approach is inspired by passivity-based control design in the sense that we exploit the natural port-Hamiltonian representation of the system dynamics and its associated shifted Hamiltonian to derive a control law together with sufficient conditions on the tuning gains that guarantee global asymptotic stability. The analysis is illustrated via detailed simulations, where accurate power sharing is manifested among the distributed generation units in the presence of load variations.**

I. Introduction

A. Motivation

Due to environmental concerns, in the last decade there has been a growing interest in augmenting the electricity generation through renewable energy sources (RES)—*e.g.,* photovoltaic and wind power. RES installations are of small capacity and geographically distributed in wide areas, contrasting with conventional, fossil-fueled power plants. In addition, the uncertain and intermittent nature of RES impose great new challenges on grid operation, particularly in control and stability assessment. To cope with these challenges, *microgrids* have been proposed as a conceptual solution [1]. Microgrids, both AC and DC, are a collection of interconnected subsystems mainly composed of Distributed Generation Units (DGUs), storage units and loads, with the salient feature of being able to operate either connected to or completely isolated from the main grid.

The implementation of DC microgrids may be more adequate than its AC counterpart in certain applications, see the discussion in [2]. For instance, data centers or industrial production lines may have an important presence of DC loads that, when interfaced with a DC-based DGU, such as PV panels or battery cells, could make an DC-AC-DC power conversion stage unnecessary, avoiding the use of typically inefficient rectifiers. By now, DC microgrids have been widely placed in trains, aircraft and spaceships and its presence can be expected to grow with the increasing deployment of DC-based DGUs [2]–[5].

In this paper we focus on primary control in DC microgrids [6]. In this setting, the predominant control scheme is power-voltage droop control, which is a decentralized proportional control that adjusts the DGU output voltage according to the change in power injection (possibly relative to a reference value) [3], [7], [8]. The objectives of this control law are voltage stabilization and power sharing [6], [7], [4]. The latter feature is essential in preventing DGUs from over stressing in the face of changing load conditions [4]. DC power is defined by the inner product of current and voltage. Therefore the use of the power injection as output feedback variable introduces a nonlinearity in the feedback loop. This fact significantly complicates the derivation of stability and tuning criteria in power-voltage droop-controlled microgrids.

As a consequence, thus far only stability results for reduced-order power-voltage droop-controlled microgrid models—in which the dynamics of the power lines are neglected—have been reported in the literature; see [7] and references therein. Yet, this assumption may not be admissible in DC microgrids, due to the fast dynamics of many (renewable) converter-interfaced distributed generation units. This has given rise to an increasing body of literature investigating control and analysis of DC microgrids under explicit consideration of the electrical network dynamics, *i.e.*, using dynamic power line models [9]–[12]. The resulting uncontrolled microgrid model can be written as a port-Hamiltonian system [13], which allows to exploit useful passivity properties in the control design [11], [12]. The closed-loop stability analyses conducted in these references rely on the explicit construction of Lyapunov (storage) functions that are closely related to the notion of *shifted Hamiltonian*. For a port-Hamiltonian system, the shifted Hamiltonian represents the Bregman distance [14] (see also [15] and [16]) of the Hamiltonian with respect to possibly non-zero equilibrium points [17], [18] and, hence, is a natural Lyapunov function candidate.

A fundamental limitation in the mentioned literature is the focus on designing controllers whose feedback variables are linear maps of the system states, *e.g.*, current or voltage variables. Then the resulting closed-loop system is also linear, which greatly simplifies the derivation of Lyapunov functions and, hence, the stability analysis. Thus, this choice of output maps is mainly due to technical reasons, while, from a practical viewpoint, using the DGU's power injections as feedback variables is much more appealing [7], [8].

[1]J.E. Machado is with Discrete Technology and Production Automation, Faculty of Science and Engineering, University of Groningen, Nijenborgh 4, 9747 AG Groningen, the Netherlands j.e.machado.martinez@rug.nl

[2]J. Schiffer is with the Fachgebiet Regelungssysteme und Netzleittechnik, Brandenburgische Technische Universität Cottbus-Senftenberg schiffer@b-tu.de.

Part of this work was supported by CONACyT from Mexico, the DAAD from Germany, and Université Paris-Saclay from France.

B. Main contributions

The main contribution of the present paper is a design procedure for power-voltage droop controllers in DC microgrids under explicit consideration of the electrical network dynamics, aiming at closing the abovementioned gap between theory and practice. Thereby, our approach is characterized by the following three main features:

1) Starting from the natural port-Hamiltonian structure of the system dynamics, we identify the associated shifted Hamiltonian as a Lyapunov function candidate. Then, inspired by passivity-based control design [15], [19], [20], we derive a dissipation equality in terms of the shifted Hamiltonian's gradient and a predefined power output relative to an equilibrium.
2) Based on the above result, we propose a power-voltage droop control and derive sufficient conditions on the controller parameters ensuring global asymptotic stability of the equilibrium of the resulting closed-loop dynamics.
3) We consider a structure-preserving microgrid model with distribution line dynamics, *i.e.*, all load and generation buses are explicitly represented in the model. This is aimed at accurately depicting general microgrid topologies and offering an alternative to the scenarios addressed in [11], [12], where loads are assumed to be connected only at generator buses, or to the application of Kron-reduction arguments [21]–[23], with which all load buses can be eliminated [3], [4].

The performance of the proposed controller in the presence of unknown load variations is illustrated via a simulation example of an 8-bus DC microgrid with three generator and five load buses.

C. Organization of the paper

The paper is organized as follows. In Section II we present the structure-preserving model of the DC microgrid and show that it can be expressed as a port-Hamiltonian system. The definition of the system output along with the problem formulation is done in Section III. After deriving a number of preliminary results, we present in Section IV our main stabilization theorem. Our analysis is illustrated in Section V with a detailed simulation example. Finally, the conclusion and directions for future work are given in Section VI.

D. Notation

We denote by $\mathbb{R}$ the set of real numbers. For an n-vector $x = (x_1, x_2, ..., x_n) \in \mathbb{R}^n$, $\langle x \rangle$ is a $n \times n$ diagonal matrix whose diagonal entries are given by x. The $n \times n$ identity matrix is denoted by I_n and $0_{m\times n}$ is the zero-matrix of size $m \times n$. For two matrices $A_1 \in \mathbb{R}^{n\times n}$ and $A_2 \in \mathbb{R}^{m\times m}$, block.diag$(A_1, A_2)$ denotes a block-diagonal matrix with A_1 and A_2 on the diagonals. For a scalar function $H : \mathbb{R}^n \to \mathbb{R}$ we denote by ∇H its transposed gradient.

II. Microgrid model in port-Hamiltonian form

The microgrid is modeled as a connected and undirected graph $G(\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is the set of vertices (*i.e.*, buses) and $\mathcal{E} = \mathcal{N} \times \mathcal{N}$ the set of edges (*i.e.*, distribution lines). Due to the structure-preserving nature of the model, we assume that the buses are partitioned into two mutually exclusive sets, *i.e.*, $\mathcal{N} = S \cup L$, with S denoting the buses at which DGUs are connected and L denoting the buses at which loads are connected. The cardinalities of these sets are conveniently denoted by n_s and n_ℓ, respectively, yielding a total of $n = n_s + n_\ell$ buses. Each edge $e_k \in \mathcal{E}$ corresponds to a distribution line consisting of a series connection of a resistance $R_{\tau k} > 0$ and an inductance $L_{\tau k} > 0$. The k-th edge connecting nodes i and j is denoted by $e_k = \{i, j\}$. We denote by n_τ the cardinality of $\mathcal{E}$. We arbitrarily assign an orientation to the edges and define the node-edge incidence matrix as

$$\mathcal{B}_{ik} = \begin{cases} +1 & \text{if } i \text{ is the positive end of } e_k, \\ -1 & \text{if } i \text{ is the negative end of } e_k, \\ 0 & \text{otherwise.} \end{cases}$$

According to the aforementioned splitting of the buses, the incidence matrix can be row-partitioned as

$$\mathcal{B} = \begin{bmatrix} \mathcal{B}_s \\ \mathcal{B}_\ell \end{bmatrix}. \tag{1}$$

The DGU at bus $i \in S$ comprises a DC voltage source interfaced to the distribution network through a Buck converter with an RLC low-pass filter at its output. The filter has the associated positive parameters R_{si}, L_{si} and C_{si}. On the other hand, each load bus is assumed to be a ZI-load, *i.e.*, a parallel connection of a constant resistance (Z) and a constant current (I) load, connected in parallel. We denote the stacked vector of load currents by $I_L \in \mathbb{R}^{n_\ell}$ and the diagonal load admittance matrix by $\mathcal{Y} = \langle R_{\ell k}^{-1} \rangle$, where $R_{\ell k} > 0$ denotes the load resistance at the k-th node, $k \in L$. Finally, the ZI-load at the k-th bus is assumed to be interfaced to the microgrid through a capacitor $C_{\ell k}$.

The dynamics of the interconnected system can be derived through basic Kirchhoff's laws (see [24]), which for the described setting results in

$$\begin{aligned} L_s \dot{i}_s &= -R_s i_s - v_s + u, \\ L_\tau \dot{i}_\tau &= -\mathcal{B}_s^\top v_s - \mathcal{B}_\ell^\top v_\ell - R_\tau i_\tau, \\ C_s \dot{v}_s &= i_s + \mathcal{B}_s i_\tau - \mathcal{Y}_{\text{shunt}} v_s, \\ C_\ell \dot{v}_\ell &= \mathcal{B}_\ell i_\tau - \mathcal{Y} v_\ell - I_L, \end{aligned} \tag{2}$$

where $i_s \in \mathbb{R}^{n_s}$ is the vector of DGU current injections, the vector of currents flowing through the distribution lines is $i_\tau \in \mathbb{R}^{n_\tau}$, and the vector of voltages of the DGU and load buses is denoted by $v_s \in \mathbb{R}^{n_s}$ and $v_\ell \in \mathbb{R}^{n_\ell}$, respectively. Also, $u \in \mathbb{R}^{n_s}$ is the vector of control variables, $\mathcal{B}_s$ and $\mathcal{B}_\ell$ are the partitions of the incidence matrix as described in equation (1). We denote by $L_{(\cdot)}$, $C_{(\cdot)}$ and $R_{(\cdot)}$ diagonal matrices of appropriate size comprising all the inductive, capacitive and resistive magnitudes of their associated elements. Moreover, $\mathcal{Y}_{\text{shunt},i} > 0$ represents the admittance of a resistor in parallel with C_{si}, as often used for passive damping provision [25], [26].

By introducing the *state* vector of the system (2) as [24]

$$x=\begin{bmatrix} L_s i_s \\ L_\tau i_\tau \\ C_s v_s \\ C_\ell v_\ell \end{bmatrix} \in \mathbb{R}^N, \quad N = 2n_s + n_\ell + n_\tau,$$

the dynamics (2) can be written in its natural port-Hamiltonian form, *i.e.*,

$$\begin{aligned} \dot{x} &= (\mathcal{J}-\mathcal{R})\nabla H(x) + \mathcal{G}u + d, \\ y_{\mathrm{n}} &= \mathcal{G}^\top \nabla H(x) = i_s, \end{aligned} \tag{3}$$

with Hamiltonian function (total energy)

$$H(x) = \frac{1}{2}x^\top \mathcal{M} x, \quad \nabla H(x) = \mathcal{M}x, \tag{4}$$

constant vector $d = \begin{bmatrix} 0_{n_s}^\top & 0_{n_\tau}^\top & 0_{n_s}^\top & -I_L^\top \end{bmatrix}^\top$, matrices $\mathcal{R} = \text{block.diag}\,(R_s,\ R_\tau,\ \mathcal{Y}_{\text{shunt}},\ \mathcal{Y})$, $\mathcal{M} = \text{block.diag}\,\left(L_s^{-1},\ L_\tau^{-1},\ C_s^{-1},\ C_\ell^{-1}\right)$, $\mathcal{G} = \begin{bmatrix} I_{n_s} & 0_{n_\tau\times n_s}^\top & 0_{n_s\times n_s}^\top 0_{n_\ell\times n_s}^\top \end{bmatrix}^\top$ and

$$\mathcal{J} = \begin{bmatrix} 0_{n_s\times n_s} & 0_{n_s\times n_\tau} & -I_{n_s} & 0_{n_\ell\times n_s} \\ 0_{n_\tau\times n_s} & 0_{n_\tau\times n_\tau} & -\mathcal{B}_s^\top & -\mathcal{B}_\ell^\top \\ I_n & \mathcal{B}_s & 0_{n_s\times n_s} & 0_{n_s\times n_\ell} \\ 0_{n_\ell\times n_s} & \mathcal{B}_\ell & 0_{n_\ell\times n_s} & 0_{n_\ell\times n_\ell} \end{bmatrix} = -\mathcal{J}^\top.$$

III. Problem Statement

The main objective of the present paper is to design a static decentralized feedback law for the system (3), which uses the power injection at each DGU as feedback variable. Hence, the output of interest is given by

$$y := p_s = \langle v_s \rangle i_s \in \mathbb{R}^{n_s}, \tag{5}$$

where $\langle v_s \rangle \in \mathbb{R}^{n_s\times n_s}$ is a diagonal matrix with v_s as its main diagonal.

In contrast to the *natural* output $y_{\mathrm{n}} = i_s$ of the system (3), the proposed measured output y in (5) is nonlinear, which significantly complicates the design task. Yet, clearly, from a power system perspective, acting on the power injections of the DGUs is much more appealing than on their current injections. Furthermore, as standard in (power system) control, we aim at regulating the (power) system output of each DGU relative to a given reference value $p_i^d \in \mathbb{R}_{>0}$.

The above discussion is formalized below.

Problem 1. Consider the system (3) with output y in (5). Design a *decentralized* controller

$$u = \gamma(x, y-p^d), \quad \gamma: \mathbb{R}^N \times \mathbb{R}^{n_s} \to \mathbb{R}^{n_s}, \quad p^d \in \mathbb{R}^{n_s}_{>0}, \tag{6}$$

such that the resulting closed-loop system possesses an asymptotically stable equilibrium point.

By decentralized we mean that each u_i has to be written only in terms of variables that are measurable at each node, namely, i_{si}, v_{si} and y_i. Decentralized static output feedback controls of the form (6) are usually termed *droop controls* in the microgrid literature [3] [7], [8].

By combining the dynamics (3) with the input u given by (6) and output y given by (5) we obtain the closed-loop system

$$\dot{x} = (\mathcal{J}-\mathcal{R})\nabla H(x) + \mathcal{G}(\gamma(x, y-p^d)) + d. \tag{7}$$

To address Problem 1, we make the following standard assumption in power system stability analysis about the existence of an equilibrium point. Possible options for verifying this assumption are given, *e.g.,* in [27] [28] and the references therein.

Assumption 1. The system (7) possesses an equilibrium point $\bar{x} \in \mathbb{R}^N$.

Clearly, a fundamental challenge when attempting to provide a solution to Problem 1 is to find a suitable Lyapunov function for establishing the desired stability claim. This aspect is further complicated since the output of interest, *i.e.*, y in (5) is *predefined*, hampering a classical passivity-based control design [19], [20], which would be a natural approach given the port-Hamiltonian structure of the dynamics (3).

Yet, a key observation for the analysis in the remainder of this paper is that nonetheless exploiting the port-Hamiltonian structure of the system (3) proves extremely useful in deriving a solution to Problem 1. More precisely, inspired by passivity-based control design [15], [19], [20], we employ the shifted Hamiltonian associated to H in (4) as a Lyapunov function candidate. Then we derive a dissipation equation in terms of the shifted Hamiltonian's gradient and the power output y relative to an equilibrium. This then instructs the particular choice of control law $\gamma(x, y-p^d)$ to render the equilibrium of the resulting closed-loop dynamics asymptotically stable (under certain sufficient conditions on the control parameters).

IV. A passivity-inspired solution to Problem 1

A. Shifted Hamiltonian and proposed control

Recall that the shifted Hamiltonian for the system (3) is defined as (see [15] and [18])

$$\mathcal{S}(x) = H(x) - H(\bar{x}) - (x-\bar{x})^\top \nabla H(x)|_{x=\bar{x}}, \tag{8}$$

where, with Assumption 1, $\bar{x}$ is an equilibrium of (3). Given the quadratic structure of the Hamiltonian (4) in the present case, the shifted Hamiltonian simplifies to

$$\mathcal{S}(x) = \frac{1}{2}(x-\bar{x})^\top \mathcal{M}(x-\bar{x}). \tag{9}$$

In addition, we can compute $\nabla\mathcal{S}(x)$ as

$$\nabla\mathcal{S}(x) = \nabla H(x) - \nabla H(\bar{x}). \tag{10}$$

We make the following observation on the time derivative of the shifted Hamiltonian $\mathcal{S}$, which is instrumental for the derivation of our proposed control.

Lemma 1. Consider the system (7) with Assumption 1. Along trajectories of the system (7), the time derivative of the shifted Hamiltonian $\mathcal{S}$ in (9) satisfies

$$\dot{\mathcal{S}} = -(\nabla\mathcal{S})^\top \mathcal{R}\nabla\mathcal{S} + (\nabla\mathcal{S})^\top \mathcal{G}\left(\gamma(x, y-p^d) - \gamma(\bar{x}, \bar{y}-p^d)\right), \tag{11}$$

where $\bar{y}$ denotes the stationary output value.

Proof. By noting that the equilibrium $\bar{x}$ satisfies

$$\dot{\bar{x}} = 0 = (\mathcal{J}-\mathcal{R})\nabla H(\bar{x}) + \mathcal{G}(\gamma(\bar{x}, \bar{y}-p^d)) + d,$$

the closed-loop system (7) can equivalently be written as

$$\dot{x} = (\mathcal{J}-\mathcal{R})\nabla\mathcal{S} + \mathcal{G}\left(\gamma(x, y-p^d) - \gamma(\bar{x}, \bar{y}-p^d)\right).$$

This implies that the time derivative of $\mathcal{S}$ in (9), along trajectories of the closed-loop system (7), is given by

$$\dot{\mathcal{S}} = (\nabla\mathcal{S})^\top \dot{x} = -(\nabla\mathcal{S})^\top \mathcal{R}\nabla\mathcal{S} + (\nabla\mathcal{S})^\top \mathcal{G}\left(\gamma(x, y-p^d) - \gamma(\bar{x}, \bar{y}-p^d)\right),$$

where we have used the fact that $\mathcal{J} = -\mathcal{J}^\top$. □

As can be seen from (11), $\dot{\mathcal{S}}$ cannot be rendered negative (semi-)definite for any arbitrary choice of $\gamma(x, y-p^d)$. Consider, therefore, the following particular choice[1] for γ in (6)

$$\gamma(x, y-p^d) := u^d - \langle k\rangle\langle v_s\rangle(y-p^d), \tag{12}$$

where the constant vector $u^d \in \mathbb{R}^{n_s}$ is a free parameter from which a desired (or nominal) steady-state can be attained, and k is the vector of control gains. The control law (12) is droop-like in the sense that it contains a proportional feedback in the power deviation $(y-p^d)$. Though in the present case this feedback term is additionally "weighted" with the local bus voltage v_s.

The next result reveals that the proposed control (12) exhibits a highly beneficial equivalence property.

Lemma 2. Consider the system (7) with Assumption 1. Choose $u = \gamma(x, y-p^d)$ with $\gamma(x, y-p^d)$ defined in (12). Then, the following equivalence holds:

$$(\nabla\mathcal{S})^\top \mathcal{G}\left(\gamma(x, y-p^d) - \gamma(\bar{x}, \bar{y}-p^d)\right) = -\begin{bmatrix}\nabla\mathcal{S}\\ y-\bar{y}\end{bmatrix}^\top \mathcal{W}\begin{bmatrix}\nabla\mathcal{S}\\ y-\bar{y}\end{bmatrix}, \tag{13}$$

where $\mathcal{W}$ is given by

$$\mathcal{W} = \begin{bmatrix} 0_{n_s\times n_s} & 0_{n_s\times n_\tau} & -\frac{1}{2}\langle k\rangle\langle p^d-\bar{y}\rangle & 0_{n_s\times n_\ell} & 0_{n_s\times n_s}\\ 0_{n_\tau\times n_s} & 0_{n_\tau\times n_\tau} & 0_{n_\tau\times n_s} & 0_{n_\tau\times n_\ell} & 0_{n_\tau\times n_s}\\ -\frac{1}{2}\langle k\rangle\langle p^d-\bar{y}\rangle & 0_{n_s\times n_\tau} & 0_{n_s\times n_s} & 0_{n_s\times n_\ell} & -\frac{1}{2}\langle k\rangle\langle \bar{i}_s\rangle\\ 0_{n_\ell\times n_s} & 0_{n_\ell\times n_\tau} & 0_{n_\ell\times n_s} & 0_{n_\ell\times n_\ell} & 0_{n_\ell\times n_s}\\ 0_{n_s\times n_s} & 0_{n_s\times n_\tau} & -\frac{1}{2}\langle k\rangle\langle \bar{i}_s\rangle & 0_{n_s\times n_\ell} & \langle k\rangle \end{bmatrix}.$$

Proof. We begin by writing useful expressions for $(y-\bar{y})$ and $\gamma(x, y-p^d) - \gamma(\bar{x}, \bar{y}-p^d)$. Considering that $y = \langle v_s\rangle i_s$ and the fact that $\langle a\rangle b = \langle b\rangle c$ for any two vectors a, b, it is possible to arrive to the equivalence

$$y - \bar{y} = \langle v_s\rangle\left(i_s - \bar{i}_s\right) + \langle \bar{i}_s\rangle\left(v_s - \bar{v}_s\right). \tag{14}$$

By following similar arguments and choosing $\gamma(x, y-p^d)$ as in (12), we can write on the other hand

$$\gamma(x, y-p^d) - \gamma(\bar{x}, \bar{y}-p^d) = \langle k\rangle\langle p^d-\bar{y}\rangle(v_s-\bar{v}_s) - \langle k\rangle\langle v_s\rangle(y-\bar{y}). \tag{15}$$

We are ready to compute the desired expression as

$$(\nabla\mathcal{S})^\top \mathcal{G}\left(\gamma(x, y-p^d) - \gamma(\bar{x}, \bar{y}-p^d))\right) = (i_s-\bar{i}_s)^\top\langle k\rangle\langle p^d-\bar{y}\rangle(v_s-\bar{v}_s) - (i_s-\bar{i}_s)^\top\langle k\rangle\langle v_s\rangle(y-\bar{y}), \tag{16}$$

where we have used the fact that (see (10))

$$\left[i_s-\bar{i}_s\ \ i_\tau-\bar{i}_\tau\ \ v_s-\bar{v}_s\ \ v_\ell-\bar{v}_\ell\right]^\top = \nabla^\top\mathcal{S} \tag{17}$$

together with equation (15). From (14) we have that the second term in the right-hand side of (16) can be written as

$$(i_s-\bar{i}_s)^\top\langle k\rangle\langle v_s\rangle(y-\bar{y}) = (y-\bar{y})^\top\langle k\rangle\left[(y-\bar{y}) - \langle \bar{i}_s\rangle(v_s-\bar{v}_s)\right]. \tag{18}$$

By substituting (18) into (16), we obtain

$$\begin{aligned}&(\nabla\mathcal{S})^\top \mathcal{G}\left(\gamma(x, y-p^d) - \gamma(\bar{x}, \bar{y}-p^d)\right)\\ &= (i_s-\bar{i}_s)^\top\langle k\rangle\langle p^d-\bar{y}\rangle(v_s-\bar{v}_s)\\ &-(y-\bar{y})^\top\langle k\rangle(y-\bar{y})\\ &+(y-\bar{y})^\top\langle \bar{i}_s\rangle\langle k\rangle(v_s-\bar{v}_s).\end{aligned} \tag{19}$$

By invoking (17), we have that the right-hand side of (19) is equivalent to (13), concluding the proof. □

Remark 1. The structure of the controller (12) resembles the so-called quadratic reactive power-voltage droop control proposed for AC microgrids in [29]. Yet, the stability analysis therein is restricted to networks with algebraic interconnections, while in the present case the power line dynamics are considered explicitly.

B. Main result

We are now in a position to present our main result.

Theorem 1. Consider the system (7) with $\gamma(x, y-p^d)$ given by (12) and Assumption 1. Choose the diagonal elements k_i of $\langle k\rangle$, such that

$$4R_{s,i}\mathcal{Y}_{\text{shunt},i} - (k_i)^2(\bar{y}_i - p_i^d)^2 > 0, \tag{20a}$$

$$k_i - (k_i\bar{i}_{s,i})^2\frac{R_{s,i}}{4R_{s,i}\mathcal{Y}_{\text{shunt},i} - (k_i)^2(\bar{y}_i-p_i^d)^2} > 0, \tag{20b}$$

for all $i = 1, 2, ..., n_s$. Then the equilibrium $\bar{x}$ is globally asymptotically stable.

Proof. To establish the claim, we choose the shifted Hamiltonian $\mathcal{S}$ in (9) as a candidate Lyapunov function, since, as can be directly seen from (9), it is globally positive definite with respect to $\bar{x}$.

Let us now compute $\dot{\mathcal{S}}$. By combining (11) with (13), we obtain

$$\dot{\mathcal{S}} = -(\nabla\mathcal{S})^\top\mathcal{R}\nabla\mathcal{S} - \begin{bmatrix}\nabla\mathcal{S}\\ y-\bar{y}\end{bmatrix}^\top \mathcal{W}\begin{bmatrix}\nabla\mathcal{S}\\ y-\bar{y}\end{bmatrix} = -\begin{bmatrix}\nabla\mathcal{S}\\ (y-\bar{y})\end{bmatrix}^\top \mathcal{Q}\begin{bmatrix}\nabla\mathcal{S}\\ (y-\bar{y})\end{bmatrix},$$

where $\mathcal{Q}$ is given by

$$\mathcal{Q} = \begin{bmatrix}\mathcal{R} & 0_{N\times n_s}\\ 0_{n_s\times N} & 0_{n_s\times n_s}\end{bmatrix} + \mathcal{W}.$$

It remains to show that, under the conditions (20), $\dot{\mathcal{S}}$ is globally negative definite with respect to $\bar{x}$. This is equivalent to the matrix $\mathcal{Q}$ being positive definite. It is possible to verify that $\mathcal{Q}$ can be written as

$$\mathcal{Q} = \begin{bmatrix}\Theta & \Omega\\ \Omega^\top & \langle k\rangle\end{bmatrix},$$

where Θ and Ω are composed by sub-block diagonal matrices. Through lengthy but straightforward computations, which rely on the Schur complement of $\mathcal{Q}$, it can be established that conditions (20) are sufficient for $\mathcal{Q}$ to be positive definite. □

The corollary below shows that the controller gains k_i can indeed always be chosen such that the conditions of

[1] With slight abuse of notation, in the sequel γ refers exclusively to the right-hand side of (12).

Theorem 1 are satisfied. In addition, the result provides a guideline on how to choose the controller gains in order for the proposed stability conditions to be verified in a broad range of operating points $(\bar{x}, \bar{y})$.

Corollary 1. For $i = 1, ..., n_s$, choose

$$0 < k_i < \begin{cases} \Gamma_{1,i} & p_i^d = \bar{y}_i \\ \min\{\Gamma_{1,i},\ \Gamma_{2,i},\ \Gamma_{3,i}\} & p_i^d \neq \bar{y}_i \end{cases} \tag{21}$$

where $\Gamma_{1,i} = 4\frac{\mathcal{Y}_{\text{shunt},i}}{\bar{i}_{s,i}^2}$, $\Gamma_{2,i} = \sqrt{\frac{4R_{s,i}\mathcal{Y}_{\text{shunt},i}}{(p_i^d-\bar{y}_i)^2}}$, and $\Gamma_{3,i} = \sqrt{\frac{R_{s,i}\left(\bar{i}_{s,i}^4 R_{s,i}+16(p_i^d-\bar{y}_i)^2\mathcal{Y}_{\text{shunt},i}\right)}{4(p_i^d-\bar{y}_i)^4}} - \frac{\bar{i}_{s,i}^2 R_{s,i}}{2(p_i^d-\bar{y}_i^2)^2}$. Then, the conditions (20) hold simultaneously for all $i = 1, ..., n_s$.

Proof. Assume $\bar{y}_i = p_i^d$, then (20a) holds for any choice of k_i, whereas (20b) holds if $0 < k_i < 4\frac{\mathcal{Y}_{\text{shunt},i}}{\bar{i}_{s,i}^2} = \Gamma_{1,i}$ for all $i = 1, 2, ..., n_s$ and the claim follows directly.

Conversely, assume that $\bar{y}_i \neq p_i^d$. Notice first that $k_i > 0$ is necessary for the satisfaction of (20) in this scenario. Also, it can be readily seen that (20a) holds if $0 < k_i < \sqrt{\frac{4R_{s,i}\mathcal{Y}_{\text{shunt},i}}{(p_i^d-\bar{y}_i)^2}} = \Gamma_{2,i}$. In order to verify (20b), observe that since $R_{s,i} > 0$, (20a) is equivalent to $\Delta_i(k_i) := 4R_{s,i}\mathcal{Y}_{\text{shunt},i} - (k_i)^2(\bar{y}_i - p_i^d)^2 > 0$, which holds by assumption. Now, an equivalent expression for inequality (20b) is obtained by multiplying both sides of it by $\frac{\Delta_i(k_i)}{k_i} > 0$, yielding $\Delta_i(k_i) - \bar{i}_{s,i}^2 R_{s,i} k_i > 0$. The satisfaction of the latter inequality, the left-hand side of which is a quadratic polynomial of k_i, is implied by the condition $0 < k_i < \frac{1}{2}\sqrt{\frac{R_{s,i}\left(\bar{i}_{s,i}^4 R_{s,i}+16(p_i^d-\bar{y}_i)^2\mathcal{Y}_{\text{shunt},i}\right)}{(p_i^d-\bar{y}_i)^4}} - \frac{1}{2}\frac{\bar{i}_{s,i}^2 R_{s,i}}{(p_i^d-\bar{y}_i^2)^2} = \Gamma_{3,i}$, which is equivalent to (20b). □

V. Simulation Example

We illustrate our theoretical developments via a simulation example of a meshed 8-bus DC microgrid with three DGUs and five purely resistive load buses, as illustrated in Fig. 1. All relevant parameters are provided in Table I.

The microgrid is assumed to operate in nominal load conditions from 0 s to 1.33 s, to be subject to a 20% step change on all the loads from 1.33 s to 2.66 s, and be restored to nominal conditions from 2.66 s. Henceforth we denote by $\hat{x}$ and $\bar{x}$ the equilibrium before and during the load change, respectively; $\hat{x}$ is to be referred as the nominal equilibrium.

The system's base voltage and power are $v^{\text{base}} = 450$ V and $p^{\text{base}} = 25$ kW, moreover we consider an admissible voltage deviation of $\pm\Delta v_{si}^{\max} = \pm 0.05 u_i^d$, *i.e.*, of a 5% (22.5 V), and maximum power injection of $p_{si}^{\max} = [24.97,\ 13.72,\ 12.09]$ kW, $i = 1, ...n_s$. We assume that all DGUs are equipped with the feedback control (12), with the gains tuned using Corollary 1, thus ensuring closed-loop global asymptotic stability of the nominal equilibrium $\hat{x}$. Moreover, for the same gains, the conditions in Corollary 1 were verified to hold in a neighborhood of $\hat{x}$ by taking incremental load changes within a ±20% deviation with respect to nominal values, hence, the equilibrium attained during the load change is also globally asymptotically stable.

The simulation results are shown in Fig. 2. We can see that after the introduction of the load step changes, there is a small transient both in the DGUs' power injections and voltages, after which all these signals become constant. Nominal conditions are then successfully restored after 2.66 s. It can also be observed that for all the DGUs the voltages remain within the prescribed domain and that, moreover, the deviation in the power injections during the load change are approximately proportional to their ratings.

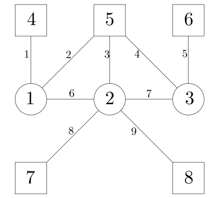

Fig. 1: Topology of the considered DC microgrid based on [30]: generation buses are denoted by circles whereas squares denote load buses.

TABLE I: Test system parameters.

DGUs	$i = 1, 2, 3.$
L_{si}	[1.0, 2.2, 1.1] mH
C_{si}	[2.6, 2.9, 1.4] mF
R_{si}	[0.19, 0.36, 0.42] Ω
$\mathcal{Y}_{\text{shunt},i}$	[3.1, 3.4, 3.7] mS
u_i^d	[1, 1, 1] pu
p_i^d	[0.62, 0.35, 0.32] pu
k_i	[0.05, 0.1, 0.13] 10^{-4}W^{-1}
Loads	$j = 4, ..., 8.$
$C_{\ell j}$	[0.91, 0.43, 0.78, 0.83, 0.7] mF
$\mathcal{Y}_j$	[0.65, 154, 0.29, 0.67, 0.52] mS
Lines	$k = 1, ..., 9.$
$L_{\tau k}$	[0.29, 0.17, 0.18, 0.30, 0.29, 0.24, 0.30, 0.25, 0.17] 10^{-5}H
$R_{\tau k}$	[8.8, ,8.8, 7.2, 4.3, 4.8, 4.5, 3.7, 4.1, 7.1] $10^{-2}\Omega$

VI. Conclusion

We have addressed the primary control of DC microgrids using a structure-preserving model with dynamic power lines. For this setting, we have derived a novel power-voltage droop feedback law together with sufficient conditions on the controller gains that ensure global asymptotic stability. This has been achieved by exploiting the inherent structural properties of the port-Hamiltonian representation of the system dynamics and using the shifted Hamiltonian as a Lyapunov function.

Our formal analysis has been illustrated via a simulation example of an 8-bus microgrid. With the aid of Corollary 1, we have verified our stability condition for realistic settings and for a wide selection of load disturbance scenarios. Moreover, accurate—yet not exact—power sharing among the DGUs has been clearly exhibited.

As future research we propose to perform a detailed steady-state analysis aimed at dropping Assumption 1, along the same lines of, for example [27], where the existence of equilibria for droop-controlled HVDC networks is addressed. In addition, the analysis will be extended to include constant-power loads, which are well-known for introducing voltage oscillations and even causing network collapse.

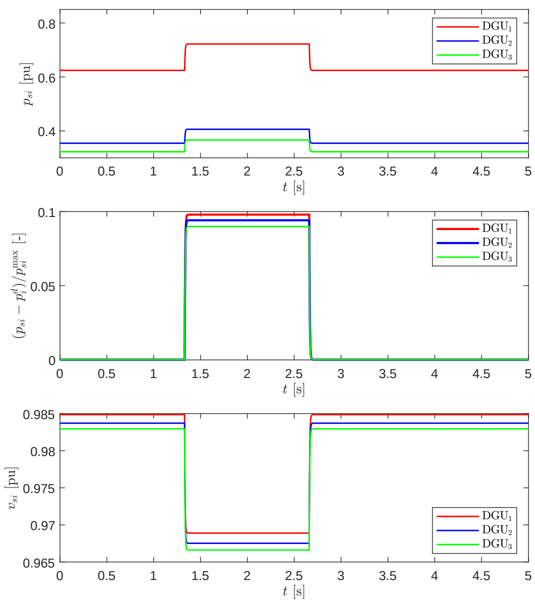

Fig. 2: Simulation results.

REFERENCES

[1] R. Lasseter, "MicroGrids," *2002 IEEE Power Engineering Society Winter Meeting*, vol. 1, pp. 305–308, 2002.

[2] J. J. Justo, F. Mwasilu, J. Lee, *et al.*, "AC-microgrids versus DC-microgrids with distributed energy resources: A review," *Renewable and Sustainable Energy Reviews*, vol. 24, pp. 387–405, 2013.

[3] J. Zhao and F. Dörfler, "Distributed control and optimization in DC microgrids," *Automatica*, vol. 61, pp. 18–26, 2015.

[4] C. De Persis, E. Weitenberg, and F. Dörfler, "A power consensus algorithm for DC microgrids," *Automatica*, vol. 89, pp. 364–375, 2018.

[5] F. Gao, R. Kang, J. Cao, *et al.*, "Primary and secondary control in DC microgrids: a review," *Journal of Modern Power Systems and Clean Energy*, vol. 7, no. 2, pp. 227–242, 2019.

[6] J. Guerrero, J. Vasquez, J. Matas, *et al.*, "Hierarchical Control of Droop-Controlled AC and DC Microgrids — A General Approach Toward Standardization," vol. 58, no. 1, pp. 158–172, 2011.

[7] T. Dragicevic, X. Lu, J. C. Vasquez, *et al.*, "DC Microgrids - Part I: A Review of Control Strategies and Stabilization Techniques," *IEEE Transactions on Power Electronics*, vol. 31, no. 7, pp. 4876–4891, 2016.

[8] F. Gao, S. Bozhko, A. Costabeber, *et al.*, "Comparative Stability Analysis of Droop Control Approaches in Voltage-Source-Converter-Based DC Microgrids," *IEEE Transactions on Power Electronics*, vol. 32, no. 3, pp. 2395–2415, 2017.

[9] M. Cucuzzella, K. C. Kosaraju, and J. M. A. Scherpen, "Voltage control of DC networks : robustness for unknown ZIP-loads," 2019. arXiv: `arXiv:1907.09973v1`.

[10] M. Cucuzzella, S. Trip, C. De Persis, *et al.*, "A Robust Consensus Algorithm for Current Sharing and Voltage Regulation in DC Microgrids," *IEEE Transactions on Control Systems Technology*, vol. 27, no. 4, pp. 1583–1595, 2019. arXiv: `1708.04608`.

[11] F. Strehle, M. Pfeifer, A. J. Malan, *et al.*, "A Scalable Port-Hamiltonian Approach to Plug-and-Play Voltage Stabilization in DC Microgrids," 2020. arXiv: `2002.05050`.

[12] P. Nahata, R. Soloperto, M. Tucci, *et al.*, "A passivity-based approach to voltage stabilization in DC microgrids with ZIP loads," *Automatica*, vol. 113, 2020.

[13] A. van der Schaft and D. Jeltsema, "Port-Hamiltonian Systems Theory: An Introductory Overview," *Foundations and Trends in Systems and Control*, vol. 1, no. 2-3, pp. 173–378, 2004.

[14] L. M. Bregman, "The relaxation method of finding the common point of convex sets and its application to the solution of problems in convex programming," *Zh. vychisl. Mat. mat. Fiz.*, vol. 7, no. 3, pp. 620–631, 1967.

[15] B. Jayawardhana, R. Ortega, E. García-Canseco, *et al.*, "Passivity of nonlinear incremental systems: Application to PI stabilization of nonlinear RLC circuits," *Systems and Control Letters*, vol. 56, pp. 618–622, 2007.

[16] C. De Persis and N. Monshizadeh, "Bregman storage functions for microgrid control," *IEEE Transactions on Automatic Control*, vol. 63, no. 1, pp. 53–68, 2018.

[17] P. Monshizadeh, J. E. Machado, R. Ortega, *et al.*, "Power-controlled Hamiltonian systems: Application to electrical systems with constant power loads," *Automatica*, vol. 109, 2019.

[18] N. Monshizadeh, P. Monshizadeh, R. Ortega, *et al.*, "Conditions on shifted passivity of port-Hamiltonian systems," *Systems and Control Letters*, vol. 123, pp. 55–61, 2019.

[19] R. Ortega, A. van der Schaft, B. Maschke, *et al.*, "Interconnection and damping assignment passivity-based control of port-controlled Hamiltonian systems," *Automatica*, vol. 38, pp. 585–596, 2002.

[20] R. Ortega, A. van der Schaft, F. Castaños, *et al.*, "Control by interconnection and standard passivity-based control of port-hamiltonian systems," *IEEE Transactions on Automatic Control*, vol. 53, no. 11, pp. 2527–2542, 2008.

[21] P. Kundur, *Power System Stability and Control*. McGraw Hill, 1994.

[22] S. Y. Caliskan and P. Tabuada, "Towards Kron reduction of generalized electrical networks," *Automatica*, vol. 50, pp. 2586–2590, 2014.

[23] A. Floriduz, M. Tucci, S. Riverso, *et al.*, "Approximate Kron Reduction Methods for Electrical Networks with Applications to Plug-and-Play Control of AC Islanded Microgrids," *IEEE Transactions on Control Systems Technology*, vol. 27, no. 6, pp. 2403–2416, 2019.

[24] S. Fiaz, D. Zonetti, R. Ortega, *et al.*, "A port-Hamiltonian approach to power network modeling and analysis," *European Journal of Control*, vol. 19, pp. 477–485, 2013.

[25] M. Cespedes, L. Xing, and J. Sun, "Constant-power load system stabilization by passive damping," *IEEE Transactions on Power Electronics*, vol. 26, no. 7, pp. 1832–1836, 2011.

[26] R. Peña-Alzola, M. Liserre, F. Blaabjerg, *et al.*, "Analysis of the passive damping losses in lcl-filter-based grid converters," *IEEE Transactions on Power Electronics*, vol. 28, no. 6, pp. 2642–2646, 2013.

[27] D. Zonetti, R. Ortega, and J. Schiffer, "A tool for stability and power-sharing analysis of a generalized class of droop controllers for high-voltage direct-current transmission systems," *IEEE Transactions on Control of Network Systems*, vol. 5, no. 3, pp. 1110–1119, 2018.

[28] A. S. Matveev, J. E. Machado Martinez, R. Ortega, *et al.*, "A Tool for Analysis of Existence of Equilibria and Voltage Stability in Power Systems with Constant Power Loads," *IEEE Transactions on Automatic Control*, 2020.

[29] J. W. Simpson-Porco, F. Dörfler, and F. Bullo, "Voltage Stabilization in Microgrids via Quadratic Droop Control," *IEEE Transactions on Automatic Control*, vol. 62, no. 3, pp. 1239–1253, 2017.

[30] D. Zonetti, A. Saoud, A. Girard, *et al.*, "Decentralized monotonicity-based voltage control of DC microgrids with ZIP loads," *IFAC-PapersOnLine*, vol. 52, no. 20, pp. 139–144, 2019.