

Inertial wave excitation and focusing in a liquid bounded by a frustum and a cylinder

Marten Klein¹, Torsten Seelig², Michael V. Kurgansky³,
Abouzar Ghasemi V.¹, Ion Dan Borcia², Andreas Will¹, Eberhard Schaller¹,
Christoph Egbers² and Uwe Harlander^{2,†}

¹Department of Environmental Meteorology, Brandenburg University of Technology
Cottbus-Senftenberg, Burger Chaussee 2, D-03044 Cottbus, Germany

²Department of Aerodynamics and Fluid Mechanics, Brandenburg University of Technology
Cottbus-Senftenberg, Siemens-Halske-Ring 14, D-03046 Cottbus, Germany

³A. M. Obukhov Institute of Atmospheric Physics, Russian Academy of Sciences, Pyzhevsky 3,
119017 Moscow, Russian Federation

(Received 29 August 2013; revised 28 March 2014; accepted 23 May 2014)

The mechanism of localized inertial wave excitation and its efficiency is investigated for an annular cavity rotating with Ω_0 . Meridional symmetry is broken by replacing the inner cylinder with a truncated cone (frustum). Waves are excited by individual longitudinal libration of the walls. The geometry is non-separable and exhibits wave focusing and wave attractors. We investigated laboratory and numerical results for the Ekman number $E \approx 10^{-6}$, inclination $\alpha = 5.71^\circ$ and libration amplitudes $\varepsilon \leq 0.2$ within the inertial wave band $0 < \omega < 2\Omega_0$. Under the assumption that the inertial waves do not essentially affect the boundary-layer structure, we use classical boundary-layer analysis to study oscillating Ekman layers over a librating wall that is at an angle $\alpha \neq 0$ to the axis of rotation. The Ekman layer erupts at frequency $\omega = f_*$, where $f_* \equiv 2\Omega_0 \sin \alpha$ is the effective Coriolis parameter in a plane tangential to the wall. For the selected inclination this eruption occurs for the forcing frequency $\omega/\Omega_0 = 0.2$. For the librating lids eruption occurs at $\omega/\Omega_0 = 2$. The study reveals that the frequency dependence of the total kinetic energy K_ω of the excited wave field is strongly connected to the square of the Ekman pumping velocity $w_E(\omega)$ that, in the linear limit, becomes singular when the boundary layer erupts. This explains the frequency dependence of non-resonantly excited waves. By the localization of the forcing, the two configurations investigated, (i) frustum libration and (ii) lids together with outer cylinder in libration, can be clearly distinguished by their response spectra. Good agreement was found for the spatial structure of low-order wave attractors and periodic orbits (both characterized by a small number of reflections) in the frequency windows predicted by geometric ray tracing. For ‘resonant’ frequencies a significantly increased total bulk energy was found, while the energy in the boundary layer remained nearly constant. Inertial wave energy enters the bulk flow via corner beams, which are parallel to the characteristics of the underlying Poincaré problem. Numerical simulations revealed a mismatch between the wall-parallel mass fluxes