Towards a Quantum Mechanical Model of the Inner Stage of Cognitive Agents

Matthias Wolff*, Markus Huber*, Günther Wirsching†, Ronald Römer*, Peter beim Graben*, and Ingo Schmitt*
*Brandenburg University of Technology Cottbus-Senftenberg, Cottbus, Germany
†Catholic University of Eichstätt-Ingolstadt, Eichstätt, Germany

***Abstract*—We present a model, inspired by quantum field theory, of the so-called inner stage of technical cognitive agents. The inner stage represents all knowledge of the agent. It allows for planning of actions and for higher cognitive functions like coping and fantasy. By the example of a cognitive mouse agent living in a maze wold, we discuss learning, action planning, and attention in a fully deterministic setting and assuming a totally observable world. We explain the relevance of our approach to cognitive infocommunications.**

***Index Terms*—cognitive systems, cognitive agents, quantum field theory, machine learning, inner stage, veridicality, attention modeling, ontology inference**

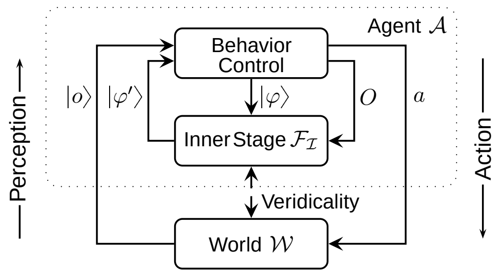

Figure 1. Perception-action cycle of a cognitive agent and the world, cf. [2]

I. Introduction

The most important aspects of cognitive agents are expressed by structure and function of the *perception-action cycle* (PAC, see Fig. 1) which models the interaction between agent and world. Central part of the agent model is the behavior controller, i.e. the agent's "brain". To provide for higher cognitive functions, the behavior controller should be equipped with an *inner stage* [1] (cf. [2], [3]) on which the agent can simulate actions and "contemplate" their effect. The inner stage helps to plan behavior and, particularly, to avoid danger for the agent.

It has been shown recently that several behavioral phenomena can be described in more detail by quantum mechanical models (e.g., [4], [5]). Hence, we propose to model the inner stage by a quantum formalism. Beforehand we will outline some foundations of quantum mechanics, namely state preparation, measurement, evolution and collapse.

Let $\mathcal{O}$ be a finite set of possible observations of the world and let us assume that a *ket-vector* $|b_o\rangle$ [6] can be readily assigned to every $o \in \mathcal{O}$ by perception as depicted in Fig. 1. To keep things simple, we define that all these kets form an orthonormal system

$$\mathcal{B} = \{|b_1\rangle, |b_2\rangle, \ldots\} \text{ with } \forall |b_i\rangle, |b_j\rangle \in \mathcal{B} : \langle b_i | b_j \rangle = \delta_{ij} \quad (1)$$

which can span a Hilbert space $\mathcal{H} = \mathrm{span}(\mathcal{B})$. A general vector $|\phi\rangle \in \mathcal{H}$ is a linear combination $|\phi\rangle = \sum_i \lambda_i |b_i\rangle$ with $\lambda_i \in \mathbb{R}$. As long as we consider a fully deterministic setting, it suffices to simplify $\mathcal{H}$ by replacing the scalar set $\mathbb{R}$ by $\{0, 1\}$.

In the quantum formalism, one pass through the PAC can be described by four steps: We start with the *preparation* of an agent state $|\varphi\rangle \leftarrow |o\rangle$ from an initial observation $|o\rangle$. To simulate an action, we apply a unitary operator O

$$|\varphi'\rangle = O|\varphi\rangle \quad \text{with } O = \textstyle\sum_{ij} \lambda_{ij} |o_i\rangle\langle o_j|, \lambda_{ij} \in \mathbb{R} \quad (2)$$

which is called *state evolution*. The result $|\varphi'\rangle$ is the predicted agent state after executing the respective real action a in the world. Then we perform that real action and receive an actual result, i.e. a new observation, $|o\rangle$. To compare prediction and observation, we perform a *measurement*

$$p\big(|\varphi'\rangle = |o\rangle\big) = \langle \varphi' | P_o | \varphi' \rangle = |\langle \varphi' | o \rangle|^2, \quad (3)$$

where $P_o = |o\rangle\langle o|$ denotes a measurement projector built from the observation $|o\rangle$. Note, that measurement does not determine the measured value, but the probability thereof. The last step—called *collapse*—changes the state of the inner stage to $|\varphi\rangle \leftarrow |o\rangle$. In the general case, collapse consists of projection and normalization. However, the restrictions we introduced simplify the description greatly. The collapsed state serves as the preparation for the next pass through the PAC.

Repeatedly passing through the PAC and comparing predicted states to observations allows building and adapting the inner stage. In the following sections we elaborate the quantum formalism by using Fock space instead of Hilbert space and by introducing some useful special operators.

II. The Inner Stage

Let us consider a mouse living in a simple $N \times M$ maze as shown for $N{=}M{=}3$ in Fig. 2. For the mouse to live, we supply bits of cheese and cups of water on some places in the maze. For the sake of simplicity, cheese and water are neverending, i.e., if the mouse eats or drinks, cheese and water are instantly replaced. The mouse can "read" the (x, y) coordinate tuple of its current position and "see" cheese and water at this position through *perceptors*. It can move to adjacent places in the maze to the north, east, south, and west through *actuators*, but it cannot directly influence the presence of water or cheese.

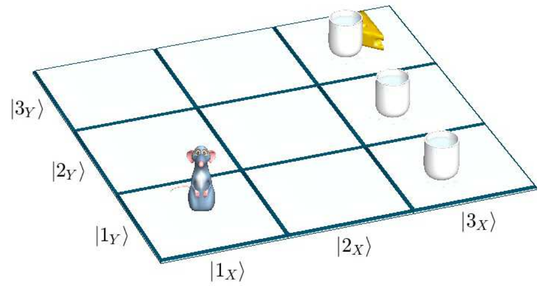

Figure 2. A simple mouse-maze world.

Thus, mouse and maze world are coupled in a perception-action cycle as shown in Fig. 1 and described in Section I.

For surviving, our mouse needs the basic capability of finding its way to cheese and water from any position in the maze world. To this end, it maintains an inner stage (see Section I), i.e. a mental model of its little world, in its mind.

A. Fock Space Model of the Inner Stage

In our maze world to be represented by the inner stage, we have x coordinates $x \in \{1, \ldots, N\}$, y coordinates $y \in \{1, \ldots, M\}$, information on cheese $c \in \{0, 1\}$ and information on water $g \in \{0, 1\}$. Firstly we create finite-dimensional coordinate Hilbert spaces, basically by just dedicating one basis vector $|n_X\rangle$ or $|m_Y\rangle$ to every possible coordinate value

$$\begin{aligned} \mathcal{H}_X &= \operatorname{span}(|1_X\rangle, \ldots, |N_X\rangle) \doteq \mathbb{R}^N && - x \text{ space}, \quad (4) \\ \mathcal{H}_Y &= \operatorname{span}(|1_Y\rangle, \ldots, |M_Y\rangle) \doteq \mathbb{R}^M && - y \text{ space}. \end{aligned}$$

We denote vectors in Hilbert spaces using Dirac's bra-ket notation [6], e.g.,[1]

$$|1_X\rangle \doteq \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad |2_X\rangle \doteq \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad \ldots, \quad |N_X\rangle \doteq \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}. \quad (5)$$

Secondly, we create two-dimensional Hilbert spaces for the binary values cheese and water

$$\begin{aligned} \mathcal{H}_C &= \operatorname{span}(|0_C\rangle, |1_C\rangle) \doteq \mathbb{R}^2 && - \text{cheese space}, \quad (6) \\ \mathcal{H}_G &= \operatorname{span}(|0_G\rangle, |1_G\rangle) \doteq \mathbb{R}^2 && - \text{water space}. \end{aligned}$$

We only consider orthonormal bases of $\mathcal{H}_X$, $\mathcal{H}_Y$, $\mathcal{H}_C$, and $\mathcal{H}_G$. Now, we define a finite-dimensional subspace of Fock space (cf. [5], [7], [8]) modeling the inner stage through

$$\begin{aligned} \mathcal{F}_\mathcal{I} &= (\mathbb{R} \oplus \mathcal{H}_X) \otimes (\mathbb{R} \oplus \mathcal{H}_Y) \otimes (\mathbb{R} \oplus \mathcal{H}_C) \otimes (\mathbb{R} \oplus \mathcal{H}_G) && (7) \\ &= \mathbb{R} && (7a) \\ &\oplus \mathcal{H}_X \oplus \mathcal{H}_Y \oplus \mathcal{H}_C \oplus \mathcal{H}_G && (7b) \\ &\oplus (\mathcal{H}_X \otimes \mathcal{H}_Y) \oplus (\mathcal{H}_X \otimes \mathcal{H}_C) \oplus (\mathcal{H}_X \otimes \mathcal{H}_G) && (7c) \\ &\quad \oplus (\mathcal{H}_Y \otimes \mathcal{H}_C) \oplus (\mathcal{H}_Y \otimes \mathcal{H}_G) \oplus (\mathcal{H}_C \otimes \mathcal{H}_G) \\ &\oplus (\mathcal{H}_X \otimes \mathcal{H}_Y \otimes \mathcal{H}_C) \oplus (\mathcal{H}_X \otimes \mathcal{H}_Y \otimes \mathcal{H}_G) && (7d) \\ &\quad \oplus (\mathcal{H}_X \otimes \mathcal{H}_C \otimes \mathcal{H}_G) \oplus (\mathcal{H}_Y \otimes \mathcal{H}_C \otimes \mathcal{H}_G) \\ &\oplus \mathcal{H}_X \otimes \mathcal{H}_Y \otimes \mathcal{H}_C \otimes \mathcal{H}_G, && (7e) \end{aligned}$$

where $\oplus$ denotes the direct sum and $\otimes$ the tensor product. The summands 7a–7e are called *sectors* and indexed from 0 to 4. Note that, in contrast to applications in quantum physics, we use a *finite*-dimensional subspace of Fock space over the field of *real numbers* and *without* symmetrization or anti-symmetrization. Similar theoretical frameworks based on Fock space are presented in [9], [10]. There, Fock space is used to represent linguistic constituent structure either through blocks of variable length or through phrase structure trees to provide a generalized theory of linguistic messages.

B. What Is or Could Be the Case: Maze and Mouse States

For our 3×3 maze example, the inner stage's Fock space $\mathcal{F}_\mathcal{I}$ comprises 144 dimensions. The maze and mouse states are (superpositions of) vectors in the 4th sector. There are 36 possible *basic situations* in a 3×3 maze with cheese and water:

$$\begin{aligned} &|1_X 1_Y 0_C 0_G\rangle && \text{– no cheese/water at pos. } (1,1) && (8) \\ &|1_X 1_Y 1_C 0_G\rangle && \text{– cheese but no water at pos. } (1,1) \\ &|1_X 1_Y 0_C 1_G\rangle && \text{– no cheese but water at pos. } (1,1) \\ &|1_X 1_Y 1_C 1_G\rangle && \text{– cheese and water at pos. } (1,1) \\ &\qquad \ldots && \text{(etc. for the remaining eight places).} \end{aligned}$$

In (8) we used the usual abbreviation of the tensor product $|xy\rangle = |x\rangle \otimes |y\rangle$. The 36-element set of basic situations will be denoted by $\mathcal{B}_\mathcal{I}$. Firstly, we define the *maze state* as the unnormalized superposition of those kets from (8) which are actually "the case" (cf. Fig. 2):

$$\begin{aligned} |w\rangle = {} & |1_X 1_Y 0_C 0_G\rangle + |2_X 1_Y 0_C 0_G\rangle + |3_X 1_Y 0_C 1_G\rangle && (9) \\ & + |1_X 2_Y 0_C 0_G\rangle + |2_X 2_Y 0_C 0_G\rangle + |3_X 2_Y 0_C 1_G\rangle \\ & + |1_X 3_Y 0_C 0_G\rangle + |2_X 3_Y 0_C 0_G\rangle + |3_X 3_Y 1_C 1_G\rangle. \end{aligned}$$

Secondly, we define the *mouse state* as one of the summands from (9), depending on the current location of the mouse which is supposed to sit on one of the maze locations at any given point of time. E.g., if the mouse sits on place $(1, 1)$ the state would be

$$|\varphi\rangle = |1_X 1_Y 0_C 0_G\rangle. \quad (10)$$

In a fully deterministic setting, i.e. no randomness in the maze and no sensor or actuator noise, the maze state $|w\rangle$ can be learned by a simple procedure: We start with the zero state $|w\rangle = 0$ and subsequently perform a random exploration[2] producing a mouse state $|o^k\rangle$ (observation) at every exploration step k. The observation is used to update the maze state

$$|w\rangle \leftarrow |w\rangle + \big(1 - \langle w|o^k\rangle\big)\, |o^k\rangle. \quad (11)$$

As the mouse can perceive its location (x, y) and the presence of cheese and water, observations are readily available.

[1]Note that the Dirac notation is symbolic and does *not* imply the choice of a particular basis.

[2]This can be a random walk trough the maze or, as we have not yet formally defined movements, by repeatedly setting the mouse on random locations.

1) Excursus on Probabilistic States: Uncertainty about the mouse state can occur due to fallacy (sensor noise), mistake (actuator noise), and environmental incidences (system noise). For example, multiple measurements of the mouse state on position (n, m) can yield different results regarding the existence of cheese and water. Let $p(n, m, 0, 0)$ be the relative frequency of measuring no cheese and no water at position (n, m) and let $p(n, m, 0, 1)$, $p(n, m, 1, 0)$ and $p(n, m, 1, 1)$ be defined analogously.

There are two principal possibilities of expressing this uncertainty: by superposition and by a density matrix. A probabilistic superposition of the example above would be

$$|\varphi\rangle = \sum_{ij\in\{0,1\}^2} \sqrt{p(n, m, i, j)}\, |n_X m_Y i_C j_G\rangle. \tag{12}$$

State preparation, evolution, measurement and collapse work exactly as described in Section I. Operators applied to the state need not be invertible or unitary. However, this superposition may interfere with the definition of the maze state in (9).

A density matrix (cf. [2], [11]) can express the same situation:

$$\rho = \sum_{ij\in\{0,1\}^2} p(n, m, i, j)\, |n_X m_Y i_C j_G\rangle\langle n_X m_Y i_C j_G|. \tag{13}$$

A measurement specified by an operator P of the state in form of a density operator is given by $\mathrm{tr}(\rho P)$. It can be shown, that if the measurement operator is $|n_X m_Y i_C j_G\rangle\langle n_X m_Y i_C j_G|$ then the measurement yields $p(n, m, i, j)$. State evolution by an operator O on a density matrix ρ is then given as a similarity transform $\rho' = O\rho O^{-1}$, provided that O is invertible.

In future work we will investigate which of the models of uncertainty should be used best.

2) Veridical, Fantasy, and Ignorance States: We call a maze or mouse state *veridical* (cf. [1]), iff it is a basic situation or a superposition of basic situations according to (8), that are actually the case in the world. By this definition, (9) and (10) are veridical. The inner stage also allows for fantasy (cf. [3]). E.g., the mouse could *imagine* to have cheese but not water at position $(1, 1)$ which is expressed by the basic situation $|\ddot{\varphi}_{11}\rangle = |1_X 1_Y 1_C 0_G\rangle$. We define a fantasy state as being *not* veridical, i.e. not the case in the world. Further, we model ignorance by unnormalized superposition, e.g., if the mouse does not know whether there is cheese or water at position (n, m), the respective ignorance state would be $|\tilde{\varphi}_{nm}\rangle = |n_X m_Y (0_C + 1_C)(0_G + 1_G)\rangle$.

Having a veridical maze state $|w\rangle$, we can turn ignorance mouse states into veridical ones by $|\varphi\rangle = P_V|\tilde{\varphi}\rangle$, where P_V denotes the *veridicality projector* defined by

$$P_V = \sum_{|b\rangle\in\mathcal{B}_\mathcal{I}} \langle w|b\rangle |b\rangle\langle b|. \tag{14}$$

The sum runs over the 36 basic situations $\mathcal{B}_\mathcal{I}$ of $\mathcal{F}_\mathcal{I}$. For location $(1, 1)$ in our example we get

$$\begin{aligned}|\varphi_{11}\rangle &= P_V|\tilde{\varphi}_{11}\rangle = P_V|1_X 1_Y (0_C + 1_C)(0_G + 1_G)\rangle \\ &= |1_X 1_Y 0_C 0_G\rangle.\end{aligned} \tag{15}$$

Applying the veridicality projector to a fantasy state yields 0.

C. Getting Around: Movement Actions

As explained above, our mouse can move in the maze but not influence the presence of cheese or water at any location. Therefore we call the location variables x and y *controllables*, and the cheese and water variables c and g *non-controllables*. If the mouse is hungry or thirsty and there is no food or water at its current location, it needs to move which, in turn, requires prior planning of a route through the maze on the mouse's inner stage. To this end the mouse needs *trial actions*, i.e. actions which are applicable on the inner stage.

With basic knowledge on the maze (variables x, y, c, g including their ranges), we can construct operators describing mouse movements.

1) Subspace Operators: On the x and y coordinate subspaces there are two possible basic operations, increment the coordinate value[3]

$$\begin{aligned}O_{Xinc} &= \sum_{n=1}^{N} |((n \mod N) + 1)_X\rangle\langle n_X|, \\ O_{Yinc} &= \sum_{m=1}^{M} |((m \mod M) + 1)_Y\rangle\langle n_Y|,\end{aligned} \tag{16}$$

and no operation

$$O_{Xnop} = \sum_{n=1}^{N} |n_X\rangle\langle n_X|,\; O_{Ynop} = \sum_{m=1}^{M} |m_Y\rangle\langle m_Y|. \tag{17}$$

For simplicity we assume the maze to be a torus, i.e. exiting it to the right or top will bring the mouse back to the left or bottom side. Decrementing is the inverse operation of incrementing

$$O_{Xdec} = {O_{Xinc}}^{-1}, \quad O_{Ydec} = {O_{Yinc}}^{-1}. \tag{18}$$

For the cheese and water variables, there are inversion

$$\begin{aligned}O_{Cinv} &= |0_C\rangle\langle 1_C| + |1_C\rangle\langle 0_C|, \\ O_{Ginv} &= |0_G\rangle\langle 1_G| + |1_G\rangle\langle 0_G|,\end{aligned} \tag{19}$$

and no operation

$$\begin{aligned}O_{Cnop} &= |0_C\rangle\langle 0_C| + |1_C\rangle\langle 1_C|, \\ O_{Gnop} &= |0_G\rangle\langle 0_G| + |1_G\rangle\langle 1_G|.\end{aligned} \tag{20}$$

As ${O_{Xinc}}^\dagger = \sum_{n=1}^{N} |n_X\rangle\langle((n \bmod N) + 1)_X|$ is the adjoint operator of O_{Xinc} above, we immediately obtain ${O_{Xinc}}^\dagger = {O_{Xinc}}^{-1}$, proving unitarity.

2) Generic and Veridical Movement Trial Action Operators: Movement on the x-y-plane in one of the cardinal directions N, E, S, and W, is described by a tensor product of suitable subspace operators on the x and y coordinate spaces. To be applicable to the Fock space of the inner stage, we need two more factors: a cheese and a water operation. The problem with the latter is that cheese and water depend on the location (x, y) and we do not know whether to use "invert" or "no-operation" for constructing a movement operator. Hence, employing the ignorance principle explained above, we use the superpositions $O_{Cinv} + O_{Cnop}$ and $O_{Ginv} + O_{Gnop}$ to define

[3]Note, that coordinates are one-based.

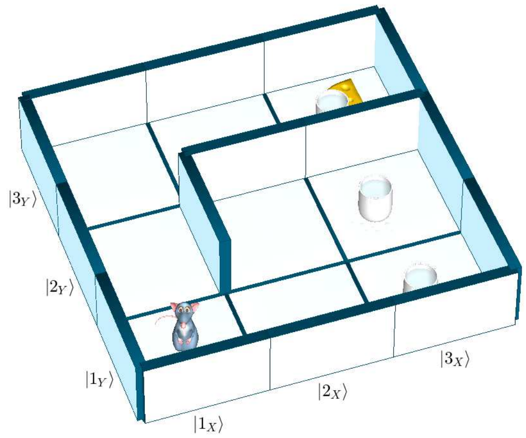

Figure 3. A simple mouse-maze world with walls.

generic east and north operators (we will take care for west and south in a moment)

$$\widetilde{O}_E = O_{X\,inc} \otimes O_{Y\,nop} \otimes (O_{Cinv} + O_{Cnop}) \otimes (O_{Ginv} + O_{Gnop}), \tag{21}$$

$$\widetilde{O}_N = O_{X\,nop} \otimes O_{Y\,inc} \otimes (O_{Cinv} + O_{Cnop}) \otimes (O_{Ginv} + O_{Gnop}). \tag{22}$$

By "generic" we mean that the so-defined operators are ignorant of cheese and water. In order to make the operators veridical, i.e. predicting the correct cheese and water state at the target location, we compute the products of the veridicality projector and the generic movement operators

$$O_E = P_V \widetilde{O}_E, \quad O_N = P_V \widetilde{O}_N. \tag{23}$$

The veridical go-west and go-south movement operators are then the inverses of the veridical go-east and go-north operators, respectively

$$O_W = {O_E}^{-1}, \quad O_S = {O_N}^{-1}. \tag{24}$$

3) Performing Trial Actions on the Inner Stage: Moving the mouse on the inner stage is just multiplying its state ket $|\varphi\rangle$ with one of the veridical movement operators O_N, O_E, O_S, or O_W or arbitrary products of them, e.g.,

$$|\varphi'\rangle = O_E\, O_E\, O_N\, O_N\, |\varphi\rangle, \tag{25}$$

which means "go two steps to the north and then two steps to the east". As the operators are veridical, the mouse can correctly predict if it will, in the real world, have cheese and/or water in result of any sequence of movements. Thus it is capable of performing route planning on the inner stage.

D. Mazes with Walls

In a torus maze without obstacles (walls), veridical movement operators (23) and (24) are unitary, i.e. reversible in the sense that going west (south) in any case "undoes" going east (north) and vice versa. Things are not so simple anymore if the maze has boundaries and walls (see Fig. 3). Going north from a place with a wall at the northern side in the real world will, at best, result in no change of position (if not in a bloody mouse nose). The respective veridical trial action operator should at least predict "no movement".

As, in our model, the mouse does not have a perceptor for walls, veridicality of trial action operators in a maze with walls can only be established by actually trying actions in the real world. Similar to learning the maze state by subsequently applying (11), we can learn a movement operator O_d by performing the action "go into direction d" and updating the corresponding operator via

$$O_d \leftarrow O_d + \big(1 - \langle o^k | O_d | \varphi \rangle\big)\, |o^k\rangle\langle\varphi|, \tag{26}$$

where φ is the mouse state before performing the action and o^k the observation after performing the kth action. To be capable of this learning method, the mouse needs to know the concept of "going into a direction" and how to perform the corresponding action in the world.

Operators obtained by applying (26) are veridical by construction but neither unitary nor, in general, invertible. The first is immediately obvious as "go-west" cannot be the inverse of "go-east" anymore, since after hitting the wall on position $(1, 1)$—and thus not changing the position—going to position $(2, 1)$ is not an undo. This is no problem in the case where the mouse state is a simple ket vector. For the use with density matrices there are at least two solutions for getting unitary movement operators back. The first one comes from spin-algebra [12] and results in a complete deletion of the mouse state. The second one emerges from quantum computation since the problem of failed reversibility in the context of walls is related to the *no-cloning* theorem, see [11, p. 23, 24]. This theorem forbids quantum circuits with *FANIN* and *FANOUT*. The typical solution to that problem is to simulate the forbidden semantics by introducing hidden wires. Hidden wires can be seen as artificial dimensions for *fantasy* states which guarantee reversibility. Since the deletion of the mouse state could be useful to represent, for example, "death by cat", it seems natural to go with the second solution for the encoding of impossible actions. Either way the resulting operators are no longer veridical, but the violation of veridicality in certain situations signals the impossibility of the action in question.

III. Attention and Ontology

The introduced concept of ignorance where some bits are unknown, can also be understood that one does not care about these bits. If we do not care about them, let us get rid of them. Therefore, we define linear operators crossing the boundaries of sectors of the inner stage. In fact we do not map to whole sectors but only to individual summands from (7), thus focusing on the information contained within those subspaces, which we will refer to as *value-spaces* from now on. So, e.g. mapping from the 4th into the 2nd sector results in dropping two bits of information. For deciding which value-spaces are helpful we use ontologies, like in Fig. 4, which can be learned from data [13].

We set $\mathcal{I} = \{X, Y, C, G\}$ and denote, for any combination $S \subset \mathcal{I}$ of coordinates, cheese and water, with $\mathcal{H}_S$ the one value-space of 7, where for every $s \in S$ a factor $\mathcal{H}_s$ exists within.

A. Focus Operators

To completely ignore cheese and water and concentrate our attention on the x-y value-space, we design the *focus operator* $F_{XY} : \mathcal{H}_\mathcal{I} \to \mathcal{H}_{\{X,Y\}}$ by

$$F_{XY} = \sum_{n=1}^{N} \sum_{m=1}^{M} |n_X m_Y\rangle\langle n_X m_Y (0_C+1_C)(0_G+1_G)|. \quad (27)$$

Since $\mathcal{H}_{\{X,Y\}}$ belongs to the 2nd sector of $\mathcal{F}_\mathcal{I}$ two pieces of information are retained (the x and y coordinate) and also two pieces of information ($2 = 4 - 2$) are lost (cheese and water).

B. Generic and Veridical Unfocus Operators

Performing the reverse operation of unfocusing also needs a post-processing step to make it veridical. We define the *generic unfocus operator* $\widetilde{U}_{XY} : \mathcal{H}_{\{X,Y\}} \to \mathcal{H}_\mathcal{I}$

$$\widetilde{U}_{XY} = \sum_{n=1}^{N} \sum_{m=1}^{M} |n_X m_Y (0_C+1_C)(0_G+1_G)\rangle\langle n_X m_Y| \quad (28)$$

which is ignorant of cheese and water. Again we get a *veridical unfocus operator* by using the veridicality projector P_V (14)

$$U_{XY} = P_V \widetilde{U}_{XY}. \quad (29)$$

Note that for any focus-unfocus-operator-pair $F = \widetilde{U}^\dagger$ holds.

C. Paying Attention

Focus operators of the form (27) allow for the reduction of dimensions under feasible circumstances. Possible scenarios could be finding cheese or water independently from one another. Whenever bits of information have no relevancy for solving a problem, we can avoid too complex computations by applying suitable focus operators. This way the computational costs can be reduced and we only have to pay for the things we center our attention on.

Using focus and unfocus operators we gain an alternative construction of movement operators: First we focus on the x-y value-space, then perform the appropriate combination of movements in x and y directions and do a veridical unfocus afterwards. The go-east operator is computed by

$$O_E = U_{XY}(O_{X inc} \otimes O_{Y nop}) F_{XY}. \quad (30)$$

Note, that going east implies *no* movement in y direction. Hence, focusing in x *and* y direction is necessary. Otherwise the meaning would be "y does not matter" which is obviously not correct. Focus operators can also be established from arbitrary value-spaces $\mathcal{H}_S$ to value-spaces $\mathcal{H}_T$ with $\mathcal{I} \supset S \supsetneq T$ whereas unfocus operators can map in opposite direction.[4]

[4]We appreciate that there is a similarity to creation and annihilation operators from quantum field theory.

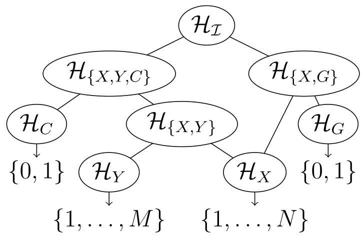

Figure 4. Ontology for the simple mouse-maze world.

D. Discovering Ontologies

Using the distinction between controllables and non-controllables we can use focus operators and projectors to discover additional structure in Fock space (cf. [13]). Let $\mathcal{N} \subset \mathcal{I}$ be the set of non-controllables and $\mathcal{M} \subset \mathcal{I}$ the set of controllables. For every non-controllable $r \in \mathcal{N}$ we search for functional dependencies on a set $D \subset \mathcal{M}$ of controllables. Therefore, $\mathcal{H}_\mathcal{I}$ has to be divided by an internal direct sum induced by $\mathcal{H}_r$. For cheese this results in the creation of the two projectors

$$P_{1_C} = \sum\nolimits_{|b\rangle \in \mathcal{B}_\mathcal{I}} \langle 1_C | F_C | b\rangle |b\rangle\langle b| \text{ and} \quad (31)$$
$$P_{0_C} = \sum\nolimits_{|b\rangle \in \mathcal{B}_\mathcal{I}} \langle 0_C | F_C | b\rangle |b\rangle\langle b|.$$

Whenever, for a set $D \subset \mathcal{M}$,

$$\langle w | P_{1_C} \widetilde{U}_D F_D P_{0_C} | w\rangle = 0 \quad (32)$$

holds, then there is a map $\zeta_{D,C} : \mathcal{H}_D \to \mathcal{H}_C$. Also applying the procedure to water and looking for the smallest set D each, we eventually get the structure depicted in Fig. 4.

E. Using Ontologies for Guiding Attention

Semantic structures as depicted in Fig. 4 encode dependencies of non-controllables on controllables and serve as a source for planning. Starting from the non-controllable in question an ontology holds the information which controllables to concentrate attention on in order to reach the desired change. Therefore, anything unrelated to the current problem can be ignored, resulting in a reduction of the search-space. Additionally, ontologies not only structure the inner stage, and thus the world as seen by a cognitive agent. They also serve as structures of communication and are widely used as such in natural language understanding and generation.

IV. Cognitive Infocommunications

From the viewpoint of semiotics, the inner stage discussed in this paper can be seen as the top of the semiotic triangle, i.e. the "concept" or "reference" aspect of symbols. Figure 5 shows the interconnected semiotic triangles of two cognitive agents, $\mathcal{A}_1$ and $\mathcal{A}_2$, playing a Horn communication game [14], with their respective inner stages $\mathcal{F}_{\mathcal{I}1}$ and $\mathcal{F}_{\mathcal{I}2}$, sharing the same world $\mathcal{W}$ and the same language $\mathcal{L}$ for communication. Veridicality of the inner stage wrt. the world $V_{\cdot\mathcal{W}}$ corresponds to "adequateness" in the semiotic triangle. Thus a veridical inner stage is a crucial premise for any communication between agents. Veridicality wrt. to language $V_{\cdot\mathcal{L}}$—"correctness" in

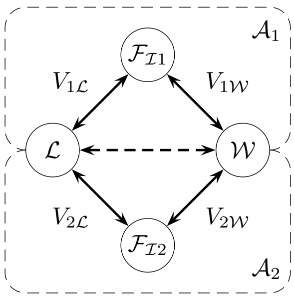

Figure 5. Interconnected semiotic triangles of two cognitive agents with inner stages $\mathcal{F}_{\mathcal{I}1}$ and $\mathcal{F}_{\mathcal{I}2}$ and veridicality relationships wrt. the world $\mathcal{W}$ ("adequateness", $V_{\cdot\mathcal{W}}$) and wrt. language $\mathcal{L}$ ("correctness", $V_{\cdot\mathcal{L}}$)

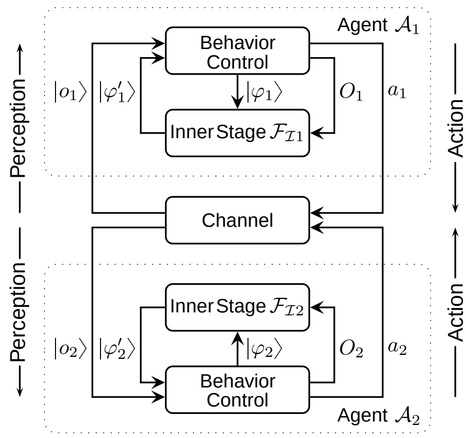

Figure 6. Interconnected perception-action cycles of two communicating cognitive agents. Here, a_n and $|o_n\rangle$ denote the intended and perceived meanings of utterances transmitted through the communication channel. The goal of communication is to synchronize the inner stages $\mathcal{F}_{\mathcal{I}1}$ and $\mathcal{F}_{\mathcal{I}2}$.

semiotic terms—can be modeled and attained using very much the same principles as described above (cf. [15], [16]). Using language communication as outlined in Fig. 6, agents can share information on the world (intra-cognitive communication [17]) which means that they do not need to learn everything from own observations. Of course, agents can also be designed to communicate with humans using a natural language (inter-cognitive communication). As the inner stage handles semantic data structures, it is independent of sensor and communication modalities. Hence, our model provides for both, sensor-sharing and sensor-bridging communication.

V. Conclusion: It's More than Mice

We intentionally chose a simplistic example demonstrating our inner stage model. Yet it covers important features of general cognitive agents: There are aspects of the world which can be perceived (variables x, y, c, g). Some of them can be manipulated by the agent (controllable variables x, y) and some cannot (non-controllable variables c, g). There has to be knowledge about the world and about the self in the mind—or, technically speaking, the behavior controller—of the agent (maze and mouse states $|w\rangle, |\varphi\rangle$, ontologies [13], trial operators O, actions a) which needs to be built-in or attained by learning from observations (e.g. (11), (26)) and by inference (e.g. [18]). This knowledge must include that actions may be unfeasible in a given context, or one-way (e.g. you would normally be unable to undo a "kill" action in the natural world and it is beneficial for action planning to know about this). Further, knowledge must be organized such that it allows focusing attention on the task at hand. For higher cognitive functions like coping [3] there needs to be some kind of fantasy, too. Finally, although not elaborated in this paper, agents must be able to cope with and to plan under uncertainty, e.g., actuator or sensor noise, or randomness in the world. As stochasticity comes naturally with the mathematics of quantum mechanics, our model provides for this, too.

References

[1] N. Bischof, *Psychologie, ein Grundkurs für Anspruchsvolle*. Verlag Kohlhammer, 2009, 2. Auflage.

[2] I. Schmitt, R. Römer, G. Wirsching, and M. Wolff, "Denormalized Quantum Density Operators for Encoding Semantic Uncertainty in Cognitive Agents," in *8th IEEE International Conference on Cognitive Infocommunications CogInfoCom 2017 Proceedings, September 11-14, 2017 Debrecen, Hungary*, 2017, pp. 165–170.

[3] M. Wolff, R. Römer, and G. Wirsching, "Towards Coping and Imagination for Cognitive Agents," in *6th IEEE International Conference on Cognitive Infocommunications (CogInfoCom), Györ (Hungary), October 9-21, 2015*, 2015, pp. 307–312.

[4] J. R. Busemeyer and P. D. Bruza, Eds., *Quantum Models of Cognition and Decision*. Cambridge University Press, 2012.

[5] P. Smolensky and G. Legendre, *The Harmonic Mind. From Neural Computation to Optimality-Theoretic Grammar*. Cambridge (MA): MIT Press, 2006, vol. 1: Cognitive Architecture.

[6] P. A. M. Dirac, "A new notation for quantum mechanics," *Mathematical Proceedings of the Cambridge Philosophical Society*, vol. 35, no. 3, pp. 416–418, 1939.

[7] V. A. Fock, "Konfigurationsraum und zweite Quantelung," *Z. Phys.*, vol. 75, pp. 622–647, 1932.

[8] M. Wolff, G. Wirsching, M. Huber, P. beim Graben, R. Römer, and I. Schmitt, "A Fock Space Toolbox and Some Applications in Computational Cognition," doi: 10.13140/RG.2.2.10025.70245, 2018.

[9] K. J. Boström, "Concepts of a quantum information theory of many letters," https://arxiv.org/abs/quant-ph/0009052, 2000.

[10] P. beim Graben, M. Huber, R. Römer, I. Schmitt, and M. Wolff:, "Der Fockraum als Labyrinth: Wissensrepräsentation und Sprachverarbeitung am Beispiel des Mouse-Maze-Problems," in *Elektronische Sprachsignalverarbeitung 2018*, A. Berton, U. Haiber, and W. Minker, Eds. TUDpress, 2018, pp. 167–174.

[11] M. A. Nielsen and I. L. Chuang, *Quantum computation and quantum information*. Cambridge university press, 2010.

[12] A. R. Edmonds, *Angular Momentum in Quantum Mechanics*. Princeton University Press, 1957.

[13] M. Huber and R. Römer, "Agenten in höheren Sphären. Quantenmechanische Situationsmodellierung am Beispiel des Mouse-Maze-Problems," in *Elektronische Sprachsignalverarbeitung 2018*, A. Berton, U. Haiber, and W. Minker, Eds. TUDpress, 2018, pp. 127–134.

[14] A. Benz, G. Jäger, and R. van Rooij, "An introduction to game theory for linguists," in *Game Theory and Pragmatics*, A. Benz, G. Jäger, and R. v. Rooij, Eds. Basingstroke: Palgrave Macmillan, 2006, pp. 1–82.

[15] M. Huber and M. Wolff, "Segmenting Sequences Semantically. Using Petri Net Transducers for the Translation from Sequential Data to Non-Sequential Models," in *Proc. ATAED*, 2017, pp. 139–157. [Online]. Available: http://ceur-ws.org/Vol-1847/paper11.pdf

[16] J. Lindemann, "Interpretation und Artikulation mit Äußerungs-Bedeutungs-Transduktoren," PhD Thesis, BTU Cottbus-Senftenberg, 2016. [Online]. Available: https://opus4.kobv.de/opus4-btu/frontdoor/index/index/docId/3891

[17] P. Baranyi, A. Csapo, and G. Sallai, *Cognitive Infocommunications (CogInfoCom)*. Springer, 2015, ISBN 9783319196077.

[18] P. Klimczak, M. Wolff, J. Lindemann, C. Petersen, R. Römer, and T. Zoglauer, "Die kognitive Heizung," in *Elektronische Sprachsignalverarbeitung 2014, Tagungsband der 25. Konferenz*, R. Hoffmann, Ed., 2014, pp. 89–96.