



Robust optimization of coordinated electricity and gas system expansion

30 October 2019

Iegor Riepin

Multistage stochastic programming

The “classical” two-stage stochastic program can be formulated as follows:

$$\min_{x \in X} \varphi(x, \omega) = c^T x + \mathbb{E}[Q(y(x, \omega))]$$

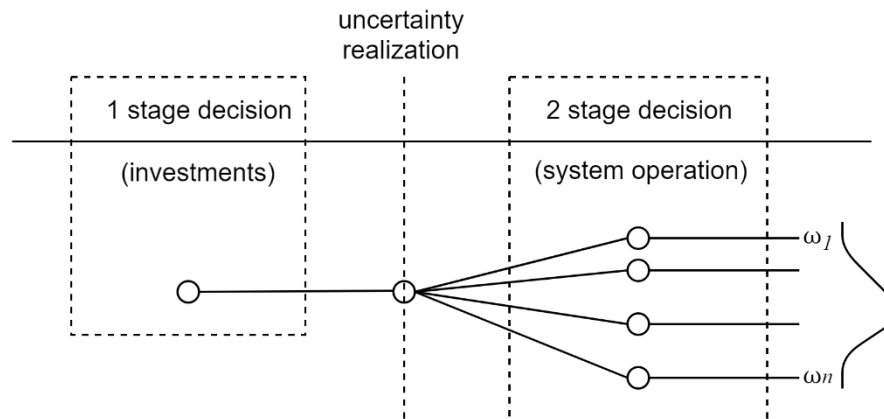
Where:

- x represent the vector of first-stage decisions
- ω represent the vector of uncertain outcomes
- $y(x, \omega)$ represent the vector of second-stage decisions

Multistage stochastic programming

The standard approach to solve this problem numerically:

- i. Assume that vector ω has a finite number of realizations (scenarios) $\omega_1 \dots \omega_n$
- ii. with respective (positive) probabilities $p_1 \dots p_n \mid \sum_1^n p = 1$



- iii. Then the problem can be reformulated with a deterministic LP equivalent

$$\min_{x, y_1, \dots, y_n} c^T x + \sum_{n=1}^N p_n Q(y_n(x, \omega))$$

Adaptive robust optimization

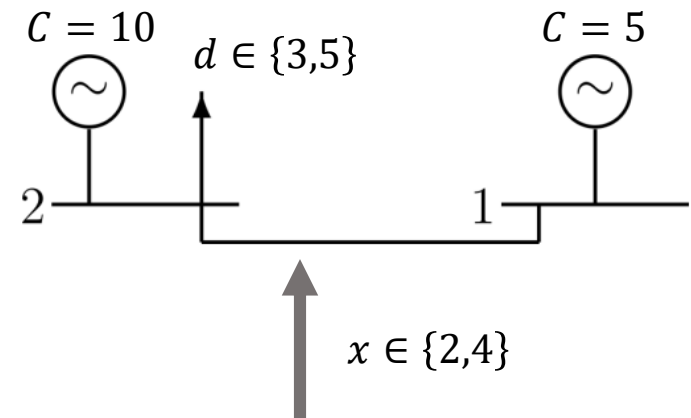
$$\min_{x \in X} \max_{u \in U} \min_{y \in Y(x, u)} f(x, y, u)$$

Adaptive robust optimization

Source: Antonio J. Conejo

A lecture on adaptive robust optimization, DTU,
13 June 2019.

<https://www.youtube.com/watch?v=Zk6y8joQLNQ>



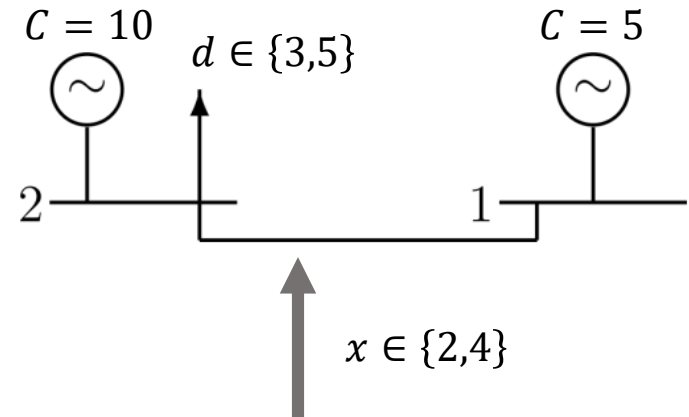
- A planner has the option of building just one of two alternative lines with a capacity of 2 or 4. Per unit cost of investment is 1.
- At node 1, a generator with per unit production cost of 5.
- At node 2, a generator with per unit production cost of 10.
- Demand is at node 2 and can take values either 3 or 5.

Adaptive robust optimization

Source: Antonio J. Conejo

A lecture on adaptive robust optimization, DTU,
 13 June 2019.

<https://www.youtube.com/watch?v=Zk6y8joQLNQ>



$$\min_{x \in \{2,4\}} \max_{d \in \{3,5\}} \min_{y_1, y_2 \geq 0} Z = 1x + 5y_1 + 10y_2$$

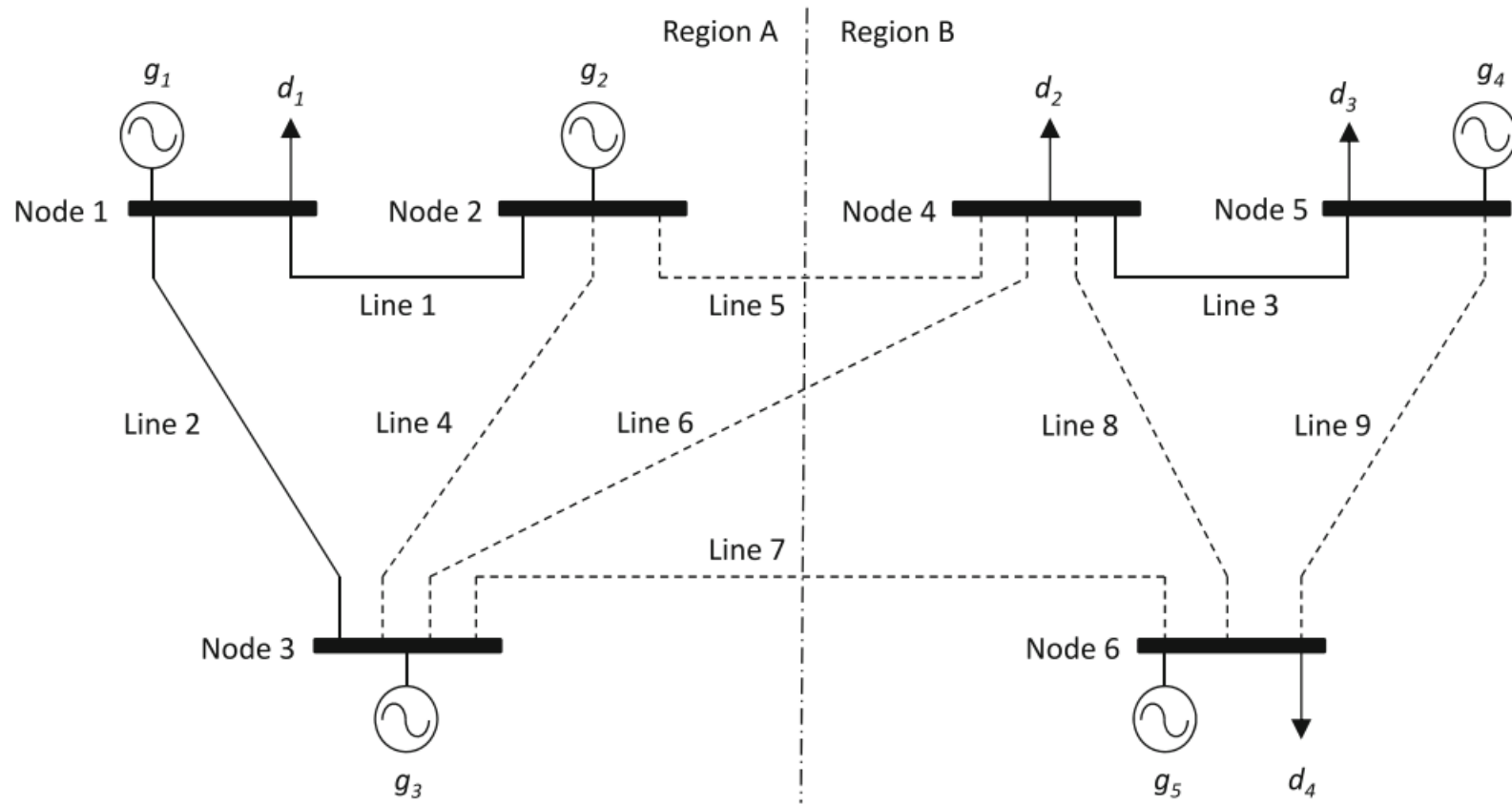
$$\text{s.t.} \quad y_1 + y_2 = 10$$

$$y_1 \leq x$$

Given the simplicity of this problem, it can be solved by enumeration.

... a few important notes before moving on.

Illustrative case: 6-node system



An illustrative case is based on Conejo, A. J., Baringo, M. L., Kazempour, S. J., & Siddiqui, A. S. (2016). *Investment in Electricity Generation and Transmission: Decision Making under Uncertainty*.

Deterministic MINLP

$$\min_{\Delta} \sum_{l \in \Omega^{L+}} \tilde{I}_l X_l^L + \sigma \left[\sum_g C_g^E P_g^E + \sum_d C_d^{LS} P_d^{LS} \right] \quad [1.a]$$

$$\Delta = \{X_l^L, P_l^L, P_g^E, P_d^{LS}, \theta_n\}$$

s.t.

$$\sum_{l \in \Omega^{L+}} \tilde{I}_l X_l^L \leq I^{\max} \quad [1.b]$$

$$X_l^L = \{0,1\} \quad \forall l \in \Omega^{L+} \quad [1.c]$$

$$\sum_{g \in \Omega_n^E} P_g^E - \sum_{l|s(l)=n} P_l^L + \sum_{l|r(l)=n} P_l^L = \sum_{d \in \Omega_n^D} (P_d^{Dmax} - P_d^{LS}) \quad \forall n \quad [1.d]$$

$$P_l^L = B_l(\theta_{s(l)} - \theta_{r(l)}) \quad \forall l \setminus l \in \Omega^{L+} \quad [1.e]$$

$$P_l^L = X_l^L B_l(\theta_{s(l)} - \theta_{r(l)}) \quad \forall l \in \Omega^{L+} \quad [1.f]$$

$$-F_l^{\max} \leq P_l^L \leq F_l^{\max} \quad \forall l \quad [1.g]$$

$$0 \leq P_g^E \leq P_d^{E\max} \quad \forall g \quad [1.h]$$

$$0 \leq P_d^{LS} \leq P_d^{Dmax} \quad \forall d \quad [1.i]$$

$$-\pi \leq \theta_n \leq \pi \quad \forall n \quad [1.j]$$

$$\theta_n = 0 \quad n: ref. \quad [1.k]$$

Deterministic MINLP

$$\min_{\Delta} \sum_{l \in \Omega^{L+}} \tilde{I}_l X_l^L + \sigma \left[\sum_g C_g^E P_g^E + \sum_d C_d^{LS} P_d^{LS} \right] \quad [1.a]$$

$$\Delta = \{X_l^L, P_l^L, P_g^E, P_d^{LS}, \theta_n\}$$

s.t.

$$\sum_{l \in \Omega^{L+}} \tilde{I}_l X_l^L \leq I^{\max} \quad [1.b]$$

$$X_l^L = \{0,1\} \quad \forall l \in \Omega^{L+} \quad [1.c]$$

$$\sum_{g \in \Omega_n^E} P_g^E - \sum_{l|s(l)=n} P_l^L + \sum_{l|r(l)=n} P_l^L = \sum_{d \in \Omega_n^D} (P_d^{Dmax} - P_d^{LS}) \quad \forall n \quad [1.d]$$

$$P_l^L = B_l(\theta_{s(l)} - \theta_{r(l)}) \quad \forall l \setminus l \in \Omega^{L+} \quad [1.e]$$

nonlinear term $\left\{ \begin{array}{l} P_l^L = B_l(\theta_{s(l)} - \theta_{r(l)}) \quad \forall l \setminus l \in \Omega^{L+} \\ P_l^L = X_l^L B_l(\theta_{s(l)} - \theta_{r(l)}) \quad \forall l \in \Omega^{L+} \end{array} \right. \quad [1.f]$

$$-F_l^{\max} \leq P_l^L \leq F_l^{\max} \quad \forall l \quad [1.g]$$

$$0 \leq P_g^E \leq P_d^{Emax} \quad \forall g \quad [1.h]$$

$$0 \leq P_d^{LS} \leq P_d^{Dmax} \quad \forall d \quad [1.i]$$

$$-\pi \leq \theta_n \leq \pi \quad \forall n \quad [1.j]$$

$$\theta_n = 0 \quad n: ref. \quad [1.k]$$

Deterministic MILP

$$\min_{\Delta} \sum_{l \in \Omega^{L+}} \tilde{I}_l X_l^L + \sigma \left[\sum_g C_g^E P_g^E + \sum_d C_d^{LS} P_d^{LS} \right] \quad \text{s.t.} \quad [1.a]$$

$$\Delta = \{X_l^L, P_l^L, P_g^E, P_d^{LS}, \theta_n\}$$

$$\sum_{l \in \Omega^{L+}} \tilde{I}_l X_l^L \leq \tilde{I}^{max} \quad [2.b]$$

$$X_l^L = \{0,1\} \quad \forall l \in \Omega^{L+} \quad [2.c]$$

$$\sum_{g \in \Omega_n^E} P_g^E - \sum_{l|s(l)=n} P_l^L + \sum_{l|r(l)=n} P_l^L = \sum_{d \in \Omega_n^D} (P_d^{Dmax} - P_d^{LS}) \quad \forall n \quad [2.d]$$

$$P_l^L = B_l(\theta_{s(l)} - \theta_{r(l)}) \quad \forall l \setminus l \in \Omega^{L+} \quad [2.e]$$

$$-F_l^{max} \leq P_l^L \leq F_l^{max} \quad \forall l \setminus l \in \Omega^{L+} \quad [2.f]$$

$$-X_l^L F_l^{max} \leq P_l^L \leq X_l^L F_l^{max} \quad \forall l \in \Omega^{L+} \quad [2.g]$$

MILP reformulation $\left\{ \begin{array}{l} -(1 - X_l^L)M \leq P_l^L - B_l(\theta_{s(l)} - \theta_{r(l)}) \leq (1 - X_l^L)M \quad \forall l \in \Omega^{L+} \quad [2.h] \end{array} \right.$

$$0 \leq P_g^E \leq P_d^{Emax} \quad \forall g \quad [2.i]$$

$$0 \leq P_d^{LS} \leq P_d^{Dmax} \quad \forall d \quad [2.j]$$

$$-\pi \leq \theta_n \leq \pi \quad \forall n \quad [2.k]$$

$$\theta_n = 0 \quad n: ref. \quad [2.l]$$

Uncertainty sets

Introducing polyhedral uncertainty set and “uncertainty budget” constraints:

$$P_g^{Emax} \in [0, \overline{P_g^{Emax}}] \quad \forall g$$

$$\frac{\sum_g (\overline{P_g^{Emax}} - P_g^{Emax})}{\sum_g (\overline{P_g^{Emax}})} \leq \Gamma^G$$

$$P_d^{Dmax} \in [\underline{P_d^{Dmax}}, \overline{P_d^{Dmax}}] \quad \forall d$$

$$\frac{\sum_d (P_d^{Dmax} - \underline{P_d^{Dmax}})}{\sum_d (\overline{P_d^{Dmax}} - \underline{P_d^{Dmax}})} \leq \Gamma^D$$

$$\Gamma^G, \Gamma^D = \{0..1\}$$

if $\Gamma^G = 0 \rightarrow P_g^{Emax} = \overline{P_g^{Emax}}$
 if $\Gamma^G = 1 \rightarrow P_g^{Emax} \in [0, \overline{P_g^{Emax}}]$
 if $\Gamma^G = 0.2 \rightarrow$ up to 20% of
 generation capacity may be
 unavailable

Creating a monster

Let's merge together:

1. Adaptive robust problem
2. MILP formulation of expansion planning problem
3. Uncertainty sets

$$\min_{X_l^L} \max_{P_g^{Emax}, P_d^{Dmax}} \min_{P_g^E, P_d^{LS}, P_l^L, \theta_n} \sum_{l \in \Omega^{L+}} \tilde{I}_l X_l^L + \sigma \left[\sum_g C_g^E P_g^E + \sum_d C_d^{LS} P_d^{LS} \right]$$

$$\sum_{g \in \Omega_n^E} P_g^E - \sum_{l|s(l)=n} P_l^L + \sum_{l|r(l)=n} P_l^L = \sum_{d \in \Omega_n^D} (P_d^{Dmax} - P_d^{LS}) \quad \forall n \quad [3.b]$$

$$P_l^L = B_l(\theta_{s(l)} - \theta_{r(l)}) \quad \forall l \setminus l \in \Omega^{L+} \quad [3.c]$$

$$P_l^L = X_l^L B_l(\theta_{s(l)} - \theta_{r(l)}) \quad \forall l \in \Omega^{L+} \quad [3.d]$$

$$-F_l^{max} \leq P_l^L \leq F_l^{max} \quad \forall l \quad [3.e]$$

$$0 \leq P_g^E \leq P_g^{Emax} \quad \forall g \quad [3.f]$$

$$0 \leq P_d^{LS} \leq P_d^{Dmax} \quad \forall d \quad [3.g]$$

$$-\pi \leq \theta_n \leq \pi \quad \forall n \quad [3.h]$$

$$\theta_n = 0 \quad n: ref. \quad [3.i]$$

$$P_g^{Emax} \in [0, \overline{P_g^{Emax}}] \quad \forall g \quad [3.j]$$

$$\frac{\sum_g (\overline{P_g^{Emax}} - P_g^{Emax})}{\sum_g (\overline{P_g^{Emax}})} \leq \Gamma^G \quad [3.k]$$

$$P_d^{Dmax} \in [\underline{P_d^{Dmax}}, \overline{P_d^{Dmax}}] \quad \forall d \quad [3.l]$$

$$\frac{\sum_d (P_d^{Dmax} - \underline{P_d^{Dmax}})}{\sum_d (\overline{P_d^{Dmax}} - \underline{P_d^{Dmax}})} \leq \Gamma^D \quad [3.m]$$

$$\sum_{l \in \Omega^{L+}} \tilde{I}_l X_l^L \leq I^{max} \quad [3.n]$$

$$X_l^L = \{0,1\} \quad \forall l \in \Omega^{L+} \quad [3.o]$$

Merging problems of level 2 and level 3

Karush–Kuhn–Tucker conditions

- Let us consider the problem:

$$\min_x F(x) \tag{4.a}$$

$$s. t. \quad g_i(x) \leq 0 \quad (\lambda_i) \quad \forall i = 1, \dots, n \tag{4.b}$$

$$h_j(x) = 0 \quad (\mu_j) \quad \forall j = 1, \dots, m \tag{4.c}$$

- For this problem, the KKT conditions are:

$$\nabla f(x) + \sum_{i=1}^n \lambda_i \nabla g_i(x) + \sum_{j=1}^m \mu_j \nabla h_j(x) \leq 0 \perp x \geq 0 \tag{4.d}$$

$$0 \geq g_i(x) \perp \lambda_i \geq 0 \quad \forall i = 1, \dots, n \tag{4.e}$$

$$0 = h_j(x) \quad \mu_j \text{ free} \quad \forall j = 1, \dots, m \tag{4.f}$$

The solution stationarity is ensured by the equation [4.d].
 Equations [4.e] and [4.f] ensure complementarity and feasibility of a solution

Merging problems of level 2 and level 3

$$\max_{\Delta^{SUB}} \sigma \left[\sum_g C_g^E P_g^E + \sum_d C_d^{LS} P_d^{LS} \right]$$

$$\Delta^M = \{P_d^{Emax}, P_d^{Dmax}, P_g^E, P_d^{LS}, P_l^L, \theta_n, \lambda_n, \phi_l^L, \phi_l^{L+}, \phi_l^{Lmax}, \phi_l^{Lmin}, \phi_l^{Emax}, \phi_l^{Emin}, P_d^{Dmax}, \phi_l^{Dmin}, \phi_l^{Nmax}, \phi_l^{Nmin}, \dots\}$$

$$P_g^{Emax} \in [0, \overline{P_g^{Emax}}] \quad \forall g \quad [5.ab]$$

$$\frac{\sum_g (\overline{P_g^{Emax}} - P_g^{Emax})}{\sum_g (\overline{P_g^{Emax}})} \leq \Gamma^G \quad [5.ac]$$

$$P_d^{Dmax} \in [\underline{P_d^{Dmax}}, \overline{P_d^{Dmax}}] \quad \forall d \quad [5.ad]$$

$$\frac{\sum_d (P_d^{Dmax} - \underline{P_d^{Dmin}})}{\sum_g (\overline{P_d^{Dmax}} - \underline{P_d^{Dmax}})} \leq \Gamma^D \quad [5.ae]$$

$$\sum_{g \in \Omega_n^E} P_g^E - \sum_{l|s(l)=n} P_l^L + \sum_{l|r(l)=n} P_l^L = \sum_{d \in \Omega_n^D} (P_d^{Dmax} - P_d^{LS}) \quad \forall n \quad [5.af]$$

$$P_l^L = B_l(\theta_{s(l)} - \theta_{r(l)}) \quad \forall l \setminus l \in \Omega^{L+} \quad [5.ag]$$

$$P_l^L = X_l^{L*} B_l(\theta_{s(l)} - \theta_{r(l)}) \quad \forall l \in \Omega^{L+} \quad [5.ah]$$

$$-F_l^{max} \leq P_l^L \leq F_l^{max} \quad \forall l \quad [5.ai]$$

$$0 \leq P_g^E \leq P_d^{Emax} \quad \forall g \quad [5.aj]$$

$$0 \leq P_d^{LS} \leq P_d^{Dmax} \quad \forall d \quad [5.ak]$$

$$-\pi \leq \theta_n \leq \pi \quad \forall n \quad [5.al]$$

$$\theta_n = 0 \quad n: ref. \quad [5.am]$$

... .. [5.tb]

For KKT we need:

1. Feasibility conditions

1. Constraints of level 2

2. Constraints of level 3

2. Stationarity conditions

3. Complementarity conditions

Merging problems of level 2 and level 3

... [tbc]

$$\sigma C_g^E - \lambda_{n(g)} + \phi_g^{Emax} - \phi_g^{Emin} = 0 \quad \forall g \quad [5.an]$$

$$\sigma C_d^{LS} - \lambda_{n(d)} + \phi_d^{Dmax} - \phi_d^{Dmin} = 0 \quad \forall d \quad [5.a0]$$

$$\lambda_{s(l)} - \lambda_{r(l)} - \phi_l^L + \phi_l^{Lmax} - \phi_l^{Lmin} = 0 \quad \forall l \setminus l \in \Omega^{L+} \quad [5.ap]$$

$$\lambda_{s(l)} - \lambda_{r(l)} - \phi_l^{L+} + \phi_l^{Lmax} - \phi_l^{Lmin} = 0 \quad \forall l \in \Omega^{L+} \quad [5.aq]$$

$$\sum_{l \setminus l \in \Omega^{L+} | s(l)=n} B_l \phi_l^L + \sum_{l \in \Omega^{L+} | s(l)=n} X_l^{L*} B_l \phi_l^{L+} - \sum_{l \setminus l \in \Omega^{L+} | r(l)=n} B_l \phi_l^L - \sum_{l \in \Omega^{L+} | r(l)=n} X_l^{L*} B_l \phi_l^{L+} + \phi_l^{Nmax} - \phi_l^{Nmin} = 0 \quad \forall n \setminus n:ref \quad [5.ar]$$

$$\sum_{l \setminus l \in \Omega^{L+} | s(l)=n} B_l \phi_l^L + \sum_{l \in \Omega^{L+} | s(l)=n} X_l^{L*} B_l \phi_l^{L+} - \sum_{l \setminus l \in \Omega^{L+} | r(l)=n} B_l \phi_l^L - \sum_{l \in \Omega^{L+} | r(l)=n} X_l^{L*} B_l \phi_l^{L+} + \phi_l^{Nmax} - \phi_l^{Nmin} - \chi^{ref} = 0 \quad n:ref \quad [5.as]$$

$$0 \leq \phi_l^{Lmax} \perp F_l^{max} - P_l^L \geq 0 \quad \forall l \quad [5.at]$$

$$0 \leq \phi_l^{Lmin} \perp P_l^L + F_l^{max} \geq 0 \quad \forall l \quad [5.au]$$

$$0 \leq \phi_l^{Emax} \perp P_d^{Emax} - P_g^E \geq 0 \quad \forall g \quad [5.av]$$

$$0 \leq \phi_l^{Emin} \perp P_g^E \geq 0 \quad \forall g \quad [5.aw]$$

$$0 \leq \phi_l^{Dmax} \perp P_d^{Dmax} - P_d^{LS} \geq 0 \quad \forall d \quad [5.ax]$$

$$0 \leq \phi_l^{Dmin} \perp P_d^{LS} \geq 0 \quad \forall d \quad [5.ay]$$

$$0 \leq \phi_l^{Nmax} \perp \pi - \theta_n \geq 0 \quad \forall n \quad [5.az]$$

$$0 \leq \phi_l^{Nmin} \perp \theta_n + \pi \geq 0 \quad \forall n \quad [5.ba]$$

For KKT we need:

1. Feasibility conditions
 1. Constraints of level 2
 2. Constraints of level 3
2. Stationarity conditions
3. Complementarity conditions

Merging problems of level 2 and level 3

... [tbc]

$$\sigma C_g^E - \lambda_{n(g)} + \phi_g^{Emax} - \phi_g^{Emin} = 0 \quad \forall g \quad [5.an]$$

$$\sigma C_d^{LS} - \lambda_{n(d)} + \phi_d^{Dmax} - \phi_d^{Dmin} = 0 \quad \forall d \quad [5.a0]$$

$$\lambda_{s(l)} - \lambda_{r(l)} - \phi_l^L + \phi_l^{Lmax} - \phi_l^{Lmin} = 0 \quad \forall l \setminus l \in \Omega^{L+} \quad [5.ap]$$

$$\lambda_{s(l)} - \lambda_{r(l)} - \phi_l^{L+} + \phi_l^{Lmax} - \phi_l^{Lmin} = 0 \quad \forall l \in \Omega^{L+} \quad [5.aq]$$

$$\sum_{l \setminus l \in \Omega^{L+} | s(l)=n} B_l \phi_l^L + \sum_{l \in \Omega^{L+} | s(l)=n} X_l^{L*} B_l \phi_l^{L+} - \sum_{l \setminus l \in \Omega^{L+} | r(l)=n} B_l \phi_l^L - \sum_{l \in \Omega^{L+} | r(l)=n} X_l^{L*} B_l \phi_l^{L+} + \phi_l^{Nmax} - \phi_l^{Nmin} = 0 \quad \forall n \setminus n:ref \quad [5.ar]$$

$$\sum_{l \setminus l \in \Omega^{L+} | s(l)=n} B_l \phi_l^L + \sum_{l \in \Omega^{L+} | s(l)=n} X_l^{L*} B_l \phi_l^{L+} - \sum_{l \setminus l \in \Omega^{L+} | r(l)=n} B_l \phi_l^L - \sum_{l \in \Omega^{L+} | r(l)=n} X_l^{L*} B_l \phi_l^{L+} + \phi_l^{Nmax} - \phi_l^{Nmin} - \chi^{ref} = 0 \quad n:ref \quad [5.as]$$

For KKT we need:

1. Feasibility conditions
 1. Constraints of level 2
 2. Constraints of level 3
2. Stationarity conditions
3. Complementarity conditions

again nonlinear terms...
 but there is a trick!

$$0 \leq \phi_l^{Lmax} \perp F_l^{max} - P_l^L \geq 0 \quad \forall l \quad [5.at]$$

$$0 \leq \phi_l^{Lmin} \perp P_l^L + F_l^{max} \geq 0 \quad \forall l \quad [5.au]$$

$$0 \leq \phi_l^{Emax} \perp P_d^{Emax} - P_g^E \geq 0 \quad \forall g \quad [5.av]$$

$$0 \leq \phi_l^{Emin} \perp P_g^E \geq 0 \quad \forall g \quad [5.aw]$$

$$0 \leq \phi_l^{Dmax} \perp P_d^{Dmax} - P_d^{LS} \geq 0 \quad \forall d \quad [5.ax]$$

$$0 \leq \phi_l^{Dmin} \perp P_d^{LS} \geq 0 \quad \forall d \quad [5.ay]$$

$$0 \leq \phi_l^{Nmax} \perp \pi - \theta_n \geq 0 \quad \forall n \quad [5.az]$$

$$0 \leq \phi_l^{Nmin} \perp \theta_n + \pi \geq 0 \quad \forall n \quad [5.ba]$$

MIP reformulation of complementarity terms

$$0 \leq a \perp b \geq 0$$

Implies:

$$a \geq 0, b \geq 0 \text{ and } ab = 0$$

It can be reformulated as:

$$a \geq 0$$

$$b \geq 0$$

$$a \leq Mu$$

$$b \leq M(1 - u)$$

Where:

- u is an auxiliary binary variable
- M is a large positive constant

Column-and-constraint generation (or Berders-primal) algorithm

Zeng, B., and Long Z. “Solving two-stage robust optimization problems using a column-and-constraint generation method.” *Oper. Res. Lett.* 41 (2013): 457-461.

Bertsimas, D. et al. “Adaptive Robust Optimization for the Security Constrained Unit Commitment Problem.” *IEEE Transactions on Power Systems* 28 (2013): 52-63.

[A book] Conejo, A. J., Baringo, M. L., Kazempour, S. J., & Siddiqui, A. S. (2016). *Investment in Electricity Generation and Transmission: Decision Making under Uncertainty*. DOI: 10.1007/978-3-319-29501-5

Master problem

$$\min_{\Delta^M} \sum_{l \in \Omega^{L+}} \tilde{I}_l X_l^L + \eta$$

$$\Delta^M = \{X_l^L, P_{l,v^i}^L, P_{g,v^i}^E, P_{d,v^i}^{LS}, \theta_{n,v^i}, \eta\}$$

$$\sum_{l \in \Omega^{L+}} \tilde{I}_l X_l^L \leq \overline{I}^{max} \quad [6.b]$$

$$X_l^L = \{0,1\} \quad \forall l \in \Omega^{L+} \quad [6.c]$$

$$\sum_{g \in \Omega_n^E} P_{g,v^i}^E - \sum_{l|s(l)=n} P_{l,v^i}^L + \sum_{l|r(l)=n} P_{l,v^i}^L = \sum_{d \in \Omega_n^D} (P_{d,v^i}^{Dmax*} - P_{d,v^i}^{LS}) \quad \forall n, \forall v^i \leq v \quad [6.d]$$

$$P_{l,v^i}^L = B_l (\theta_{s(l),v^i} - \theta_{r(l),v^i}) \quad \forall l \setminus l \in \Omega^{L+}, \forall v^i \leq v \quad [6.e]$$

$$P_{l,v^i}^L = X_l^L B_l (\theta_{s(l),v^i} - \theta_{r(l),v^i}) \quad \forall l \in \Omega^{L+}, \forall v^i \leq v \quad [6.f]$$

$$-F_l^{max} \leq P_{l,v^i}^L \leq F_l^{max} \quad \forall l, \forall v^i \leq v \quad [6.g]$$

$$0 \leq P_{g,v^i}^E \leq P_{d,v^i}^{Emax*} \quad \forall g, \forall v^i \leq v \quad [6.i]$$

$$0 \leq P_{d,v^i}^{LS} \leq P_{d,v^i}^{Dmax*} \quad \forall d, \forall v^i \leq v \quad [6.i]$$

$$-\pi \leq \theta_{n,v^i} \leq \pi \quad \forall n, \forall v^i \leq v \quad [6.j]$$

$$\theta_{n,v^i} = 0 \quad n: ref. \forall v^i \leq v \quad [6.k]$$

$$\eta \geq \sigma \left[\sum_g C_g^E P_{g,v^i}^E + \sum_d C_d^{LS} P_{d,v^i}^{LS} \right] \quad \forall v^i \leq v \quad [6.l]$$

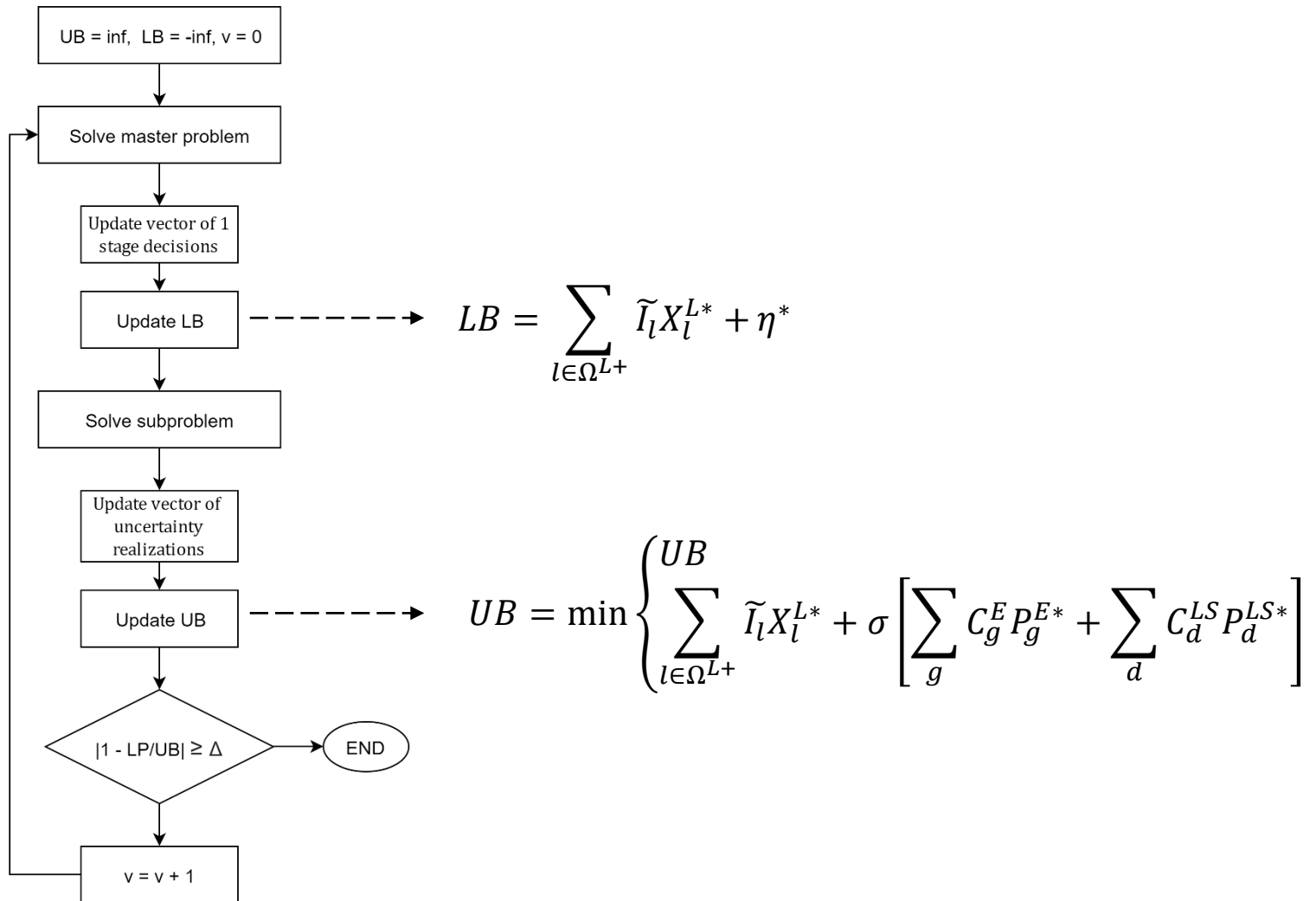
Three things to consider:

1. Auxiliary variable η
2. Constraints [6.d] – [6.l] have index v
3. P_{d,v^i}^{Dmax*} and P_{d,v^i}^{Emax*} are fixed
(they are results of a subproblem)

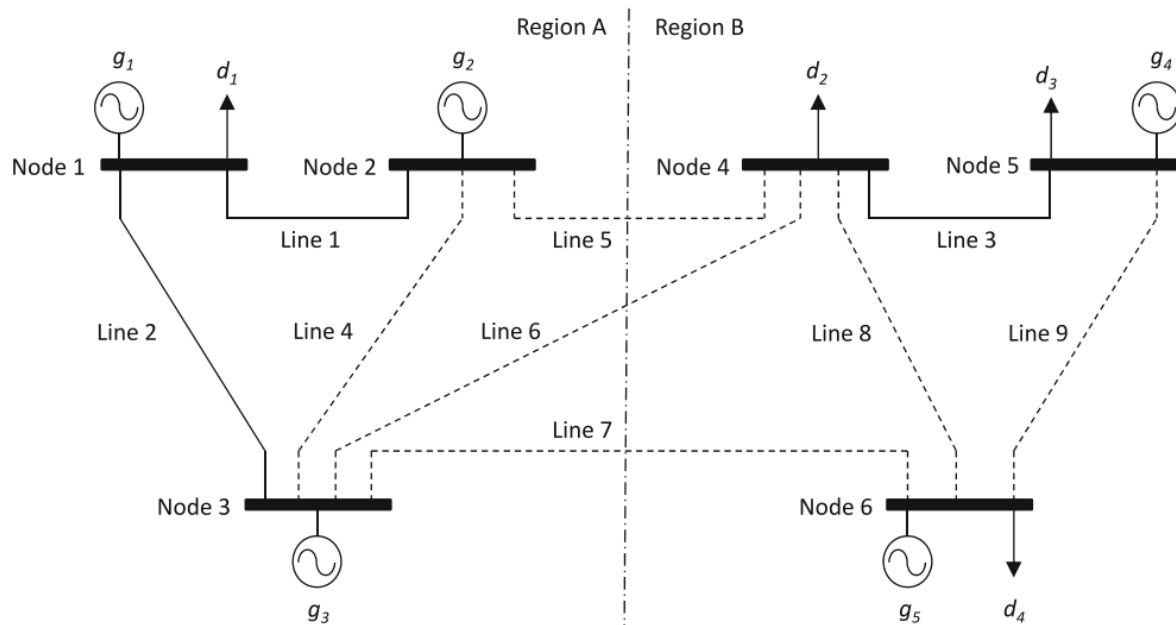
A **subproblem** is a level 2&3 merged problem [5.aa] - [5.ba]
in which a vector of first-stage decisions X_t^{L*} is fixed to a
result of the master problem

(this time no equations 😊)

Decomposition procedure



Modelling results: sensitivity to uncertainty budget



| Line | B_l [S] | F_l^{max} [MW] | \tilde{I}_l [\$] · 10 ⁶ |
|------|-----------|------------------|--------------------------------------|
| 4 | 500 | 150 | 0.7 |
| 5 | 500 | 200 | 1.4 |
| 6 | 500 | 200 | 1.8 |
| 7 | 500 | 200 | 1.6 |
| 8 | 500 | 150 | 0.8 |
| 9 | 500 | 150 | 0.7 |

| Γ^D | Γ^G | | | | |
|------------|------------|-------|-------|-------|----------|
| | 0 | 0.2 | 0.4 | 0.6 | 0.8 |
| 0 | 15 17 | 15 17 | 15 17 | 14 17 | 14 17 19 |
| 0.2 | 15 17 | 15 17 | 15 17 | 14 17 | 14 17 19 |
| 0.4 | 15 17 | 15 17 | 15 17 | 14 17 | 14 17 19 |
| 0.6 | 15 17 | 15 17 | 15 17 | 14 17 | 14 17 19 |
| 0.8 | 15 17 | 15 17 | 15 17 | 14 17 | 14 17 19 |

Summing up

Conclusions

- i. RO allows for dropping assumptions that a finite number of uncertainty realizations exist with respective (known) probabilities.
- ii. RO is particularly suitable when decisions are costly and protection against the worst-case scenario is a must.
- iii. RO allows for robustness control.
- iv. Accuracy of RO models generally does not depend on the accuracy of the uncertainty description.

Next steps (?)

- i. Updating the test case by adding gas system components.
- ii. Integrating gas system objective/constraints/linkage into the model.
- iii. Thinking it over...