Assessing historical bridge bearings –
The development of approaches to the "tangential problem"

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ABSTRACT

The paper deals with the structural analysis of bridge bearings, in particular with the analysis of the historical steel roller bearings. Bridge bearings usually need to be assessed as part of the assessment of a whole structure. They can not simply be replaced for economical reasons or because the whole bridge is classified as a monument. But even if the historical bearings look sound and there is visually no need to replace them, engineers encounter difficulties in proofing the ability of these in coping with present-day or future loads.

The paper discusses the historical approaches to deal with the tangential problem occurring in the contact area between rollers and bearing plates and which characterised mainly the sizing of the bearings. It introduces and compares the different design approaches from the empirically based beginnings to the more scientifically based designs.

Introduction

With the number of existing bridge structures having to be maintained or refurbished we find the historical steel bridge bearings re-entering the engineers' field of perception. Due to their sturdiness they are often found to be in sound condition despite having served for more than a century in spite of often negligent maintenance and increasing loads. To assess their load-bearing capacity one usually has to take recourse to those static calculations arrived at in the second half of the 19th century which accompanied the process of the constructive maturing of the steel bearing technology.

Two problems substantially shaped the development of this kind of bearing technology both in terms of constructive design and of static calculation, firstly the "friction problem" occurring between segments of the bearing moving against each other and secondly the "tangential problem” with the transfer of immense pressure forces in areas of extremely minute proportions. The latter question is the subject of this paper.

After a brief presentation of the problem the decisive historical approaches to cope with the tangential problem will be discussed and analyzed vis-à-vis the requirements existing according to contemporary calculations. This will be exemplified by discussing the proportioning of rollers for moveable bridge bearings.

The tangential problem

In general terms technical mechanics define a tangential problem to occur when forces are transmitted in the contact area of planes differing in curvature. This applies in various forms in the field of engineering, for example between a wagon-wheel and its rails. Similarly different possibilities exist for contact in the case of bridge bearings ranging from rollers touching planes to the contact between a segment from a sphere and a convex curvature with another one having concave shape. Theoretically the first case will produce a line of contact, whereas the latter will yield a point. In the both cases the total amount of the bridge bearing pressure has to be transferred onto the base by means of only a few of these minute contact areas. Depending on the load the contact area will expand until a balance of forces is achieved. Once the bridge is unloaded again the contact areas will shrink back to line or point respectively if the stresses working upon the contact areas have remained within the material’s range of linear resilience. However, the performance of realistically sized bearings has shown to yield local plastifications.

The aim of every design approach was to represent as realistic as possible the state of stress in the contact area to limit the unavoidable extent of the plastifications produced. Flattenings of a larger size would diminish the ability of the superstructure for both rotation and translation which in turn would question the fitness for purpose of the bearings. (Fig.1)

Sizing approaches

The initial form of iron bearings were plane bearings with their upper and lower parts sharing an area of direct contact or – in the case of rollers put between them – upper and lower bearing part kept in coplanar distance defined by the rollers’ diameter. (Fig.2, left) Bearings of this kind were already used in suspension bridges at the beginning of the 19th century. They reached their constructive peak in the middle of the century when Conway- and Britannia Bridge were opened. Plane bearings were superseded by rotating bearings gradually substituting the former.

A substantial drawback of the plane bearing is that the demarcation of its bearing point is rather imprecise. Depending on the adjustment of the bearing and the deflection of the superstructure an indeterminate number of rollers will be activated resulting in a rather haphazard transfer of load. Inserting wooden bolsters to guarantee a certain capacity of the whole bearing for rotation to ensure a rather planar transfer of load did not significantly improve the situation. (Wetzk 2006)

The sizing of the rollers in these early iron bearings evolved "…with the aid of rather blunt rule-of-thumb calculations differing substantially from each other and very rarely based on experimentation" (Bleich 1924, p.557). Results of such investigation had been published by *Louis Vicat* (1786—1861) as early as 1833. He found out that the load causing a roller to break when pressed between two planar bodies amounted to a third of that exerted upon a prism of the base identical to the diameter of the roller (Kollmar 1919, p.41). To what extend these results were appropriated for the bearings of larger iron girder bridges remains open to speculation for the time being. But because the transfer of load provided by planar bearings could only be guessed at, structural calculations were of little avail. This indeterminacy was reflected in the experiences regarding the efficiency of this kind of support. Those deployed in Conway- and Britannia-Bridge were described as highly successful achievements: "The timber placed beneath and above the bed-plates has been found perfectly efficient in preserving them from fracture under any unequal strain, and the rollers and balls are found to move freely during changes of temperature" (Clark 1850, p.602). On the other hand in the case of the plans resulting in the bridge crossing the river Kinzig near the south-west German town of Offenburg (1852/53) roller bearings were dispensed with, "because rollers were found to be pressed into the upper and lower plates of the bearing, thus inhibiting displacement instead of allowing for it" (Becker 1853, p.183).

The practical adjustment of the theoretically calculated bearing point for all feasible loads became only possible through the application of rotating devices. Rotating bearings provided an almost even distribution of the pressure onto the rollers. (Fig.2, right) This clarity in the flow of the loads justified the striving for more precise information regarding the load-carrying capacity of the rollers. Concerning the bearings implemented in his bridge crossing the river Brahe near Czersk (1861) the engineer *Johann Wilhelm Schwedler* (1823—1892) observed the following rule, presumably the result of related experiments. "The pressure of an even cast-iron plane onto a cast-iron roller may amount to 1 Zentner [=50 kg] per inch in diameter and per inch in length, without the resilience of the material being subject to changes" (Schwedler 1861, p.599). Ten years on and founded upon new experimentation the cast-iron rollers used in the bridge crossing the Danube near Mariaort (1871) were considered to be safe with the same amount of load per cm squared in diameter and length only. (Baldermann 1873, p.201)

But the new challenges faced in bridge building could no longer be solved by merely using empirical and experimental procedures. Previous experiences with what then already had to be counted as small scale bridges were of little value. Practical engineers had to rely increasingly upon theoretical considerations, whereas theoreticians in turn were stimulated by the new challenges in engineering. Also for the size of the bearings one can observe "…the desire to have more reliable formulae at one's disposal to calculate the diameters for the rollers" (Bleich 1924, p.557).

Starting with *Claus Koepke's* (1831-1911) equation in 1869 a number of approaches were published which tried to permit calculations founded upon theoretical considerations as how to tackle with the problems of tangency. (Fig.3) Supported by the evaluations made by *Kollmar* (Kollmar 1919, p.46) the following Tab.1 surveys the most important approaches summarizing the data in relation to the material to be used:

Table 1: Required radius of the rollers (number *n*, length *l*, radius *r*)
in relation to the load *A* to be transferred (Load [t], Radius and Length [cm])

Year	Published by	Cast iron n·l·r =	Cast steel/ Steel castings n·l·r =	Mild steel n·l·r =	Reference
1869	Koepke	48 A	13…18 A	-	Koepke 1861, p.61
1869	Fraenkel	21…25 A	14…16.5 A	-	Fraenkel 1869, p.196
1875	Winkler	10…19.5 A	8…15.5 A	-	Winkler 1875, p.277
1876	Baentsch	57 A	26 A	-	Kollmar 1919, p.46
1876	Laissle/Schuebler	16…25 A	-	-	Kollmar 1919, p.46
1882	Steiner	15 A	12 A	-	Schaeffer 1882, p.505
1888	Haeseler	16.5…23 A	18 A	-	Kollmar 1919, p.46
1889	Tetmajer	16.5 A	8 A	12.5 A	Tetmajer 1889, p.148
1889	Bach	20 A	9 A	-	Bach 1889, p.477
1893	Deslandres	22.5 A	28 A	-	Weyrauch 1894, p.572

Beyond them it will be referred to additional approaches in Grashof (1878, p.49), Kübler (1874) and Willmann (1886)

These results which differ substantially from each other were products of a vast variety of theoretical presuppositions corresponding only to a limited extent to the practical realities and proving without claim for precision. Already in the year 1894 *Johann Jakob Weyrauch* (1845—1917) a German mathematician cum engineer referred to the most common and widely used calculations as "…makeshift devices only employed for want of better orientation" (Weyrauch 1894, p.134). After a thorough comparison and analysis of the different approaches and their results he declared that "…in the face of such differences in the assumptions of the most prominent experts the conclusion may not be unwarranted, that the calculation of bridge bearings so far has lacked any scientific basis whatsoever. The bearings have lasted up to now, that is the only thing going for them" (Weyrauch 1894, p.135). Thumb rules according to the formulae in Tab.1 would still be published by the beginning of the 20th century, for instance *Vianello's* equation, "…which is in accord with well-built examples but can not be scientifically justified" (Vianello 1905, p.476).

In the year 1881 the physicist *Heinrich Rudolf Hertz* (1857—1894) had devised a new strictly scientific solution to describe the processes occurring in the contact area of two differently curved bodies (Hertz 1881). It was subsequently published for application in the constructive field under the title "Concerning the mutual compression of elastic bodies with curved surfaces" (Keck 1884), but it would remain virtually unnoticed. Only after *Weyrauch* had recommended *Hertz*' presentation a decade later as a useful approach regarding the tangential problem, it became established as a standard procedure for calculating steel bearings. Commencing with models about the distribution of electricity in two bodies touching, *Hertz* calculated the stress and the deformation in the contact area of firm bodies. From the general case of an elliptical contact area he derived the two extreme cases of a circular area of pressure and a rectangular one respectively. Simplified formulae could in turn be employed to infer for the practical use of calculating roller sizes.

The contact pertinent to this presentation is the one occurring between a cylinder and a plane (i.e. between bearing roller and bearing plate) according to the formulae devised by *Hertz* yielded the maximum stress in the contact area:

$$\sigma_H = 0{,}418 \cdot \sqrt{\frac{p \cdot E}{r}} \tag{1}$$

where p = linear load, E = modulus of elasticity and r the radius of the rollers.
The approach developed by *Hertz* was primarily based upon the following assumptions:

1.) The contact areas of the bodies are infinitesimally small in relation to their surfaces.
2.) Reaction must be normal to the contact surface.
3.) The bodies have to be elastic, homogeneous and isotropic.
4.) Strain must be within the elastic limit.

Hertz' theoretical assumptions were not strictly applicable to roller bearings. If for example bearings were chosen to have feasible proportions, plastifications in the contact area could not be avoided. These were accompanied by cold hardenings in the contact zone representing inhomogeneities. *Hertz* already anticipated possible discrepancies between his theoretical model and practical applications: “It may, however, occur that the real bodies do not comply with the concept of homogeneity we have based our reasoning upon; for the strength of the bodies’ materials near the surface which we are concerned with here, is well-known often to differ substantially from that in their cores” (Hertz 1882, p.195).

Based upon a body of experiments and theoretical reasonings the engineer *Ewald Rasch* (1871—1927) came to reject the validity of *Hertz*’ equations for the sizing of bearings in 1915: “The latter lead to a […] structural underestimation of ball- as well as roller-bearings. Calculations dealing with these important structural elements should not use them.” (Rasch 1915, p.2) These claims of *Rasch* corroborated results from experiments already commissioned by the *Association of German Engineers (VDI)* in 1905/06. These had shown that even if the loads on the bearings were increased by three to four times to what could be expected in practice no dangerous deformations could be observed at the bearings (Stamer 1915, p.58). However, the work of *Rasch* as well as that of others, sometimes based on a different theoretical approach (e.g. Fleischhauer 1965) was unnoticed in the practical field.

In spite of this critique it was the formulation of *Hertz* which served as the design base for efforts in standardisation for bridge bearings in 1920s Germany (Karig 1923) and they continue to do so today when tangential problems are being calculated.

Comparison of the different approaches for sizing

For the structural assessment of historical bridges it is particularly interesting to compare the results of those rule-of-thumb calculations used until the early 20th century with those formulae developed by *Hertz*. For this purpose we will transform Eq.1 so as to fit the format of those formulae employed in Tab.1.

The linear load *p* will be substitute by:

$$p = \frac{A}{n \cdot l} \tag{2}$$

where *A* = load to be transferred and *n, l* = number, length of the rollers.
This results in:

$$n \cdot l \cdot r = 0{,}175 \cdot \frac{E \cdot A}{\sigma_H{}^2} \tag{3}$$

where the values depend on E = modulus of elasticity of the material employed, A = load to be transferred and on σ_H = *Hertzian* pressure.

For a long time there was no clarity regarding the *allowable* amount of stress in the contact area calculated with the *Hertzian* formulae. It seemed plausible to rely on the material characteristics derived from tension tests. But already in 1894 *Weyrauch* pointed to the incommensurability of these assumptions. The results of these tests would not represent the conditions in the contact area where "…a spatially extremely finite compression will cease as soon as the robustness has been regained by means of enhancing the area of contact as well as the elastic limit and the compressive yield point… In order to avoid permanent deformations at the rollers, for larger bridges numbers of rollers would have to be chosen that cannot provide an even distribution of load" (Weyrauch 1894, p.141). The following Tab.2 surveys the development of the *Hertzian* pressure used for calculations from the first wary recommendations by *Weyrauch* (1894) to the late 20th century.

Table 2: Development of the recommended / allowable pressure σ_H for the *Hertzian* formulae

Year	Published by/as	Cast iron t/cm²	Cast steel/ Steel castings t/cm²	Mild steel t/cm²	Reference
1894	Weyrauch	2.0…3.0	3.0…4.8	2.0…3.0	Weyrauch 1894, p.142
1919	Kollmar	-	6.0	-	Kollmar 1919, p.55
1922	Schaper	4.0	6.5	5.0	Schaper 1922, p.665
1937	DIN 1050	5.0	8.5	-	DIN 1050, 1937, pl. 3
1990	DIN 18 800 - 1	5.0	8.5	6.5	DIN 18000, 1990,

Thus Tab. 1 can be extended by combining Eq.3 with the values shown in Tab.2. Table 3 will be limited to the values (average values) of the mainly used material (cast iron, steel-castings/cast steel). The rule by *Baentsch* (1876) will be disregarded as an outlier; the discussed sizings used by *Schwedler* (1861) and for the Danube bridge near Mariaort (1871) as well as *Vianello*'s thumb rule (1905) will be added for comparison. The following Fig.4 will present the historical development in graphic terms.

Table 3: Forward projection of the adjusted Tab. 1 on the basis of the results following Hertz

Year	Published by/as	Cast iron	Cast steel/ steel castings

		n·l·r =	n·l·r =
1861	Schwedler	62.5 A	-
1869	Koepke	48 A	13…18 A
1869	Fraenkel	21…25 A	14…16.5 A
1871	Bridge Mariaort	13.5 A	-
1875	Winkler	10…19.5 A	8…15.5 A
1876	Laissle/Schuebler	16…25 A	-
1882	Steiner	15 A	12 A
1888	Haeseler	16.5…23 A	18 A
1889	Tetmajer	16.5 A	8 A
1889	Bach	20 A	9 A
1893	Deslandres	22.5 A	28 A
1905	Vianello	10 A	10 A
Forward projection for the Hertzian formulae			
	with E [t/cm^2] =	*1000*	*2200*
1894	Weyrauch	28 A	25.5 A
1919	Kollmar	-	10.5 A
1922	Schaper	11 A	9 A
1937	DIN 1050	7 A	5.5 A
1990	DIN 18 800 - 1	7 A	5.5 A

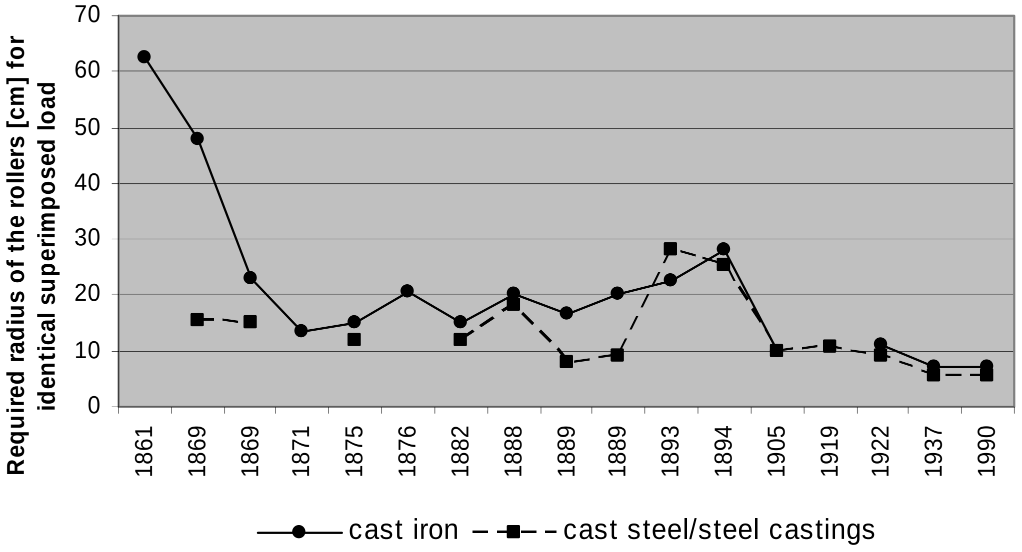

Figure 4: Development of the radii of the rollers according to the calculations

Analysis of the comparison

An evaluation of the design formulae of bearings across this period of one and a half centuries has to point to the rapid developments in metallurgy occurring in that very period which in turn influenced the requirements for sizing. The compatibility of materials as well as sizing therefore decreases with growing temporal distance. Thus the present assessment has to count as a qualitative one.

Across the whole period under consideration, however, the development towards using smaller rollers can be discerned, and this holds for both materials. One significant deviation from this trend shows in the last quarter of the 19th century which comes as a surprise because this epoch is characterized by the most substantial advances in metallurgy. Without further research any interpretation of this counterintuitive observation has to face the pitfalls of speculation, in particular since the engineers were provided with considerable range in some of the calculative approaches (see Tab.1). On the other hand, two reasons for some anxiety on the engineers' part can be assumed: The growing awareness of the damaging impact of repeated load will have shaped the desire for safety as well as the memory of spectacular collapses, for example the Taybridge disaster in December 1879 with 75 victims and the bridge failure crossing the river Birs in Münchenstein with another 72 persons dead in June 1891. In combination with the absence of reliable experiments regarding the load bearing capacity of rollers (Weyrauch 1894, p.140) this could have been the background for *Weyrauch'*s cautionary recommendations for a first approach based upon the *Hertzian* formulae (1894).

It was at the turn of the century that experiments investigating the stress- and deformation behaviour of bearing plate and -roller warranted higher limits for allowable *Hertzian* pressure which then led to reducing the radii of the bearing rollers. The research performed by *Crandall / Marston* (Crandall and Marston, 1894) and in the name of the *VDI* (Stamer, 1915) may serve as examples. In the context of the latter *Vianello* could devise his rule of thumb (1905), whose claims would diverge but insignificantly from those later on derived from the *Hertzian* formulae.

One of the reasons for the unsteady development in the curve depicting the cast steel / steel casting (culminating in *Deslandres*' suggestion from 1893 to implement steel rollers larger than those made of cast iron – see Fig.4) may be attributed to the lack of definition with the terms cast steel (Gussstahl)/steel castings (Stahlguss). During the second half of the 19th century these could denominate clearly different materials, for example products made by pouring liquid wrought iron directly into forms as well as products made of cast iron processed in the cupola furnace by adding steel waste. Although attempts can be discerned to clarify the terminology as early as 1880 (Ledebur, 1889), it was only in the 20th century that a clear definition could be established. The values displayed in Tab.1 depend of course on the respective author's definition of the material which is rarely ever explicitly named in the formulae. In many cases these information were only supplied by *Kollmar* for his evaluation from the year 1919. Altogether these indeterminacies for (cast)steel rollers rule out any precise interpretation. Only the assertion is warranted that, regardless of the individual author's interpretation of the material most calculations required considerably larger steel rollers than a calculation based on *Hertz* would have proposed.

Summary

The growing interlacing of theory and practice in bridge building that began in the middle of the 19th century allowed for an increasingly scientific treatment in the field of structural engineering - for instance the tangential problem that occurs where the rollers touch the bearing plates in the case of bridge bearings. Commencing in the 1860s a proliferation of rule-of-thumb calculations can be observed, however, they still lacked what counts as sound principles according to the concept of science today. The equations of the physicist *Johann Heinrich Hertz* initiated the move towards a scientific definition. Although the value of the *Hertzian* formulae for practical application for calculating the sizes of bearings was soon contested, they are still used nowadays to solve tangential problems.

One reason for the long-standing popularity of the rule-of-thumb calculations even in technical books of reference, it is safe to assume, was their simplicity. The approach put forth by *Emil Winkler* (1835—1888) was still being applied in the 1920s (Bleich, 1924, p.557). The bearings of the majority of bridges built by then will most likely have been derived from calculations based on such guidelines.

The qualitative comparison of these historical calculations for sizing confirms that these old bearings still have reserves in load bearing capacity if compared with the calculations arrived at via the *Hertzian* equations. The load capacities diverge considerably and among other factors they depend on the rollers' material. Most calculations will indicate the cast iron rollers to be sufficiently sized even under the increased impact of contemporary traffic. Steel rollers will yield similar totals. From this point of view the increased demands exerted on the bridges by today's traffic may only very rarely necessitate the bearings actually to be replaced. However, negligent maintenance among other reasons may have caused considerable damages. Thus it is essential to check the relevance of any such impairment with regard to the load bearing capacity and the fitness for purpose of these bearings. Certain unclearities inherent in the extant historical structural material have been pointed out. Information as well as assumptions pertinent to the structural assessment of historical bearings should be compared with laboratory tests. To devise an appropriate method of investigation for both fabric and material analysis using careful methods is currently being developed as part of a scientific project at Brandenburg Technical University in Cottbus.

REFERENCES

Bach, C.v. 1889. Walzen*. Zeitschrift des Vereins deutscher Ingenieure* 33, p. 476 – 478

Bach, C. 1894. Versuche über die Formänderung der Rollen (Cylinder, Walzen) und Platten von Rollenlagern. *Zeitschrift des Vereins deutscher Ingenieure* 38, p. 199

Baldermann, O. 1873. Die Eisenbahnbrücke über die Donau bei Maria-Ort.
Allgemeine Bauzeitung 38, p. 193-205

Bäuerle, A. 1875. Zur Bestimmung der Größe der Rollen oder Pendel in den Rollenauflagern…
Zeitschrift des Vereins deutscher Ingenieure 19, p. 161-2

Becker, M. 1853. Die eiserne Brücke über die Kinzig bei Offenburg.
Allgemeine Bauzeitung 18, p. 179-88, pl. 567-71

Bleich, Fr. 1924. *Theorie und Berechnung der eisernen Brücken*. Berlin: Springer.

Clark, E. 1850. *The Britannia and Conway Tubular Bridges*. Vol. I, II. London: Day & Son.

Crandall, C.L. and Marston, A. 1894. Friction Rollers. *Transactions of the ASCE* 32, Aug., p. 99-129,
and Discussion in *Transactions of the ASCE* 32. Sept., p. 270-7

DIN 1050: *Berechnungsgrundlagen für Stahl im Hochbau*. July 1937

DIN 18800 – 1*: Stahlbauten; Bemessung und Konstruktion*. Nov. 1990

Fleischhauer, K.H. 1965. Die Tragfähigkeit von Zylinderrollen u. Kugeln. *VDI-Zeitschrift* 107, p.821-2

Fraenkel, W. 1869. Construction eiserner Fachwerkträgerbrücken. *Civil-Ingenieur* 15, p. 177-98, pl. 14

Grashof, F. 1878. *Theorie der Elastizität und Festigkeit*. Berlin: Rudolf Gaertner.

Hertz, H. 1881. Über die Berührung fester elastischer Körper.
Journal für reine und angewandte Mathematik 92, p.156-71

Hertz, H.1882. Über die Berührung fester elastischer Körper und über die Härte.
In: Lenard, Ph.1988. *Schriften vermischten Inhalts von Heinrich Hertz*. Vaduz: Sändig.

Karig, J. 1923. Vorschlag für ein einheitliches Brückenlager. *Bautechnik* 1, p.357-360, 416-20, 428-33

Keck 1884. Theoretische Untersuchungen – Ueber die gegenseitige Zusammendrückung elastischer …
Zeitschrift des Architekten- und Ingenieur-Vereins zu Hannover 30, p. 201-3

Koepke 1869. Ueber die Kompression von Körpern mit gekrümmten Oberflächen.
Deutsche Bauzeitung 3, p. 61-2

Kollmar, A. 1919. *Auflager und Gelenke*. Berlin: Ernst & Sohn.

Kübler, J. 1874. Bestimmung der Größe der Rollen oder Pendel in den Rollenauflagern der Brücken.
Zeitschrift des Vereins deutscher Ingenieure 18, p.235-7, incl. correction (Bäuerle 1875)

Ledebur, A.1889. Läßt sich im Cupolofen Stahlguß erzeugen? *Stahl und Eisen* 9, p.333-4

Rasch, E. 1915. Über die Berechnung der an Kugel- und Rollenlagern…*Eisenbau* 6, p.1-8

Schaeffer, Th. and Sonne, E. (eds.) 1882. *Handbuch der Ingenieurwissenschaften*. Vol. II*: Der Brückenbau*. Leipzig.

Schaper, G. 1922. *Eiserne Brücken*. Berlin: Ernst & Sohn.

Schwedler, W. 1861. Der eiserne Überbau der Brahe-Brücke bei Czersk. *Zeitschrift für Bauwesen* 11, p. 579-602

Stamer 1915. *Versuche mit Rollenlagern*. In: Verein Deutscher Ingenieure (ed.). *Forschungsarbeiten auf dem Gebiete des Ingenieurwesens*. Berlin: Selbstverlag.

Tetmajer, L. 1889. *Die Baumechanik. II: Die angewandte Elasticitäts- und Festigkeitslehre*. Zürich: Zürcher und Furrer.

Vianello, L. 1905. *Der Eisenbau*. München: Oldenburg.

Wetzk, V. 2006. Bridge bearings – A Historical Survey. In: Dunkeld, M. et al. (eds). *Proceedings of the Second International Congress on Construction History*, *Cambridge, 29 March – 2 April 2006*. vol. 3, p. 3333-55

Weyrauch 1894. Ueber die Berechnung der Brücken-Auflager. *Zeitschrift des Architekten- und Ingenieurvereins zu Hannover* 40, p.131-44, 571-74

Willmann, L.v. 1886. Beitrag zur Berechnung der Rollvorrichtung für Brückenverschiebungen. *Zeitschrift des Vereins deutscher Ingenieure* 30, p. 914-5

Winkler, E. 1875. *Die Gitterträger und Lager gerader Träger*. 2.Ed. In: Winkler, E. (ed.). *Vorträge über Brückenbau. Eiserne Brücken*. II. Wien: Carl Gerold's Sohn.