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Forschungsseminar Analysis und Optimierung Sommersemester 2026

Im Rahmen des Forschungsseminars spricht am **Donnerstag, den 21. Mai 2026**

Prof. Peter Takac
Universität Rostock

Titel: An Elliptic Spectral Problem with Concave and Convex Nonlinearity: Uniqueness and Multiplicity of Positive Solutions

Inhalt: Population models with a diffusive spread of a population following a nonlinear growth pattern have a long history. Selecting, for the growth, a mixture of a concave nonlinearity at low population densities and a convex one at high densities, in the late 1970s and early 1980s a number of researchers have developed powerful methods of partial differential equations and functional analysis to treat such problems assuming a *uniform, space-independent* growth throughout the domain of the habitat. A typical result was the possibility of *two positive equilibria* of a lower and a higher population density, pointwise ordered throughout the habitat. In our presentation we will construct a similar pair of equilibria in a model with *space-dependent growth*: concave in one subdomain and convex in the other one, linear on the boundary (or region) between the two subdomains.

We will discuss the question of **existence** and **multiplicity** of *positive solutions* to the semilinear elliptic Dirichlet problem

$$-\Delta u = \lambda u(x)^{q(x)-1} + f(x, u(x)) \quad \text{for } x \in \Omega; \quad u = 0 \quad \text{on } \partial\Omega, \quad (1)$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with the boundary of class $C^{1,\alpha}$, $\lambda \in \mathbb{R}$ a spectral parameter, and $f(x, u) = |u|^{r-1}u$ is a **signed r -power** ($r > 0$) of the unknown function of (a positive variable) $u \in (0, \infty)$ which depends on the point $x \in \Omega$; $r = q(x) - 1$, for instance.

We will briefly present basic methods for treating the semilinear problem (1) with a **convex** and **concave** nonlinear reaction $f(x, \cdot) : s \mapsto |s|^{q(x)-2}s : \mathbb{R}_+ \subset \mathbb{R} \rightarrow \mathbb{R}$ which (for $s \geq 0$) is **convex** in a nonempty open subset $\Omega_+ \stackrel{\text{def}}{=} \{x \in \Omega : q(x) > 2\}$ and **concave** in another nonempty open subset $\Omega_- \stackrel{\text{def}}{=} \{x \in \Omega : q(x) < 2\}$ of a bounded domain $\Omega \subset \mathbb{R}^N$. Here, $\lambda \in \mathbb{R}_+$ is a nonnegative

spectral parameter which decides about the existence and multiplicity of positive weak solutions (at least two) to problem (1) in case we take $f \equiv 0$. Our main contribution is a method how to handle the interplay between *convex* and *concave* nonlinearities in two disjoint nonempty open subsets of a domain Ω (connected in \mathbb{R}^N), as opposed to the classical works assuming a nonlinearity $f(s)$ being *concave* for small values of $s \in \mathbb{R}_+$ and *convex* for large $s \in \mathbb{R}_+$, uniformly in Ω .

The Dirichlet Laplace operator being linear, we observe that the elliptic equation in problem (1) is **convex** (**concave**, respectively) at a given point $x \in \Omega$, provided $q(x) \geq 2$ ($1 < q(x) \leq 2$). This is a typical problem with variable powers (exponents).

Finally, if time permits, we will discuss also the classical question of **uniqueness** for a related problem with the $p(x)$ -Laplacian provided $q(x) \leq \text{const}_1 < \text{const}_2 \leq p(x)$ holds for all $x \in \Omega$, and the **multiplicity** of large solution branches bifurcating from infinity as $\lambda \searrow 0+$.

Weitere Details zum Seminar: Der Vortrag findet in Raum HG 3.35 des Zentralcampus Cottbus der BTU statt.