State Observation of Power Systems Equipped with Phasor Measurement Units: The Case of Fourth Order Flux-Decay Model

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***Abstract*—The problem of effective use of Phasor Measurement Units (PMUs) to enhance power systems awareness and security is a topic of key interest. The central question to solve is how to use this new measurements to reconstruct the *state* of the system. In this paper we provide the first solution to the problem of (globally convergent) state estimation of multimachine power systems equipped with PMUs and described by the *fourth order* flux-decay model. This work is a significant extension of our previous result, where this problem was solved for the simpler third order model, for which it is possible to recover *algebraically* part of the unknown state. Unfortunately, this property is lost in the more accurate fourth order model, and we are confronted with the problem of estimating the *full* state vector. The design of the observer relies on two recent developments proposed by the authors, a parameter estimation based approach to the problem of state estimation and the use of the Dynamic Regressor Extension and Mixing (DREM) technique to estimate these parameters. The use of DREM allows us to overcome the problem of lack of persistent excitation that stymies the application of standard parameter estimation designs. Simulation results illustrate the latter fact and show the improved performance of the proposed observer with respect to a locally stable gradient-descent based observer.**

***Index Terms*—Dynamic state estimation, power system operation, phasor measurements, synchronous generator.**

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I. Introduction

POWER systems are experiencing major changes and challenges, such as an increasing amount of power-electronics-interfaced equipment, growing transit power flows and fluctuating (renewable) generation, see [24]. Therefore power systems are operated under more and more stressed conditions and, thus, closer to their stability limits as ever before. In addition, as detailed in [12], their dynamics become faster, more uncertain and also more volatile. Hence, fast and accurate monitoring of the system states is crucial in order to ensure a stable and reliable system operation, see [27]. This, however, implies that the conventional monitoring approaches based on steady-state assumptions are no longer appropriate and instead novel *state observers*[1] tools have to be developed, see [27], [22].

By recognizing this need, the design of state observers has become a very active research area in the past years. The interest has been further accelerated by the growing deployment of PMUs, see [23]. The vast majority of the reported results on this matter rely on the use of *linear* systems-based theories, *e.g.*, the use of Kalman filters, whose performance is assessed only via simulations, see [7], [17], [25], [27]. As thoroughly discussed in [10]. this approach suffers from several major drawbacks.

Recently [10], the authors provided a globally convergent solution to the state observation problem for the case when the generators are modelled by the classical *third order* flux-decay model. Instrumental for the solution of the problem was the observation that, for this model, it is possible to recover *algebraically* part of the unknown state. It is widely recognized [11], [22], [25] that to improve the precision of the model, it is necessary to include additional dynamic effects, leading to a fourth order model. Unfortunately, for this case, the algebraic reconstruction of part of the state is *impossible*, and we are confronted with the problem of estimating the *full* state vector.

In this paper we provide the first solution to the state observation problem for multimachine power systems described by the fourth order model. The design of the observer relies on two recent developments proposed by the authors, first, a generalization of the Parameter Estimation-based Observer (GPEBO) [14], which translates the problem of state estimation into one of *parameter* estimation. Second, the use of the DREM technique [1], [16] to estimate these parameters. These two theoretical developments are instrumental to solve the current problem. GPEBO was used in [15] for the design of observers for bio-chemical reactors and the simplest problem of state estimation of third-order power systems with *measurement* of the rotor angle. The latter, practically restrictive assumption, is removed here significantly widening the applicability of the result. Thanks to the use of DREM it is possible to overcome the problem of lack of persistent excitation that stymies the application of standard observer designs.

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[1]In the power systems community, to distinguish it from the steady-sate case, the qualifier“dynamic” is added to the state observation problem, and it is sometimes called “dynamic state estimation”.

In the theoretical part of the paper we restrict ourselves to the study of a single generator. As shown in [10], thanks to the incorporation of the PMUs, for the purposes of observer design it is possible to treat multimachine systems as a set of decentralized single machines, hence our result can be extended in a straightforward way to the multimachine case. In the interest of brevity we omit the details of this generalization, and refer the interest reader to [10, Section II] for the details. However, the simulation results, presented in Section VI, include the multimachine case, and show the improved performance of the proposed observer with respect to a locally stable gradient-descent based observer.

II. Mathematical Model and Problem Formulation

We consider the well-known fourth-order model of the single machine system given by [7, eq. (1)], see also [19, Chapter 5.4] and [11, Chapter 11.1.7.1] ,

$$\dot{x}_1 = x_2 \tag{1a}$$
$$\dot{x}_2 = -a_0 x_2 + b_0(u_1 - y_5) \tag{1b}$$
$$\dot{x}_3 = -a_2 x_3 + b_2 y_2 \sin(x_1 - y_1) \tag{1c}$$
$$\dot{x}_4 = -a_1 x_4 + b_1 y_2 \cos(x_1 - y_1) + c_1 u_2, \tag{1d}$$

where the *unknown* state and input variables are defined as

$$x := \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^\top = \begin{bmatrix} \delta & \omega & E'_d & E'_q \end{bmatrix}^\top$$
$$u := \begin{bmatrix} u_1 & u_2 \end{bmatrix}^\top = \begin{bmatrix} P_m & E_f \end{bmatrix}^\top,$$

with δ the rotor angle, ω the shaft speed, E'_d and E'_q the direct and quadrature axis internal voltages, respectively, P_m the mechanical power and E_f the field voltage, and we defined the positive constants

$$a_0 := \frac{\omega_0 D}{2H},\ a_1 := \frac{1}{T_{d0'}}\frac{x_d}{x'_d},\ a_2 := \frac{1}{T_{q0'}}\frac{x_q}{x'_q},\ b_0 := \frac{\omega_0}{2H}$$
$$b_1 := \frac{1}{T_{d0'}}\frac{(x_d - x'_d)}{x'_d},\ b_2 := \frac{1}{T_{q0'}}\frac{(x_q - x'_q)}{x'_q},\ c_1 := \frac{1}{T_{d0'}}.$$

with ω_0 the nominal synchronous speed, D the damping factor, H the inertia constant, T'_{d0} and T'_{q0} the direct and quadrature axis transient open-circuit time constant, x_d and x_q the direct and quadrature axis reactances and x'_d and x'_q the direct and quadrature axis transient reactances, respectively.

The PMU *measurements* are

$$y := \begin{bmatrix} y_1 & y_2 & y_3 & y_4 & y_5 & y_6 \end{bmatrix}^\top = \begin{bmatrix} \theta_t & V_t & \phi_t & I_t & P_t & Q_t \end{bmatrix}^\top, \tag{2}$$

where

$$y_4^2 = \frac{1}{x_q'^2}(x_3^2 + x_4^2 + y_2^2 - 2y_2[x_4\cos(x_1 - y_1) + x_3\sin(x_1 - y_1)]) \tag{3a}$$
$$y_5 = \frac{y_2}{x'_q}[x_4 \sin(x_1 - y_1) - x_3\cos(x_1 - y_1)] \tag{3b}$$
$$y_6 = \frac{y_2}{x'_q}[x_4\cos(x_1 - y_1) + x_3\sin(x_1 - y_1) - y_2] \tag{3c}$$

with θ_t the terminal voltage phase angle, V_t the terminal voltage magnitude, ϕ_t the terminal current phase angle, I_t the terminal current magnitude, P_t the terminal active power and Q_t the terminal reactive power.

We underscore the presence of the terminal bus voltage $y_2 = V_t$ that, in a multimachine scenario, captures the effect of the interconnection among the machines, see [10, Section II] for details.

To formulate the observer problem we make the following assumptions on systems prior knowledge and the available measurements.

Assumption 1. The signals u are *measurable* and the electrical subsystem parameters $(a_1, a_2, b_1, b_2.c_1)$ are *known*.[2]

Problem Formulation: Consider the SMIB power system (1) with measurable outputs (2), (3), verifying Assumption 1. Design an observer

$$\dot{\chi} = F(\chi, u, y),\ \hat{x} := H(\chi, u, y)$$

such that $\lim_{t\to\infty} \tilde{x}(t) = 0$, where we, generically, define the estimation *errors* $\tilde{(\cdot)} := \hat{(\cdot)} - (\cdot)$.

Remark 1. The model (1a)-(1d) is obtained making the standard assumption that the *stator resistance is zero*. Moreover, the expressions in (3b) and (3c) are obtained by neglecting transient saliency, thus assuming that the direct- and quadrature-axis transient reactances, x'_d and x'_q, respectively, are *equal*. The latter assumption, commonly referred to as neglecting transient saliency, is introduced for example in the standard power systems book [19, Chap. 7.5.1].

Remark 2. The expression for y_4 given in (3a) can be derived using the direct- and quadrature-axis currents I_d and I_q, respectively, defined as

$$I_d := \frac{1}{x'_d}[x_4 - y_2\cos(x_1 - y_1)],$$
$$I_q := \frac{1}{x'_q}[-x_3 + y_2\sin(x_1 - y_1)],$$

with $x'_d = x'_q$.

III. 3rd and 4th Order Models: A Fundamental Difference

As indicated in Section I, in [10] we present a globally convergent solution to the state observation problem for the case when the generators are modeled by the classical *third order* flux-decay model given by[3]

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -a_1 x_2 + a_2[P_m - \mathcal{Y} V x_3 \sin(x_1)]$$
$$\dot{x}_3 = -a_3 x_3 + a_4 V\cos(x_1) + E_f,$$

where the *unknown* state is defined as

$$x := \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^\top = \begin{bmatrix} \delta - \theta_t & \omega & E'_q \end{bmatrix}^\top,$$

[2] As usual in observer problems [3] we assume that u is bounded and such that all state trajectories are *bounded*.

[3] To avoid cluttering, and with some abuse of notation, we keep the same symbols for the 3rd and the 4th order models.

with $\mathcal{Y} > 0$ is the susceptance of the network admittance, and V is the terminal voltage. All other parameters of the model a_i, $i = 1, 4$, are constant. The *measurements*, which are provided via PMUs, are defined as

$$\begin{aligned} y_1 &= V \\ y_2 &= \mathcal{Y} V x_3 \sin(x_1) \\ y_3 &= \mathcal{Y} V x_3 \cos(x_1) - \mathcal{Y} V^2 \\ y_4^2 &= \mathcal{Y}^2 [x_3^2 + V^2 - 2V x_3 \cos(x_1)], \quad (4) \\ y_5 &= f_t, \quad (5) \end{aligned}$$

where $y_1 > 0$ is the terminal voltage, y_2 is the active power, y_3 is the reactive power, y_4 is the terminal current and y_5 the terminal voltage frequency.

In [10] it was shown that it is possible to algebraically reconstruct the states x_1 and x_3 from the measurements y as follows.

Proposition 1. [10] The states x_1 and x_3 can be determined *uniquely* from the PMU measurements (4) via

$$x_3 = \sqrt{\frac{y_4^2 + 2\mathcal{Y} y_3}{\mathcal{Y}^2} + y_1^2}, \; x_1 = \arcsin\left(\frac{y_2}{\mathcal{Y} y_1 x_3}\right).$$

This result is essential for the solution of the state observation problem, which now reduces to the observation of x_2 only. We will show now that, unfortunately, this fundamental property of the state-to-output map is *lost* for the 4th order model (2). The proposition below establishes this fact by proving that the mapping from states to measurements is *non-injective*. We recall that injectivity of a mapping $N(v)$ is *equivalent* to the existence of a mapping $N^I(\cdot)$ such that $N^I(N(v)) = v$. This, in its turn, is equivalent to the following implication:

$$\forall v_a, v_b, \; v_a \neq v_b \;\Rightarrow\; N(v_a) \neq N(v_b).$$

Proposition 2. Consider the SMIB model (1) with PMU measurements (2) and (3).

F1 There exists a *measurable* signal $Y = \text{col}(Y_1, Y_2, Y_3) \in \mathbb{R}^3$ and a mapping $N : \mathbb{R}^3 \to \mathbb{R}^3$ such that

$$Y = N(x_1, x_3, x_4). \quad (6)$$

F2 The mapping $(x_1, x_3, x_4) \mapsto Y$, which is given by

$$N(x_1, x_3, x_4) := \begin{bmatrix} x_3^2 + x_4^2 & e^{Jx_1} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} \end{bmatrix}^\top, \quad (7)$$

with $J := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, is *non-injective*.

Proof. To avoid cluttering, let us define the measurable signal

$$z_0 := y_6 + \frac{y_2^2}{x_q'}. \quad (8)$$

From (3b) and (3c) we get after some simple calculations

$$\begin{aligned} y_5 x_4 + z_0 x_3 &= \frac{y_2}{x_q'} (x_3^2 + x_4^2) \sin(x_1 - y_1) \\ z_0 x_4 - y_5 x_3 &= \frac{y_2}{x_q'} (x_3^2 + x_4^2) \cos(x_1 - y_1). \quad (9) \end{aligned}$$

Note also that from (3) we have

$$x_3^2 + x_4^2 = (x_q')^2 y_4^2 + 2x_q' y_6 + y_2^2 =: Y_1, \quad (10)$$

which is the first identity in (6), (7). We underscore that Y_1 is bounded away from zero.

We now to complete the proof of the fact **F1**. From (3b) and (3c) we get

$$\begin{aligned} \begin{bmatrix} -\frac{x_q'}{y_2} y_5 \\ \frac{x_q'}{y_2} y_6 + y_2 \end{bmatrix} &= \begin{bmatrix} \cos(x_1 - y_1) & -\sin(x_1 - y_1) \\ \sin(x_1 - y_1) & \cos(x_1 - y_1) \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} \\ &= e^{J(x_1 - y_1)} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}. \end{aligned}$$

The proof is completed defining

$$\begin{bmatrix} Y_2 \\ Y_3 \end{bmatrix} := e^{Jy_1} \begin{bmatrix} -\frac{x_q'}{y_2} y_5 \\ \frac{x_q'}{y_2} y_6 + y_2 \end{bmatrix}. \quad (11)$$

We proceed now to prove that the mapping $N(x_1, x_3, x_4)$ is non-injective. To simplify the notation we define the vector $v := \text{col}(x_1, x_3, x_4)$. To prove non-injectivity of the mapping $N(v)$, we will show that there exists two vectors $v^a \in \mathbb{R}^3$ and $v^b \in \mathbb{R}^3$, which are *different*, but such that $N(v^a) = N(v^b)$. Consider the two vectors

$$v^a = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}^\top, \; v^b = \begin{bmatrix} \frac{\pi}{2} & 1 & -1 \end{bmatrix}^\top.$$

Some simple calculations yield

$$N(v^a) = N(v^b) = \begin{bmatrix} 2 & 1 & 1 \end{bmatrix}^\top,$$

completing the proof. □□□

IV. Reparameterization of the Electrical Dynamics

In this section we propose a reparameterization of the electrical dynamics which is *linear* in x_3 and x_4.

Lemma 1. Consider the SMIB model (1) with PMU measurements (2) and (3). There exists a matrix of measurable signals $\mathcal{M} \in \mathbb{R}^{2\times 2}$ such that

$$\begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \mathcal{M} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ c_1 u_2 \end{bmatrix}. \quad (12)$$

Proof. Replacing (10) in (9) and rearranging terms we get

$$\begin{aligned} y_2 \sin(x_1 - y_1) &= \frac{x_q'}{Y_1} (y_5 x_4 + z_0 x_3) \\ y_2 \cos(x_1 - y_1) &= \frac{x_q'}{Y_1} (z_0 x_3 - y_5 x_3). \end{aligned}$$

Replacing the latter equations in (1c) and (1d) and defining the matrix

$$\mathcal{M} := \begin{bmatrix} -a_2 + \frac{b_2 x_q'}{Y_1} z_0 & \frac{b_2 x_q'}{Y_1} y_5 \\ -\frac{b_1 x_q'}{Y_1} y_5 & -a_1 + \frac{b_1 x_q'}{Y_1} z_0 \end{bmatrix}. \quad (13)$$

completes the proof of claim **C1**. □□□

Remark 3. From (12) and the first component in (7), that is $Y_1 = x_3^2 + x_4^2$, we see that we are dealing with a subsystem with linear dynamics but *nonlinear* state-output map. To the best of our knowledge [2, Chapter 5], [3, Chapter 3] there is no systematic way to design state observers for this class of systems.

V. Proposed State Observer

A corollary of Proposition 2 is that there are two possibilities to reconstruct the states (x_1, x_3, x_4). The first option is to find an observer for x_1 and get (x_3, x_4) from the components Y_2 and Y_3 of (7). Alternatively, we can estimate (x_3, x_4) and—as shown in the proposition below—obtain x_1 from simple trigonometric relations. The state x_2, having the same dynamics of the third order model studied in [10], can be reconstructed with the I&I observer of [10, Lemma 1].

Although the first approach looks simpler, the design of an observer for x_1 is still an open problem. Therefore, in this section we take the second route and design a GPEBO observer [15] for the states (x_3, x_4). To enhance readability we divide the presentation of the observer in two parts, first—in the spirit of PEBO [14] that translates the problem of state observation into one of parameter estimation—the derivation of a *nonlinear regression equation* required for the parameter estimation is given. Then, we invoke DREM [1] to carry out the latter task with weak excitation requirements.

A. Derivation of the regression equation for parameter estimation

Lemma 2. Consider the reparameterized electrical dynamics (12) and the output map (7). Define the dynamic extension

$$\dot{\xi} = A(t)\xi + \begin{bmatrix} 0 \\ c_1 u_2 \end{bmatrix} \tag{14a}$$

$$\dot{\Phi} = A(t)\Phi, \; \Phi(0) = I_2, \tag{14b}$$

where we defined the time-varying matrix[4]

$$A(t) := \begin{bmatrix} -a_2 + \frac{b_2 x_q'}{Y_1(t)} z_0(t) & \frac{b_2 x_q'}{Y_1(t)} y_5(t) \\ -\frac{b_1 x_q'}{Y_1(t)} y_5(t) & -a_1 + \frac{b_1 x_q'}{Y_1(t)} z_0(t) \end{bmatrix}. \tag{15}$$

P1 There exists a constant vector $\theta = \mathrm{col}(\theta_1, \theta_2) \in \mathbb{R}^2$ such that

$$\begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = \xi + \Phi\theta. \tag{16}$$

P2 There exists measurable signals $y_E \in \mathbb{R}$ and $\psi \in \mathbb{R}^5$ that verify the regression equation

$$y_E = \psi^\top \Theta, \tag{17}$$

where we defined the constant vector

$$\Theta := \mathrm{col}(\theta_1, \theta_2, \theta_1\theta_2, \theta_1^2, \theta_2^2). \tag{18}$$

P3 Define the observer

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_3 \\ \hat{x}_4 \end{bmatrix} = \begin{bmatrix} \arcsin\left(\frac{1}{Y_1}\begin{bmatrix} Y_3 & -Y_2 \end{bmatrix}(\xi + \Phi\hat{\theta})\right) \\ \xi + \Phi\hat{\theta} \end{bmatrix}, \tag{19}$$

where $\hat{\theta}$ is an estimate of the parameter θ. The following implication is true

$$\lim_{t\to\infty} \tilde{\theta}(t) = 0 \;\Rightarrow\; \lim_{t\to\infty} \begin{bmatrix} \tilde{x}_1(t) \\ \tilde{x}_3(t) \\ \tilde{x}_4(t) \end{bmatrix} = 0. \tag{20}$$

[4]That is, the evaluation of the matrix $\mathcal{M}$, given in (13), along the trajectories of the system outputs.

Proof. Define the error signal

$$\epsilon := \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} - \xi \tag{21}$$

and taking into account (12), (13), (14a) and (15) we obtain a linear time-varying (LTV) system $\dot{\epsilon} = A(t)\epsilon$. Now, from (14b) we see that Φ is the *state transition matrix* of the ϵ system. Consequently, there exists a *constant* vector $\theta \in \mathbb{R}^2$ such that

$$\epsilon = \Phi\theta,$$

namely $\theta = \epsilon(0)$. We now have the following chain of implications

$$\begin{aligned} \epsilon = \Phi\theta \Leftrightarrow \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} &= \xi + \Phi\theta & (\Leftarrow (21)) \\ \Rightarrow\; x_3^2 + x_4^2 &= |\xi + \Phi\theta|^2 & (\Leftarrow |\cdot|^2) \\ \Leftrightarrow\; Y_1 &= |\xi + \Phi\theta|^2 & (\Leftarrow (7)). \end{aligned}$$

Notice that the right hand side of the first equivalence above proves property **P1**. The proof of the property **P2** follows developing the right hand side square above, rearranging terms and defining

$$y_E := Y_1 - |\xi|^2, \; \psi := \begin{bmatrix} 2(\Phi_{11}\xi_1 + \Phi_{21}\xi_2) \\ 2(\Phi_{12}\xi_1 + \Phi_{22}\xi_2) \\ 2(\Phi_{11}\Phi_{12} + \Phi_{21}\Phi_{22}) \\ \Phi_{11}^2 + \Phi_{21}^2 \\ \Phi_{12}^2 + \Phi_{22}^2 \end{bmatrix}.$$

The proof of the implication for the errors $\mathrm{col}(\tilde{x}_3, \tilde{x}_4)$ is obvious from (16) and the definition of $\mathrm{col}(\hat{x}_3, \hat{x}_4)$ in (19). To prove the claim for $\tilde{x}_1$ notice that using (7) and the definition of e^{Jx_1}, we get

$$x_3 Y_3 - x_4 Y_2 = (x_3^2 + x_4^2)\sin(x_1) = Y_1 \sin(x_1),$$

from which we obtain

$$x_1 = \arcsin\left(\frac{x_3 Y_3 - x_4 Y_2}{Y_1}\right).$$

□□□

B. DREM parameter estimator

In view of the implication (20) the remaining task to complete the observer design is to generate a consistent estimate for θ. Towards this end, we dispose of the regressor equation (17) that, unfortunately, is *nonlinear* in the unknown parameters θ. Treating Θ as the unknown vector, it is possible to view (17) as an *overparameterized* linear regression equation (LRE) to which we can directly apply a classical gradient descent estimator, that is

$$\dot{\hat{\Theta}} = -\Gamma\psi(\psi^\top\hat{\Theta} - y_E), \; \Gamma > 0. \tag{22}$$

However, this approach has the following fundamental shortcoming. It is well-known [18, Theorem 2.5.1] that a necessary and sufficient conditions for global (exponential) convergence of the gradient estimator equation (17), (22) is that the regressor ψ satisfies a stringent *persistent excitation* requirement [18, Equation 2.5.3], which is not possible to satisfy in normal operation of the power system because of the overparameterization. To avoid this difficulty we propose here to use a DREM estimator that has the unique feature of generating 5 new, one-dimensional linear regression equations to *independently* estimate each of the parameters. This feature allows, on one hand, to estimate *only* the parameters θ and, on the other hand, to relax the excitation assumptions that guarantee its convergence. This fact is illustrated in the simulations of Section V. For further details on DREM the interested reader is refered to [1], [16].

The first step to apply DREM to (17) is to introduce a *linear, single-input* 5*-output, bounded-input bounded-output (BIBO)–stable* operator $\mathcal{H}$ and define the vector $Y_E \in \mathbb{R}^5$ and the matrix $\Psi \in \mathbb{R}^{q\times q}$ as

$$\begin{aligned} Y_E &:= \mathcal{H}[y_E] \\ \Psi &:= \mathcal{H}[\psi^\top]. \end{aligned} \tag{23}$$

Clearly, because of linearity and BIBO stability, these signals satisfy

$$Y_E = \Psi\Theta. \tag{24}$$

At this point the key step of regressor "mixing" of the DREM procedure is used to obtain a set of 5 *scalar* equations as follows. First, recall that, for any (possibly singular) $q \times q$ matrix M we have $\text{adj}\{M\}M = \det\{M\}I_q$, where $\text{adj}\{\cdot\}$ is the adjunct (also called "adjugate") matrix. Now, multiplying from the left the vector equation (24) by the *adjunct matrix* of Ψ, we get

$$\mathcal{Y}_i = \Delta\Theta_i, \quad i \in \{1, 2, \ldots, 5\} \tag{25}$$

where we have defined the signals

$$\begin{aligned} \Delta &:= \det\{\Psi\} \in \mathbb{R} \\ \mathcal{Y} &:= \text{adj}\{\Psi\}Y_E \in \mathbb{R}^5. \end{aligned} \tag{26}$$

The availability of the *scalar* LREs (25) is the main feature of DREM that distinguishes it with respect to all other estimators and allows us to obtain significantly stronger results. Indeed, in DREM we propose the gradient-descent estimators[5]

$$\dot{\hat{\Theta}}_i = \gamma_i\Delta(\mathcal{Y}_i - \Delta\hat{\Theta}_i),$$

with $\gamma_i > 0$, which gives rise to the scalar error equations

$$\dot{\tilde{\Theta}}_i = -\gamma_i\Delta^2\tilde{\Theta}_i,$$

whose explicit solution is

$$\tilde{\Theta}_i(t) = e^{-\gamma_i\int_0^t \Delta^2(s)ds}\tilde{\Theta}_i(0). \tag{27}$$

From direct inspection of (27) we conclude the following *equivalence*

$$\lim_{t\to\infty}\tilde{\Theta}_i(t) = 0 \quad\Leftrightarrow\quad \lim_{t\to\infty}\int_0^t \Delta^2(s)ds = \infty,$$

and convergence can be made *arbitrarily fast* simply increasing the gains γ_i.

[5]In the sequel, the quantifier $i \in \{1, 2, \ldots, 5\}$ is omitted for brevity.

Remark 4. It is clear from the definition of Θ in (18) that we are only interested in the first and second components of this vector.

Remark 5. In [8] it was observed that, using Cramer's rule, the computation of the adjunct matrix $\text{adj}\{\Psi^\top\}$ can be avoided. Indeed, the elements of the vector $\mathcal{Y}$ can be computed as

$$\mathcal{Y}_i = \det\{\Psi_{Y,i}\} \tag{28}$$

where the matrix $\Psi_{Y,i}$ is obtained replacing the i-th column of Ψ by the vector Y.

C. Main result

We are now in position to present the main result of this paper, a *globally convergent* observer for the state of the SMIB power system (1) with measurable outputs (2), with the required excitation conditions been rather weak. As indicated before, the state x_2 can be reconstructed with the I&I observer of [10, Lemma 1] and is omitted for brevity.

Proposition 3. Consider the SMIB power system (1), (2) verifying Assumption 1. Fix a stable transfer matrix[6]

$$\mathcal{H}(s) = \begin{bmatrix} 1 \\ \frac{d_2}{s+d_2} \\ \vdots \\ \frac{d_5}{s+d_5} \end{bmatrix}, \; d_i > 0, \; d_i \neq d_j, \; \forall i \neq j. \tag{29}$$

Let the state observer be defined by (7), (8), (14), (15), (19), (21), (23) and (26) together with the parameter estimators

$$\dot{\hat{\theta}}_k = -\gamma_k\Delta(\Delta\hat{\theta}_k - \mathcal{Y}_k), \; \gamma_k > 0, \; k = 1, 2. \tag{30}$$

If $\Delta \notin \mathcal{L}_2$ then

$$\lim_{t\to\infty}\begin{bmatrix}\tilde{x}_1(t) \\ \tilde{x}_3(t) \\ \tilde{x}_4(t)\end{bmatrix} = 0,$$

with all signals bounded.

Proof. Given the derivations of Subsection V-B we get the parameter estimator error equations

$$\dot{\tilde{\theta}}_k = -\gamma_k\Delta^2\tilde{\theta}_k, \; k = 1, 2.$$

Clearly, with the standing assumption on Δ, we have that $\tilde{\theta}(t) \to 0$. The proof is completed invoking (20).

□□□

Remark 6. Although the construction of DREM allows for the use of general, LTV, BIBO-stable operators $\mathcal{H}$, for the sake of simplicity we consider here the use of simple LTI filters. Moreover, we take the first element of the matrix to be the identity. See [16] for more general versions of DREM.

[6]The latter condition on the constants d_i is necessary to avoid the possibility of Ψ been singular.

Symbol	Description	Value	Unit
D	Damping factor	2	pu
H	Inertia constant	23.64	sec
k	Tuning parameter	80	-
T'_{d0}	Direct-axis transient open-circuit time constant	8.96	sec
x_d	Direct-axis reactance	0.146	pu
x'_d	Direct-axis transient reactance	0.0608	pu
Y	Stator admittance	16.45	pu
ω_s	Nominal synchronous speed	314.16	rad/sec

TABLE I: Parameters for the SMIB system (1).

VI. Simulation Results

In this section we present some simulations that illustrate the performance of the observer of the states (x_3, x_4) of Proposition 3, which combines GPEBO with DREM. For the sake of comparison we also show the simulation results of GPEBO with the overparameterized parameter estimator (22) and a simple *state* observer directly derived from optimization considerations. We consider the cases of the classical single-machine infinite bus and a benchmark multimachine example.

A. Single Machine Infinite Bus

We simulated the system (1) with the parameters $a_0 = 13.2893$, $a_1 = 0.268$, $a_2 = 7.7462$, $b_0 = 6.6447$, $b_1 = 0.1564$, $b_2 = 4.5204$, $c_1 = 0.1116$, obtained from Table I with $u_1 = u_2 = 0.1$. The systems initial conditions were set to $x(0) = \text{col}(0.1, 0.2, 0.4, 0.3)$. The initial conditions of *all* the states of the three observers were set to zero.

1) GPEBO+DREM of Proposition 3: The parameters of the transfer matrix (29) were chosen as $d_2 = 2,\ d_3 = 4,\ d_4 = 6,\ d_5 = 8$.

Figure 1 shows the transients of x_3 and x_4 and their observed values $\hat{x}_3$ and $\hat{x}_4$ with the adaptation gains $\gamma_1 = \gamma_2 =: \gamma$ and different values for γ. As expected, increasing γ speeds-up the convergence—interestingly, without generating overshoots.

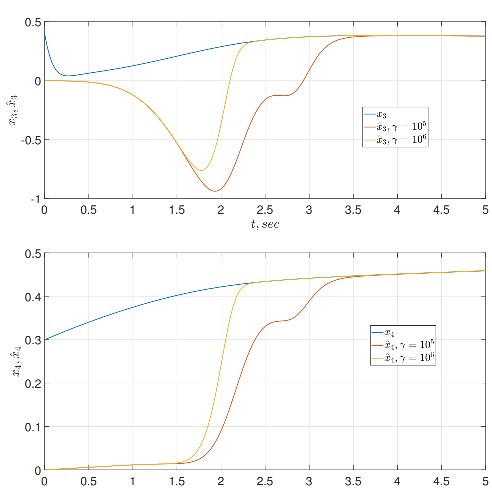

Fig. 1: Transients of x_3, x_4 and $\hat{x}_3$, $\hat{x}_4$ of GPEBO+DREM of Proposition 3 for different values of γ

2) GPEBO with overparameterized estimator (22): In this subsection we show that the standard gradient estimator (22) for the overparameterized regression is inadequate. To assess the quality of the estimation we define the vector

$$e := \begin{bmatrix} \Theta_1\Theta_2 - \Theta_3 & \Theta_1 - \Theta_4^2 & \Theta_2 - \Theta_5^2 \end{bmatrix}^\top.$$

From the definition of the vector Θ in (18) we have that $e \equiv 0$. In Figure 2 we show the transients of the estimated vector

$$\hat{e} := \begin{bmatrix} \hat{\Theta}_1\hat{\Theta}_2 - \hat{\Theta}_3 & \hat{\Theta}_1 - \hat{\Theta}_4^2 & \hat{\Theta}_2 - \hat{\Theta}_5^2 \end{bmatrix}^\top \tag{31}$$

with the adaptation gains $\Gamma = 10^6 I_5$ and $\Gamma = 10^8 I_5$, which does not converge to zero—proving that the parameters do not converge to their true values. Several different values of Γ were tried, observing always an erroneous behavior.

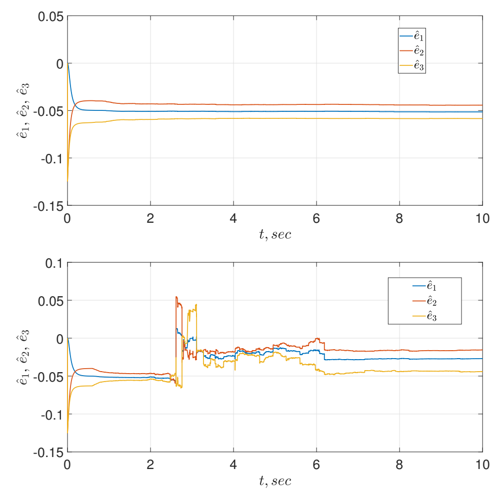

Fig. 2: Transients of the components of $\hat{e}$ for $\Gamma = 10^6 I_5$ and $\Gamma = 10^8 I_5$ of the overparameterized estimator (22)

3) Gradient-descent state estimation algorithm: In this subsection we propose to design gradient-descent algorithms, directly for observation of the *states* (x_3, x_4), proceeding from the state-to-output map (10). The gradient descent-based approach to state observation was, apparently, first proposed in [20], and has been pursued recently by several researchers [4], [6], [9], [13], [21].

The construction proceeds as follows. Given the criterion

$$\mathcal{T}(x_3, x_4) := \frac{1}{4}[Y_1 - (x_3^2 + x_4^2)]^2,$$

with Y_1 given in (10), propose an observer

$$\begin{bmatrix} \dot{\hat{x}}_3 \\ \dot{\hat{x}}_4 \end{bmatrix} = -\Gamma\nabla\{\mathcal{T}(\hat{x}_3, \hat{x}_4)\} + \left(A(t) \begin{bmatrix} \hat{x}_3 \\ \hat{x}_4 \end{bmatrix} + \begin{bmatrix} 0 \\ c_1 u_2 \end{bmatrix} \right)$$

where $\nabla\{\cdot\}$ denotes the gradient operator, and $\Gamma \in \mathbb{R}^{2\times 2}$ positive definite. That is,

$$\begin{bmatrix} \dot{\hat{x}}_3 \\ \dot{\hat{x}}_4 \end{bmatrix} = \Gamma[Y_1 - (\hat{x}_3^2 + \hat{x}_4^2)] \begin{bmatrix} \hat{x}_3 \\ \hat{x}_4 \end{bmatrix} + \left(A(t) \begin{bmatrix} \hat{x}_3 \\ \hat{x}_4 \end{bmatrix} + \begin{bmatrix} 0 \\ c_1 u_2 \end{bmatrix} \right) \tag{32}$$

The *local* stability properties of this observer can be studied using the Taylor-expansion based analysis proposed in [21]. To the best of the authors' knowledge no result of *global* stability of this kind of observers has been reported in the literature.

Figure 3 shows the transients of x_3 and x_4 and their observed values $\hat{x}_3$ and $\hat{x}_4$ with $\Gamma = \gamma I_2$ and different values[7] of γ. Interestingly, the state estimation errors converge to zero, even for large initial conditions errors. However, the transient behavior is significantly slower that the one of GPEBO+DREM—notice the difference in time scales.

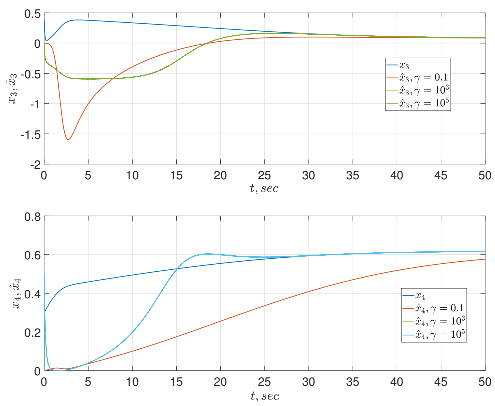

Fig. 3: Transients of x_3, x_4 and $\hat{x}_3$, $\hat{x}_4$ of the gradient descent observer (32) for different values of $\Gamma = \gamma I_2$

B. Multi-Machine Power System

We simulated the well-known New England IEEE 39 bus system, with the parameters provided in [5]. All synchronous generators are represented by the fourth-order flux-decay model (1) and are equipped with automatic voltage regulators and power system stabilizers according to [5]. To monitor the system, we assume that a PMU is installed at the terminal bus of generator 6.

As a test case we used minor load variations in the system. The resulting frequency variations are within 60 ± 0.020 [Hz] and hence consistent with those during regular operation of transmission grids [26].

1) GPEBO+DREM and algebraic observer of Proposition 3: The parameters of the transfer matrix (29) were chosen as $d_2 = 2,\ d_3 = 4,\ d_4 = 6,\ d_5 = 8$. Different values were chosen for the adaptation gains $\gamma_i = \gamma$. In Figure 4 the simulation results for x_1, x_2, x_3 and the state estimation of the observer of Proposition 3 are shown. As seen from the figure consistent estimation of the state variables is achieved.

2) GPEBO with overparameterized estimator (22): The overparameterized estimator was simulated using different values for the adaptation gain $\Gamma = \gamma I_5$. The elements of the error vector defined in (31) are given in Figure 5, showing that convergence is not achieved.

3) Gradient-descent state estimation algorithm (32): In Figure 6 the simulation results for the gradient-descent state estimation algorithm introduced in (32) are shown using different values for the gain $\Gamma = \gamma I_2$. As seen from the figure the transient behavior is very good, mainly due to the

[7]The plots of $\gamma = 10^3$ and $\gamma = 10^5$ are overlapped

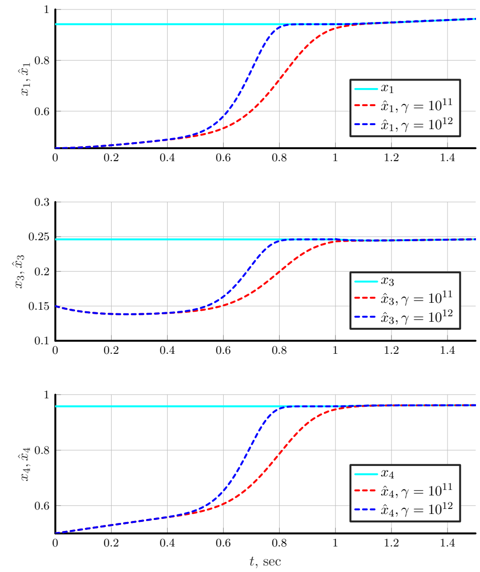

Fig. 4: Transients of the GPEBO+DREM of Proposition 3, with different values of γ, for generator 6 in the presence of load variations.

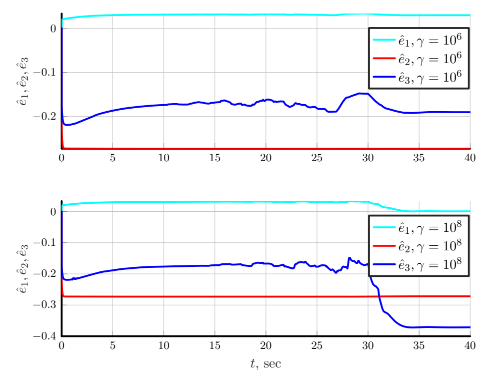

Fig. 5: Transients of the error vector (31) of the overparameterized estimator (22) for generator 6 in the presence of load variations.

rapid change of the state variables x_3 and x_4 due to the load variation that provide the required excitation to estimate the gradient.

We have also done simulations for both observers in the case when $x'_d \neq x'_q$ that, as indicated in Remark 1, is an assumption instrumental to obtain the equations (3b) and (3c). In both cases, a significant steady-state error appeared exhibiting the high sensitivity of both observers to this critical assumption.

VII. Conclusion And Future Research

We have proposed a globally convergent observer for the state estimation, from PMU measurements, of multimachine power systems described by the widely popular fourth order model (1). It is shown that we can concentrate on the observation of the states (x_3, x_4) and compute x_1 from an explicit algebraic equation. The observer has only a few tuning gains: the time constants of the LTI filters $\mathcal{H}$ in (29) and the adaptation gains γ_i. The former must be selected related to

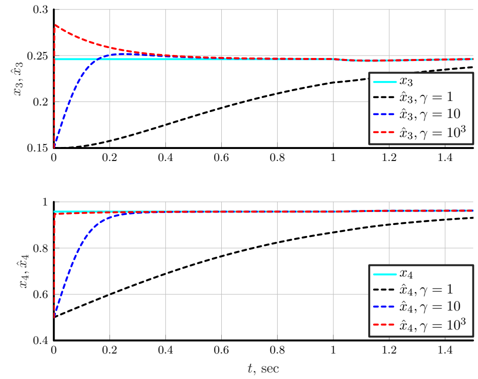

Fig. 6: Transients of the gradient descent observer (32) of generator 6 in the presence of load variations.

the bandwidth of the process, while the latter determine the rate of convergence of the parameter estimator.

For the observation of (x_3, x_4) we have also proposed a gradient-descent based observer that, in spite of the lack of a global convergence proof, performs quite well in a realistic multimachine scenario. A topic of current research is to assess the convergence properties of this observer—beyond the local analysis based on linearization of [21].

To extend the range of application of the proposed observer to *salient pole* synchronous generators, we are currently working on an alternative solution that allows us to relax the assumption on the direct- and quadrature-axis transient reactances being equal and zero stator resistance indicated in Remark 1.

Another interesting possibility motivated by (7) is to design a gradient-descent observer for (x_1, x_3, x_4) fixing a cost function

$$\mathcal{T}_N(x_1, x_3, x_4) := \left| Y - \begin{bmatrix} x_3^2 + x_4^2 \\ e^{Jx_1} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} \end{bmatrix} \right|^2,$$

and going in the direction of descent of the gradient—with respect to (x_1, x_3, x_4)—of this cost function. We hope to be able to report this result in the near future.

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