Trial application of Zarka's method under cyclic loading

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Introduction

If a mechanical structure is to be designed for operation under cyclic loading, primarily two kinds of failure must be guarded against:

- low cycle fatigue which may occur due to strains cycling between two states (controlled by the strain range exceeding twice the yield limit)
- ductility exhaustion which may occur due to accumulating strain from one load cycle to another.

These two kinds of failure are local failure modes so that strains need to be calculated and then assessed by comparison with code allowables such as the 1%, 2% and 5% strain limits set by the ASME nuclear codes. Elastic-plastic strains can be calculated by incremental (or step-by-step or evolutive) analyses. Unfortunately, this can be extremely costly if thousands of cycles are required to achieve shakedown.

Therefore, simplified elastic-plastic analysis methods are desired allowing to obtain specific information at reduced effort, nevertheless accounting for the main features controlling strain such as kinematic hardening. Zarka's method, early versions of which are available since twenty years, appears promising to provide both strain ranges and accumulated strains in the saturated cycle, i.e. after shakedown has been achieved. However, several attempts to use this method in the nuclear industry failed to qualify the method as a reliable analysis tool. This was due to several reasons:

- the publications describing the method were written in a highly scientific language the design engineers in industry were not familiar with
- in some cases Zarka's method provided excellent results (compared with incremental analyses), but bad ones in others.

Nevertheless, there remained some interest to uncover the potential of this method. For that purpose some calculations are performed for simple configurations of structure and loading (so that the structural response can be interpreted relatively easily). More insight into the performance of the method may thus be gained in terms of computational steps to be followed, the numerical effort required, the quality of the results obtained, and the sensibility with respect to material data and load level.

Zarka's method is described in many publications of Zarka and his co-workers, e.g. in [1] and [2]. Reviews can be found in [3] and [4]. The basic idea is to redefine the elastic-plastic problem by an equivalent elastic problem with suitably defined modified elastic material parameters and initial strains. This requires estimating (and iteratively improving) the geometry of the plastic zone and of transformed internal variables. A particular class of material models is admitted, the simplest of which is the linear kinematic hardening model.

Examples

Calculation of Strain Ranges

Calculation of strain ranges is presented for the following configurations:

- truss of two different cross-sections subjected to cyclic displacement controlled extension
- beam of ideal sandwich profile subjected to cyclic displacement controlled anchor movement
- beam of solid rectangular cross section subjected to cyclic displacement controlled anchor movement
- plane stress element subjected to biaxial cyclic strain controlled loading.

Due to the simplicity of the configurations of geometry and loading the results could be obtained by hand-calculations. Parameter studies were performed to investigate the effects of material parameters and of the load level. In addition, some Finite Element analyses were performed to check the effect of space discretisation. Only one-parameter loading was considered. The material was assumed to be linear kinematic hardening, corresponding to the material model most frequently used in industry. As far as multiaxial stress states were considered, a Mises yield surface was adopted. The results are compared with the exact results obtained via step-by-step analyses.

Calculation of Strain Accumulation

Calculation of accumulated strains was performed adopting the same assumptions (for example concerning the material model) as for calculating strain ranges. The following configurations of geometry and loading were investigated:

- a two-bar structure (two trusses in parallel) subjected to a constant force and cyclic temperature variation
- the Bree problem, i.e. a thin-walled tube subjected to constant internal pressure and cyclically varying linear temperature distribution across the wall

- plane stress element subjected to constant stress controlled loading in one direction and cyclic strain controlled loading in the other direction.

The results are presented in terms of ratcheting interaction diagrams and strains accumulated when shakedown is achieved. The analyses were performed by hand calculation allowing parameter studies to be performed for various hardening parameters and any combination of load level of the constant and the cyclic loading.

Discussion

Speed of convergence (in terms of number of modified elastic analyses required) and quality of the results obtained with Zarka's method were found to depend on details of material parameters (hardening modulus) and load level. In general, the elastic-plastic strain range as well as the ratcheting interaction diagrams and the accumulated strains were within a few percent of the exact results after one or two modified elastic analyses. For a trilinear stress strain curve (instead of a bilinear one) some more modified elastic analyses may be required. The Finite Element analyses showed that space discretisation may cause the method to become numerically unstable if there is only little hardening along with a high load level.

It is concluded that Zarka's method is, in general, a powerful tool to calculate elastic-plastic strain ranges for fatigue analyses, at least if load cases can be grouped to pairs forming one-parameter cyclic loadings (corresponding to widely used practice in industry). This is partly because the simplest material model Zarka's method can make use of, i.e. linear kinematic hardening, is capable of reflecting the main features of a saturated stress strain hysteresis loop. However, this is not true with respect to elastic-plastic strain accumulation, since the (unlimited) linear kinematic hardening model cannot describe some of the essential features of material ratcheting. Thus, Zarka's method may be used to investigate structural ratcheting, but should be used with caution if material ratcheting is likely to occur as well.

References

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