Use of Zarka's Method at FHL

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Theory

Implementation in ANSYS

Experience

Cracow

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Background, History

in nuclear industry:

elastic-plastic response required for life assessment under cyclic loading:

→ **strain range $\Delta\varepsilon$**

→ **cyclic accumulated (ratchet-) strain ε_{acc}**

→ **displacements at end of life**

20 years ago:

only rough estimates available as „simplified" elastic-plastic methods (knock-down factors), without capturing effects of:

→ **individual geometry of structure**

→ **kind of loading (thermal, displacement-, force-controlled, ...)**

→ **hardening of material**

Background, History (ctd.)

by end of seventies Zarka's method came up, claiming it was able to account for these effects

publication of some examples obtained by Zarka with Zarka's method revealed:

- → superior quality of results (approximations almost exact)
- → obtained with little numerical effort (few lin. elastic analyses)

however: method remained largely obscure:

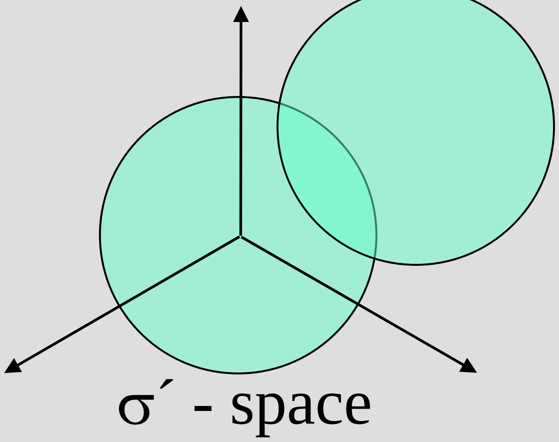
- → description of theory appeared to be not complete
- → some assumptions seemed to be heuristically motivated
- → some applications outside Zarka's team showed poor results

question:

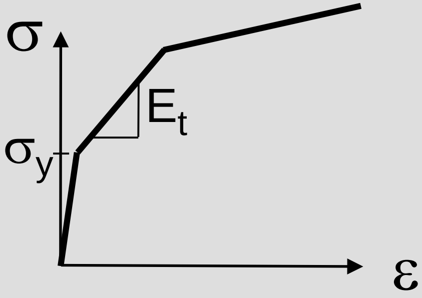
- **why** that ambiguous?
- **when** advantageous?

Outline of Zarka's method

material model:
- yield surface must exist
- hardening required:
  - » purely kinematic
  - » linear or multilinear
- temperature dependence:
  only size of yield surface, i.e. σ_y (not E, ν, E_t)



⇒"infinite" ratcheting ruled out:
limit state = ES or PS,
associated with "finite" ratcheting

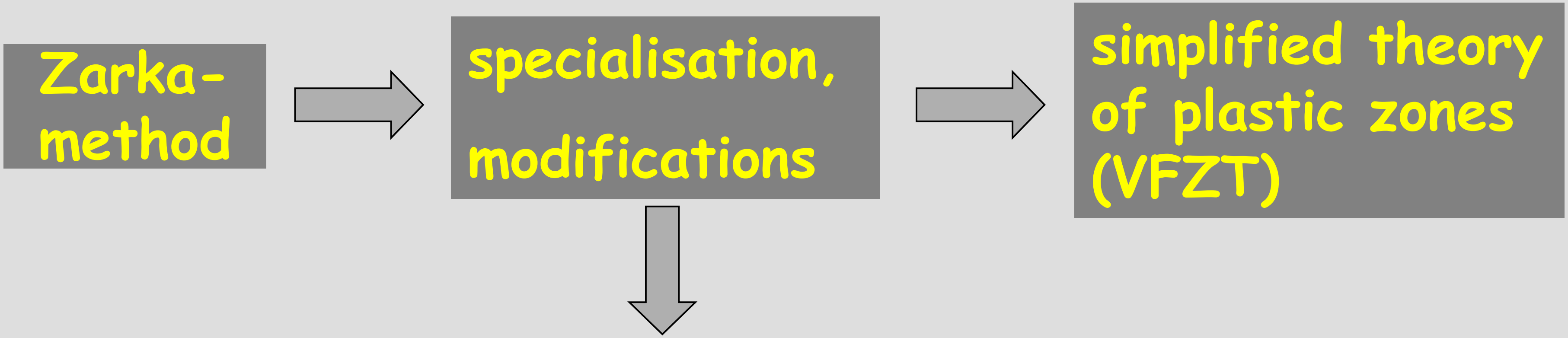


simplified method:

only partial information:
- no evolution of σ, ε with cycles
- only post-shakedown quantities
- loss in accuracy (Vp, σ, ε)

only reduced effort:
- direct method
- few elastic analyses sufficient
  + some "local" calculations

Specialisation, Modification



Zarka-method

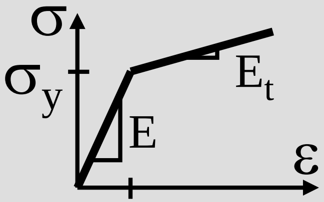
specialisation, modifications

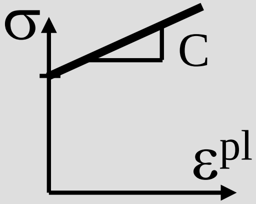
simplified theory of plastic zones (VFZT)

- moderate levels of strain (e.g. no deep drawing)
- isotropic material
- no strength differential effects (tensile = compressive)
- linear and bilinear kinematic hardening (bilin., trilin. $\sigma-\varepsilon$)
- Mises yield surface
- temperature-dependent yield stress
- mainly cyclic loading between 2 extreme states of loading
- logic of iterative improvement: identifying Vp, estimating TIV Y

Reformulation of "Exact" Plastic Theory

here: - uniaxial stress state
- bilinear σ–ε-diagram (kinem. hardening)
- monotonic loading





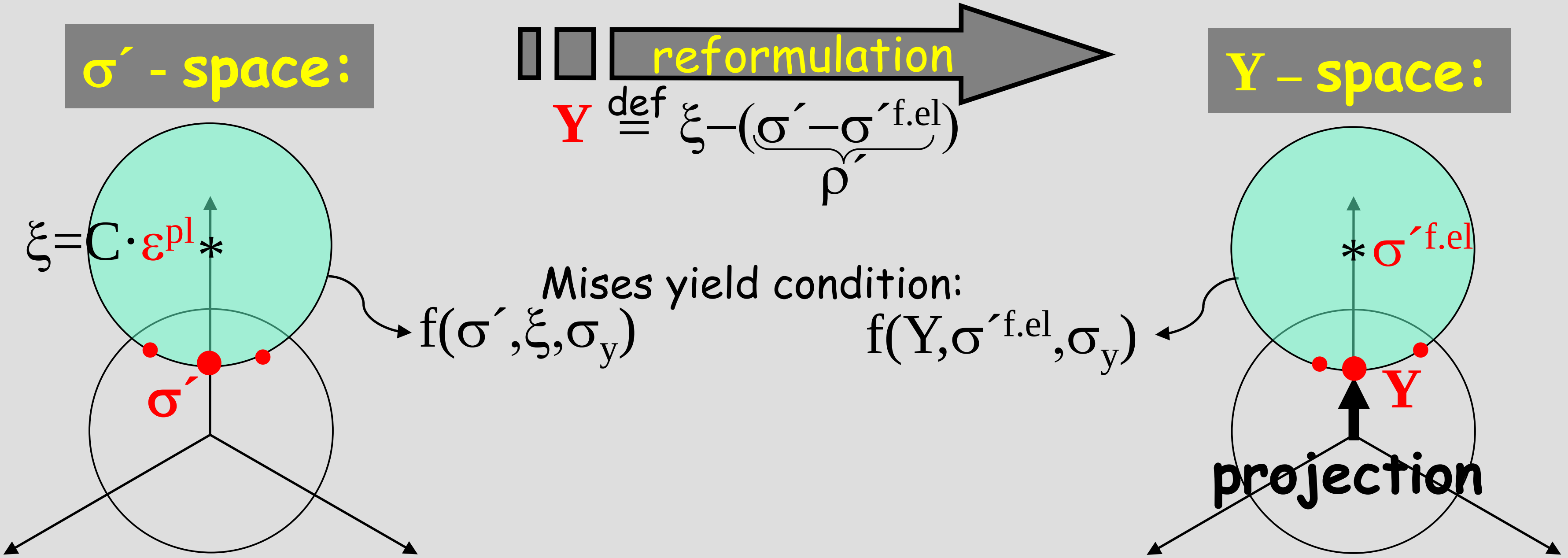
kind of analysis	**in plastic zone Vp**	**in el. zone Ve** (=V-Vp)	**load applied**
exact (el-pl)	$\varepsilon^{el\text{-}pl} = \sigma^{el\text{-}pl}/E + \xi/C$	$\varepsilon^{el\text{-}pl} = \sigma^{el\text{-}pl}/E$	yes
fictitious elastic (f.el)	$\varepsilon^{f.el} = \sigma^{f.el}/E$	$\varepsilon^{f.el} = \sigma^{f.el}/E$	yes
difference	$\underbrace{\varepsilon^{el\text{-}pl}-\varepsilon^{f.el}}_{\varepsilon^*}=(\underbrace{\sigma^{el\text{-}pl}-\sigma^{f.el}}_{\rho})/E+\underbrace{\xi}/C$ $\xi \overset{def}{=} \rho+\mathbf{Y}$	$\underbrace{\varepsilon^{el\text{-}pl}-\varepsilon^{f.el}}_{\varepsilon^*}=(\underbrace{\sigma^{el\text{-}pl}-\sigma^{f.el}}_{\rho})/E$	no
modif. elast. (**mea**)	$\Rightarrow \boldsymbol{\varepsilon^* = \rho\cdot(1/E^*) + \varepsilon_0}$ with $1/E^*=1/E+1/C$ and $\varepsilon_0=\mathbf{Y}/C$	$\Rightarrow \boldsymbol{\varepsilon^* = \rho\cdot(1/E)}$	initial strain ε_0 in Vp

→ residual state can be obtained by 1 mea with modif. elastic material data (E*) and modif. loading (ε_0), if **Vp** and **Y** are known

→ idea of Zarka-Method: estimation of **Vp** and **Y**

Mon. Loading: Estimation of Y

here: bilinear σ–ε-diagram (lin. kinem. hardening)



σ´ - space:

reformulation

Y – space:

$$\mathbf{Y} \overset{\text{def}}{=} \xi-(\underbrace{\sigma'-\sigma'^{f.el}}_{\rho'})$$

Mises yield condition:

$f(\sigma',\xi,\sigma_y)$

$f(Y,\sigma'^{f.el},\sigma_y)$

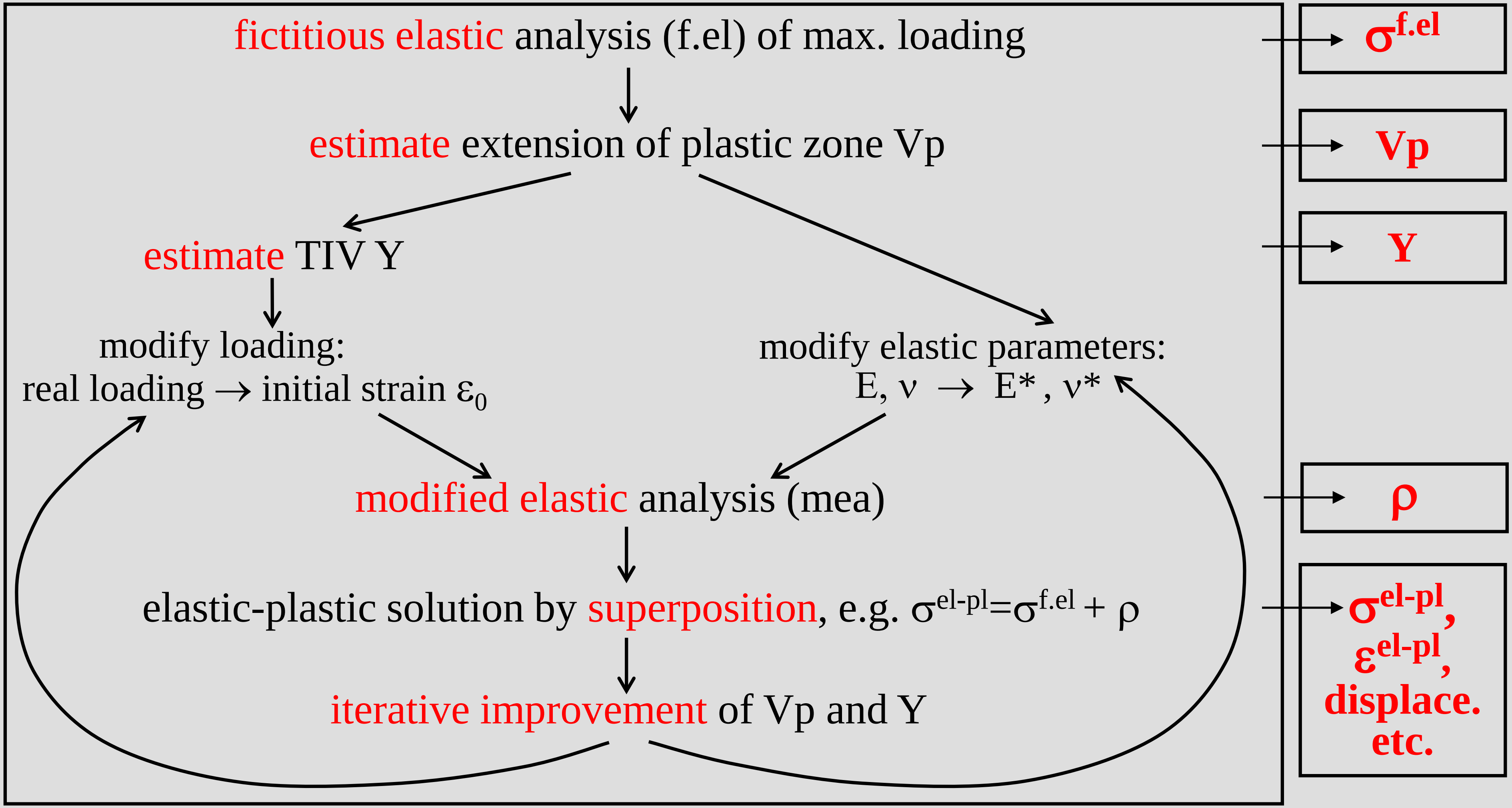
σ´ is unknown;
estimation not possible, since
center ξ (i.e. ε^{pl}) also unknown

Y is unknown;
but estimation possible,
since center $\sigma'^{f.el}$ is known

uniaxial stress: Y exactly known by projection; only Vp unknown

multiaxial stress: Vp and Y unknown; local estimation of Y sufficient, since field effects of Y are relieved compared to σ (=heuristic)

Mon. Loading: Algorithm



fictitious elastic analysis (f.el) of max. loading

estimate extension of plastic zone Vp

estimate TIV Y

modify loading:
real loading → initial strain ε_0

modify elastic parameters:
$E, \nu \rightarrow E^*, \nu^*$

modified elastic analysis (mea)

elastic-plastic solution by superposition, e.g. $\sigma^{el\text{-}pl}=\sigma^{f.el} + \rho$

iterative improvement of Vp and Y

$\sigma^{f.el}$

Vp

Y

ρ

$\sigma^{el\text{-}pl}$,
$\varepsilon^{el\text{-}pl}$,
displace.
etc.

numerical effort required:

→ few linear analyses (fictitious elastic and modified elastic)
→ local calculations

Mon. Loading: Iterative Improvement

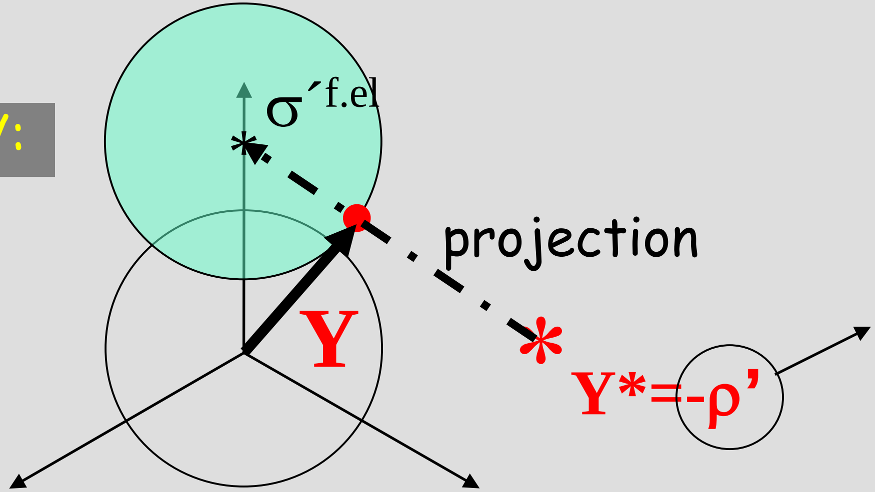
iterative improvement of Vp:

$$\sigma_{v}^{el-pl} \begin{Bmatrix} > \\ \le \end{Bmatrix} \sigma_{y} \rightarrow \begin{Bmatrix} Vp \\ Ve \end{Bmatrix}$$

with σ_{ij}^{el-pl} from previous iteration

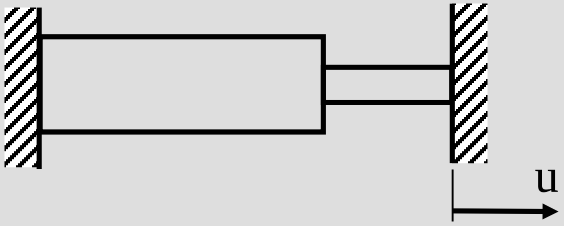
iterative improvement of Y:

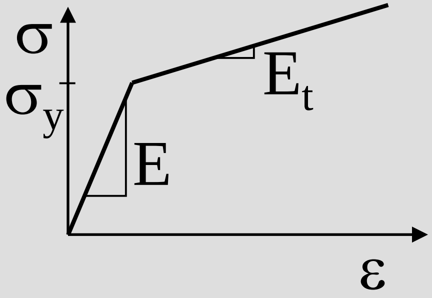
Y – space:



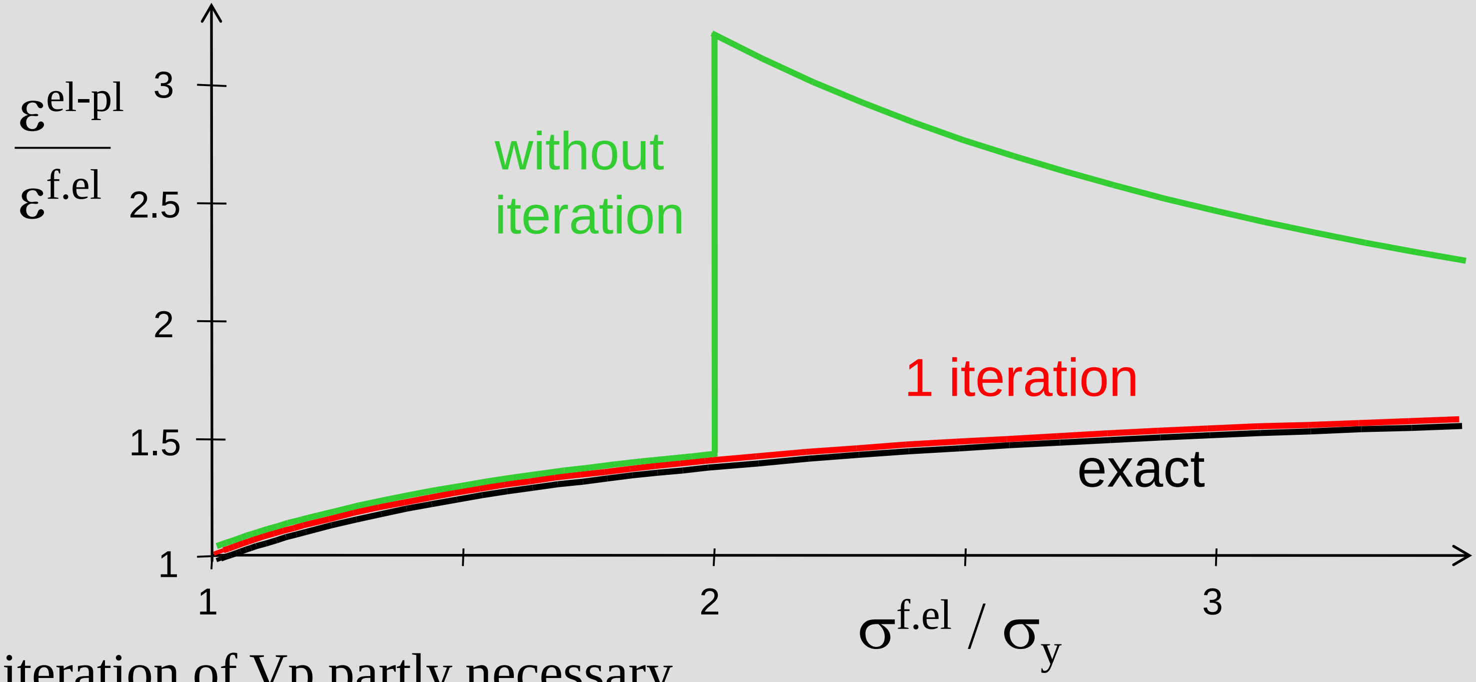
with ρ_{ij} from previous iteration

Example 1: Mon. Loading: 2 Bars in Series



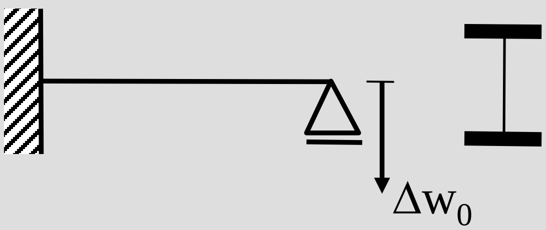


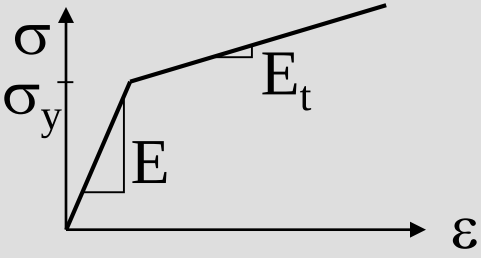
- hand calculation ⇒ analytical solution
- e.g. for specific configuration (A_1/A_2, l_1/l_2; E_t/E):



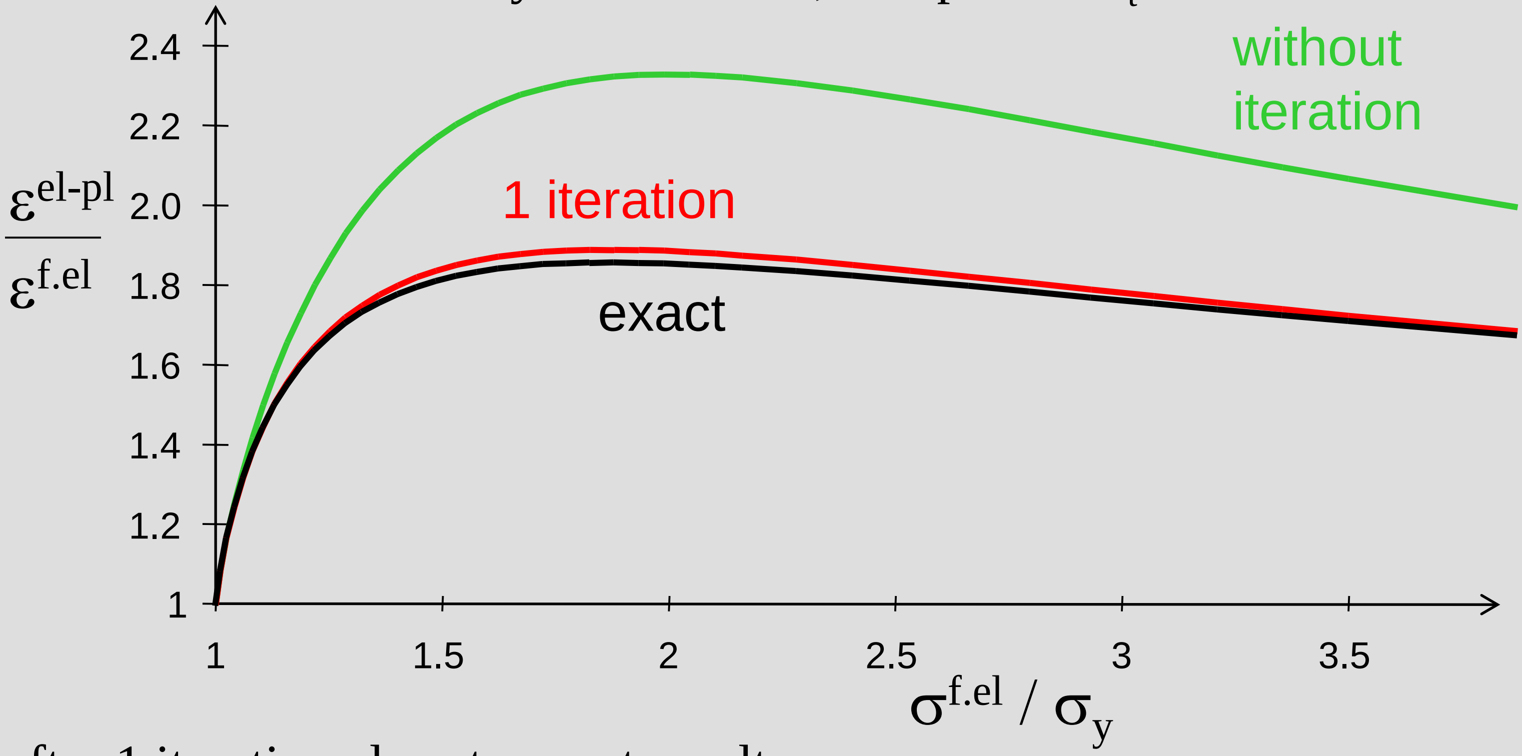
→ iteration of Vp partly necessary
→ exact results obtained after maximum 1 iteration

Example 2a: Mon. Loading: Beam of ideal I-Profile



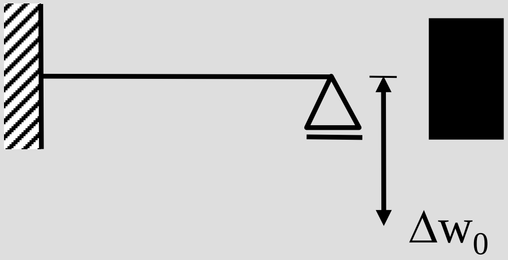


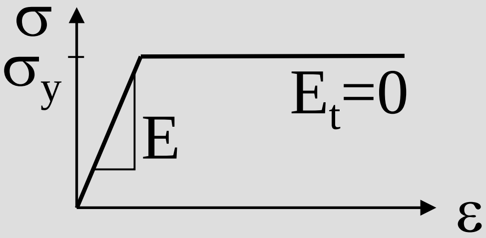
- hand calculation ⇒ analytical solution; for specific E_t/E:



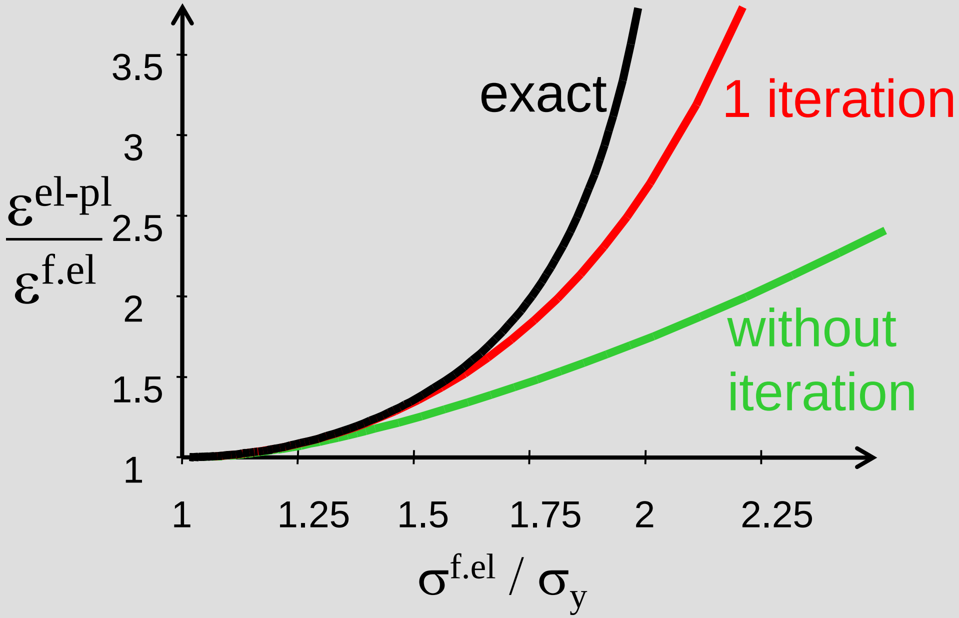
→ after 1 iteration close to exact results

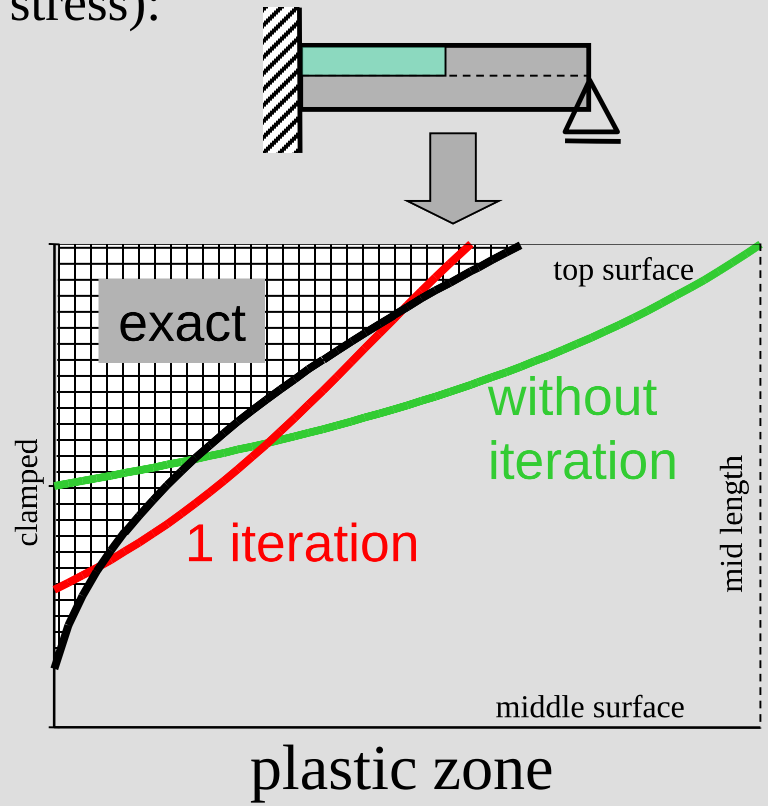
Example 2b: Mon. Loading: Beam of rect. Cross Section





hand calculation possible for $E_t=0$ (via initial stress):

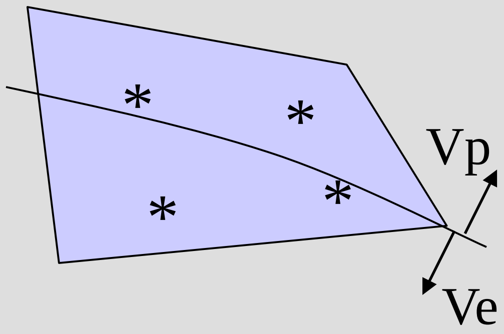




Implementation in FE-codes

implementation in any commercial FE code (ANSYS, ABAQUS, ADINA, ...) usually not easy:

- modification of elastic material properties and of loading (initial strain) required during iterative solution
- access to elastic material properties and application of initial strain usually inconsistent: E → element, ε_0 → Gauss or nodal point
- exact definition of Vp somewhat arbitrary (Vp, if $\sigma_v > \sigma_y$ in > 50% of Gauss-points)



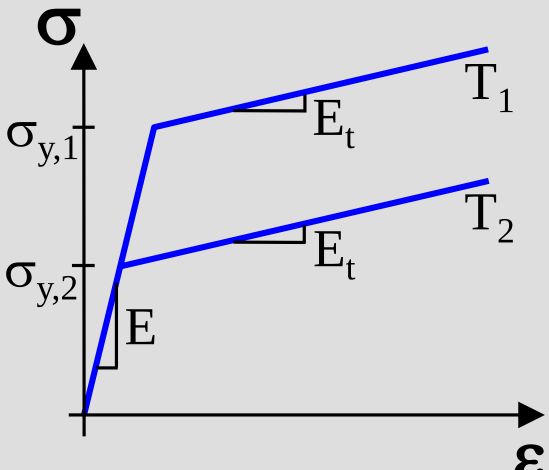
implementation at FHL in ANSYS :

for general structures (1D, 2D, 3D):

- bilinear $\sigma - \varepsilon$
- T-dependent σ_y
- mon. + cyclic loading (2 states of loading)

extension for specific structures (trusses, beams):

- trilinear $\sigma - \varepsilon$
- increm. projection in ES, if > 2 states of loading



Ways of Implementation in ANSYS

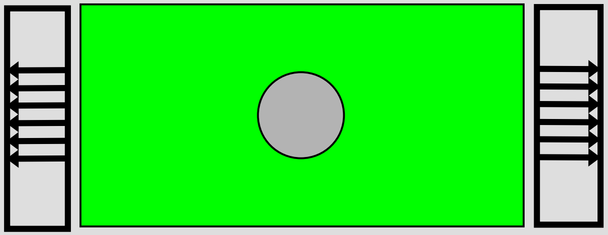
1. VEFZOT: APDL-Macro (=ANSYS parameter language; slow)
   initial strain simulated by:
   * thermal strain at nodal points
   * orthotropic coefficient of thermal expansion (→ deviatoric)
   * orthotropic directions changing from el. to el. →sometimes poor qual.

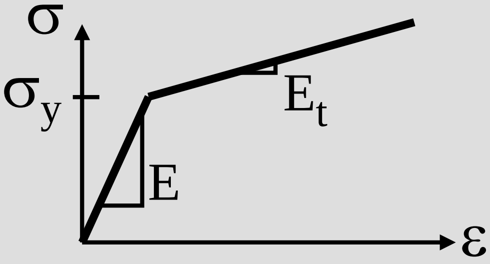
2. Userpl.f: FORTRAN user subroutine (compiling and linking required)
   initial strain simulated by:
   * inelastic strain at Gauss points
   * formally equilibrium iterations performed, although not necessary

3. IS-Macros (APDL-Macro) or Ustress.f (FORTRAN-subroutine):
   initial strain simulated by initial *stress* at Gauss points

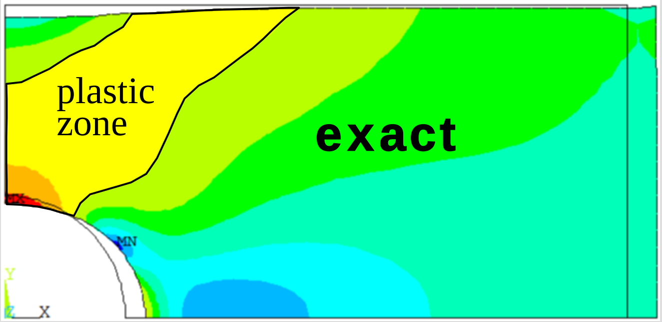
4. Particular implementation for trusses and bending beams:
   initial strain simulated by ts and Δt across section

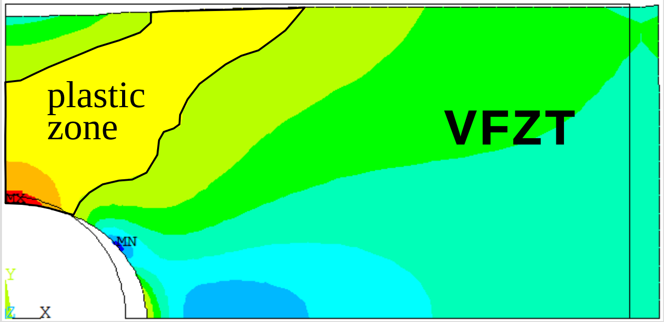
Example 3: Mon. Loading: Hollowed Plate



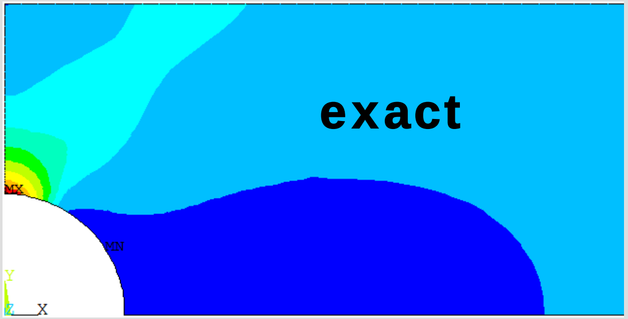


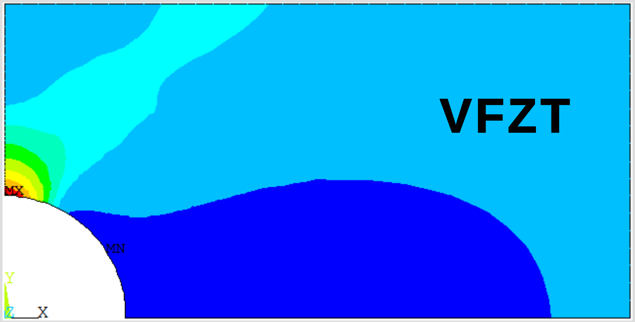
displace-
ment
+
Mises
effective
stress



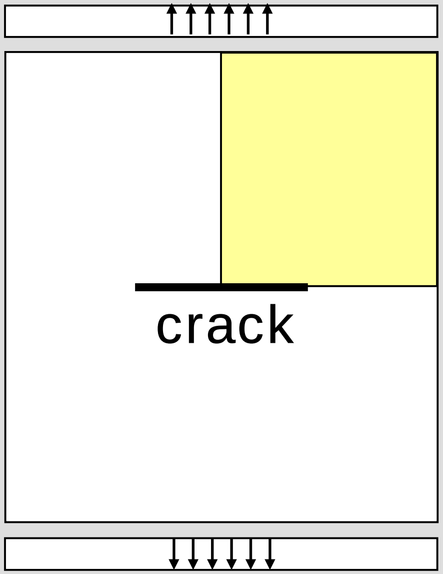


axial
strain

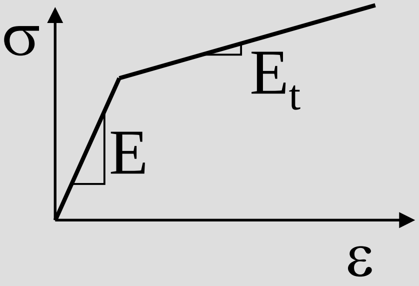




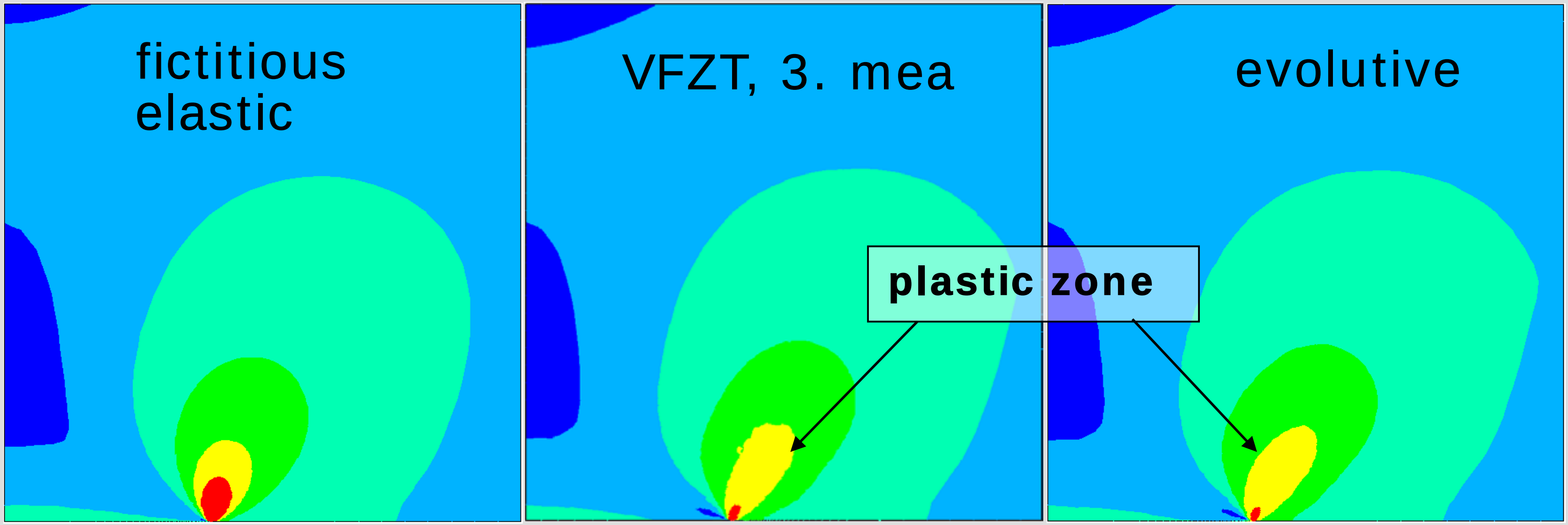
Example 4: Mon. Loading: Crack in a Plate



monotonic loading



Mises effect. stress:

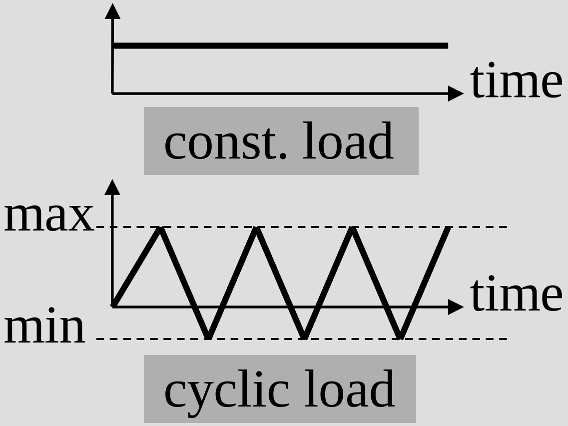

fictitious elastic

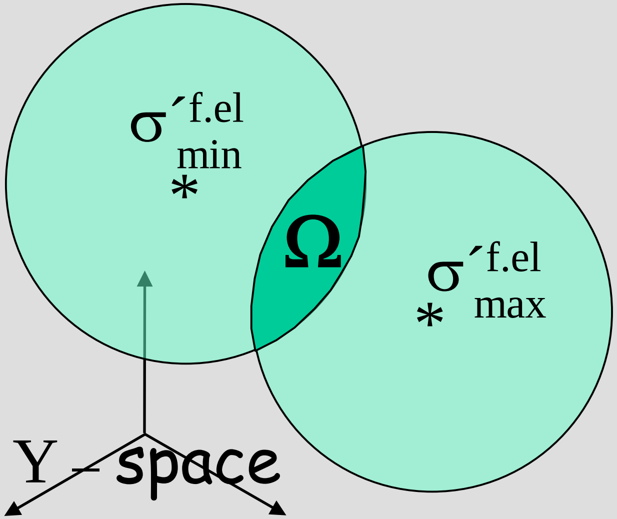
VFZT, 3. mea

evolutive

Cyclic Loading: ES: Estimation of Y

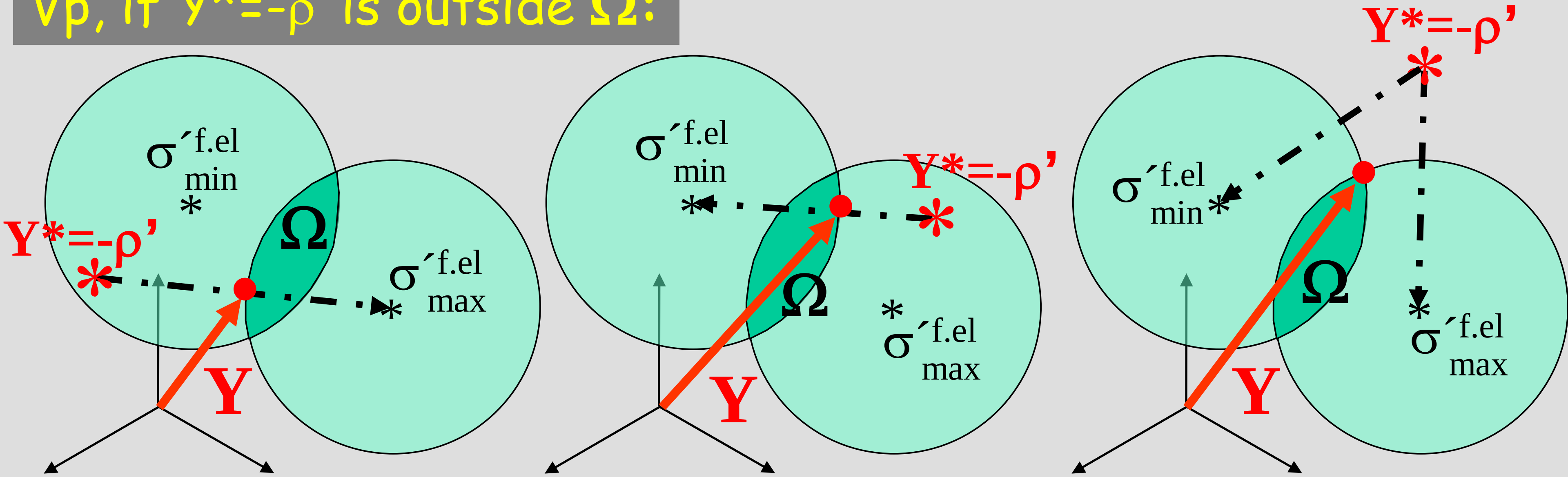
load histogram:



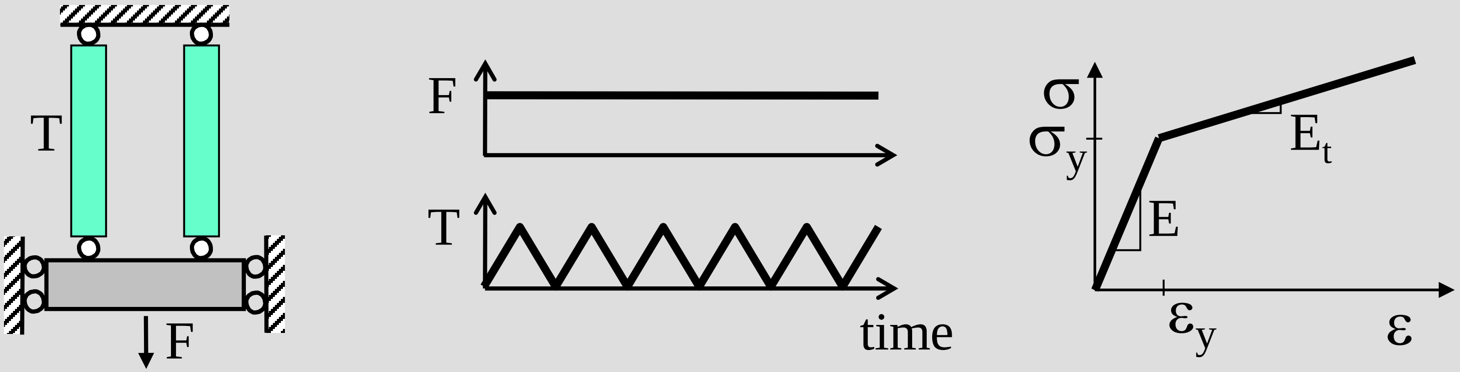


elastic shakedown (ES): if Ω exists at each point of structure

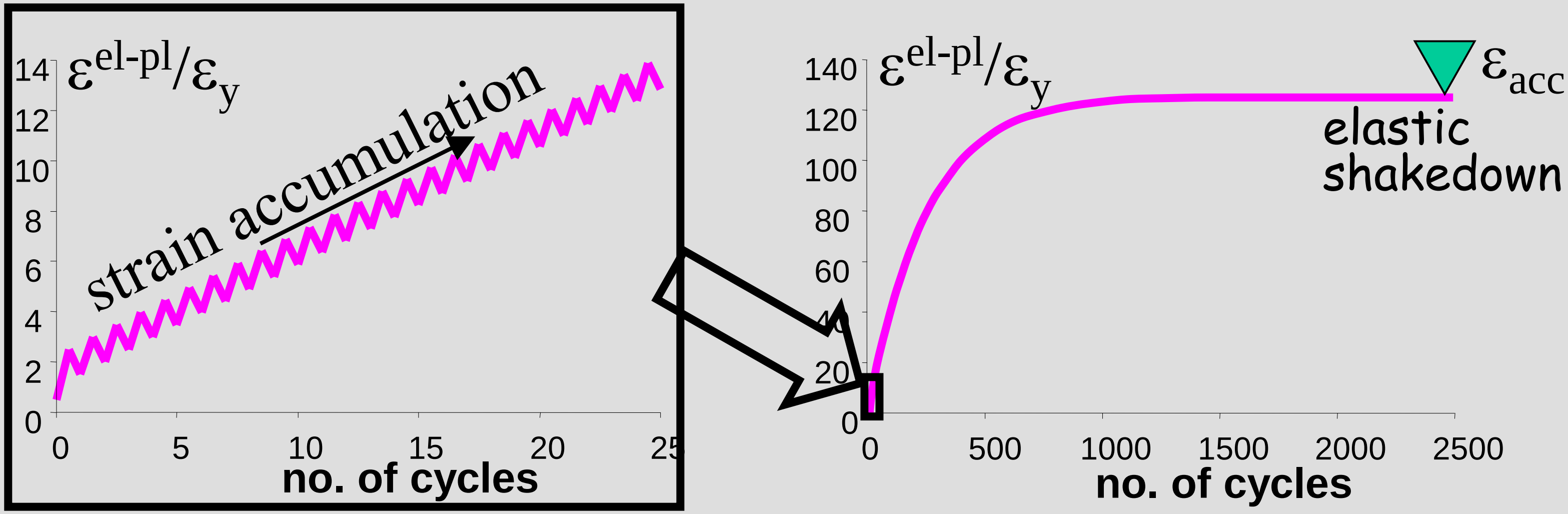
Vp, if $Y^*=-\rho'$ is outside Ω:



Example 5: ES: 2 Bars Parallel



exact:



numerical effort required to achieve ES:

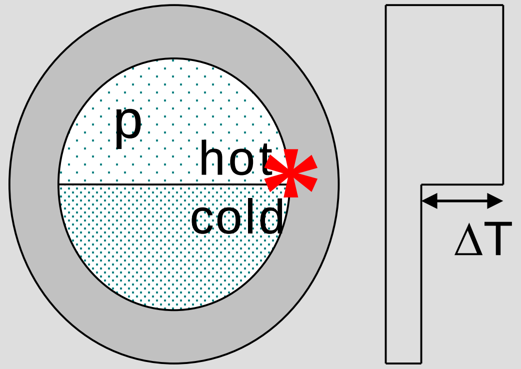
→ **exact**: 1000 cy. * 10 loadsteps * 5 iterations

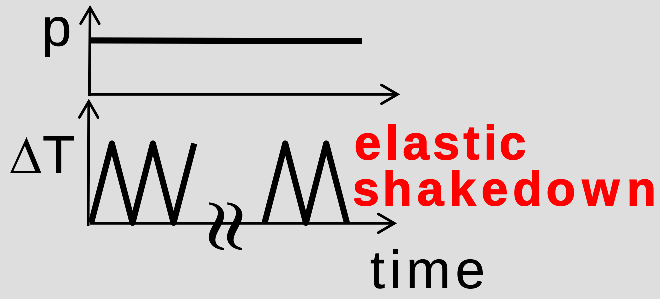
~ ca. 50 000 linear elast. anal.

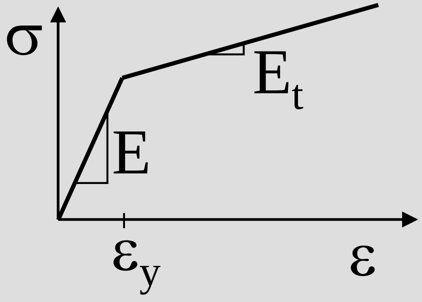
→ **VFZT**: 2 f.el. + 2 mea = 4 linear elast. anal.

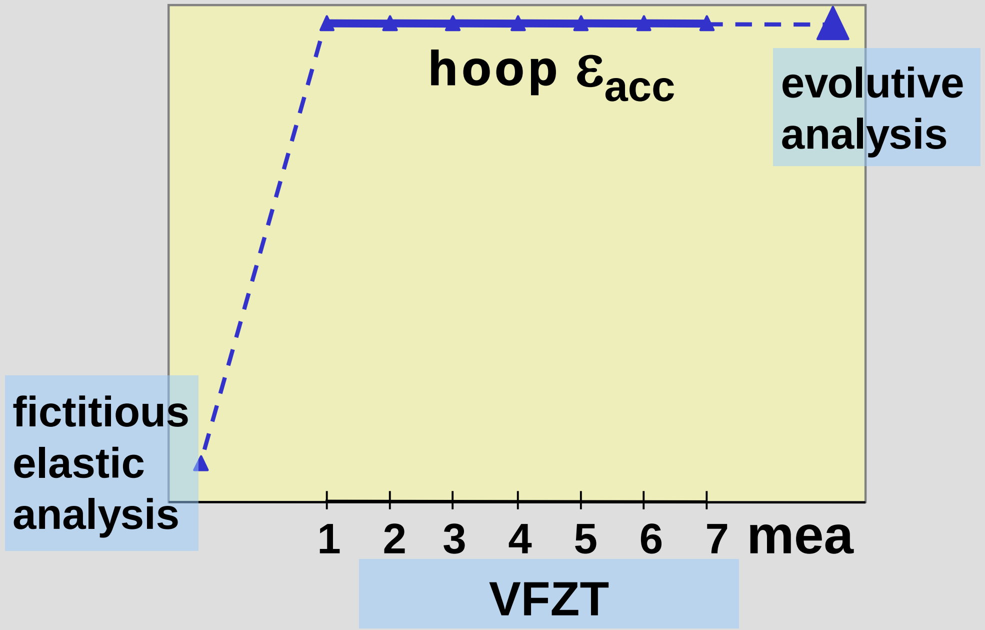
identical results

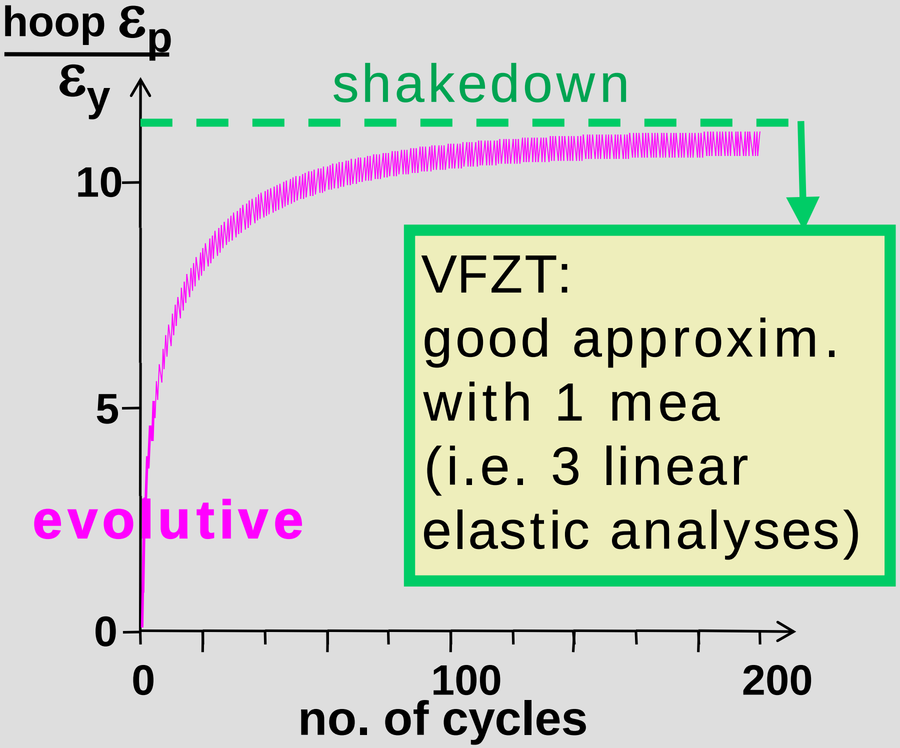
Example 6: ES: Thermal Stratification







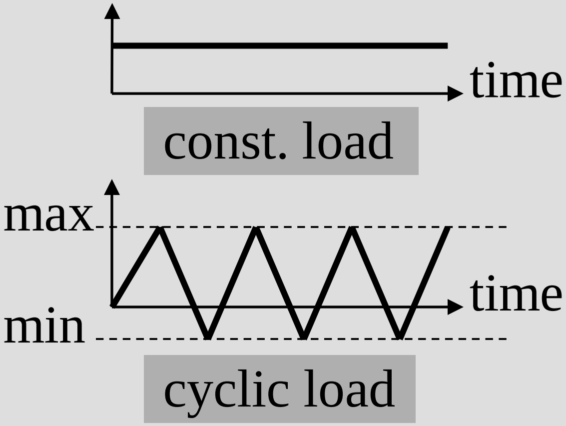


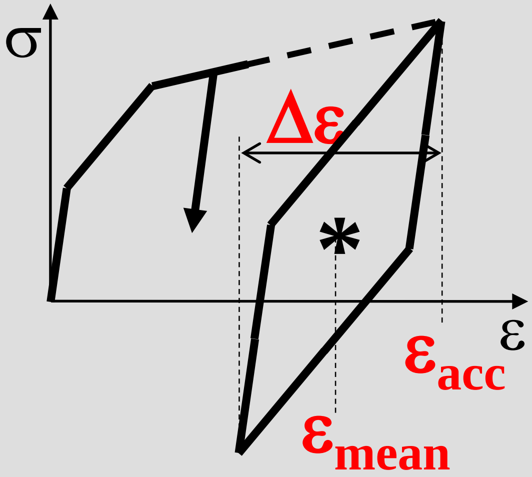


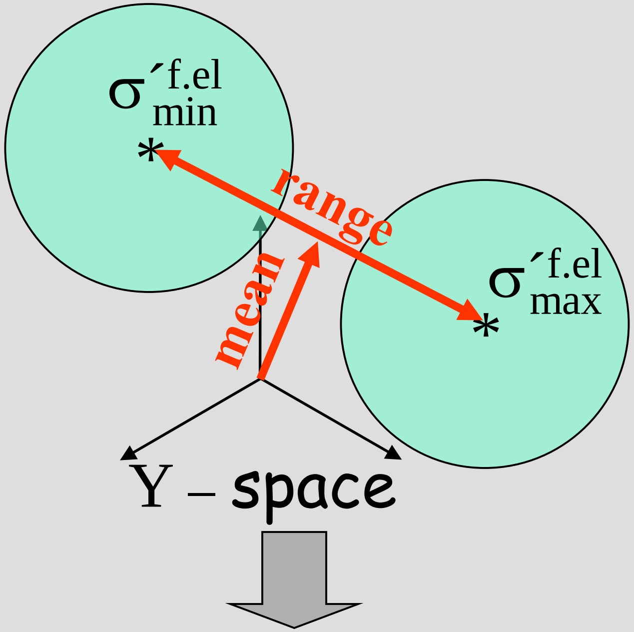
VFZT:
good approxim.
with 1 mea
(i.e. 3 linear
elastic analyses)

Cyclic Loading: PS: Estimation of ΔY

load histogram:

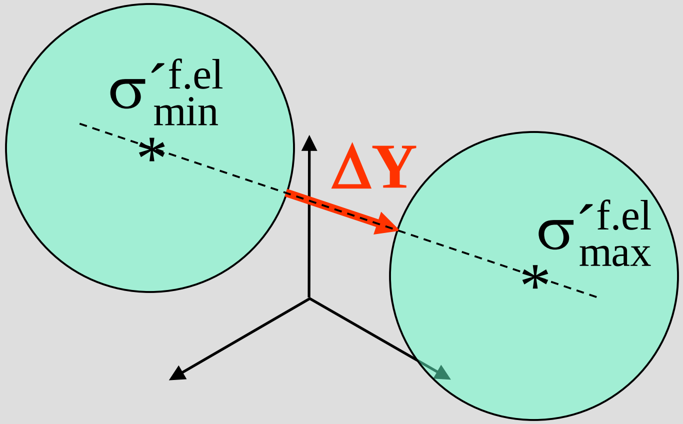


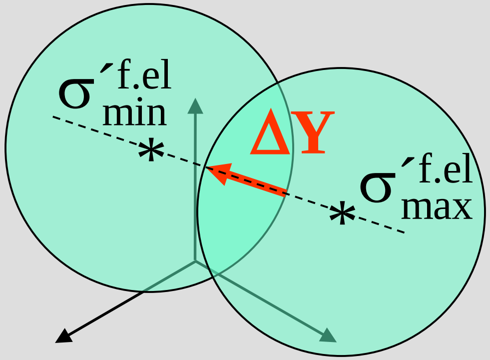




plastic shakedown (PS): if there is no intersection at least at 1 point of structure

range values: Vp, if $\Delta\sigma_v > 2\sigma_y$:

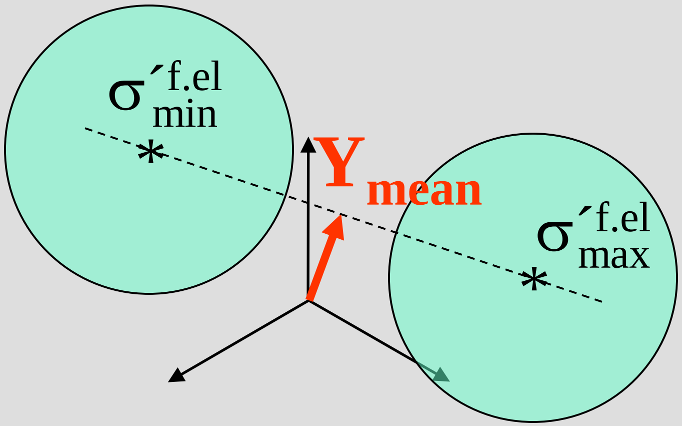




Cyclic Loading: PS: Estimation of Y_{mean}

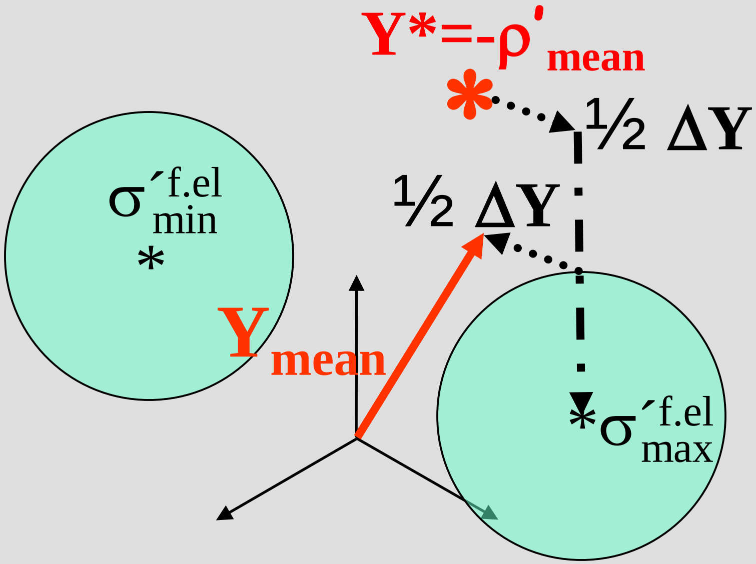
mean values: Vp, if $\sigma_{v,min} > \sigma_y$ or $\sigma_{v,max} > \sigma_y$:

in Vp,Δ:

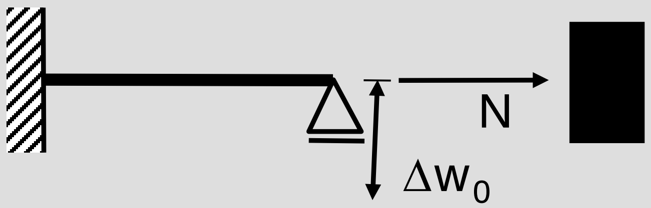


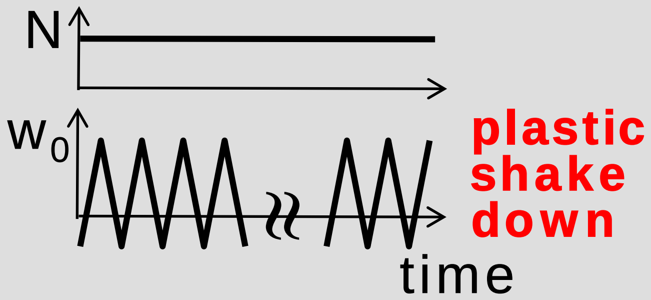
in Ve,Δ:

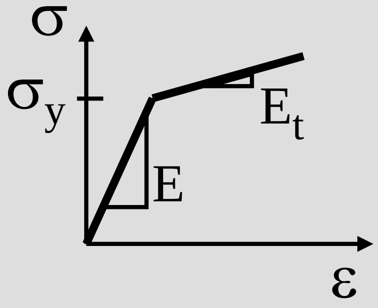
projection of $Y^* \pm \frac{1}{2} \Delta Y$ on max (or on min, if closer):

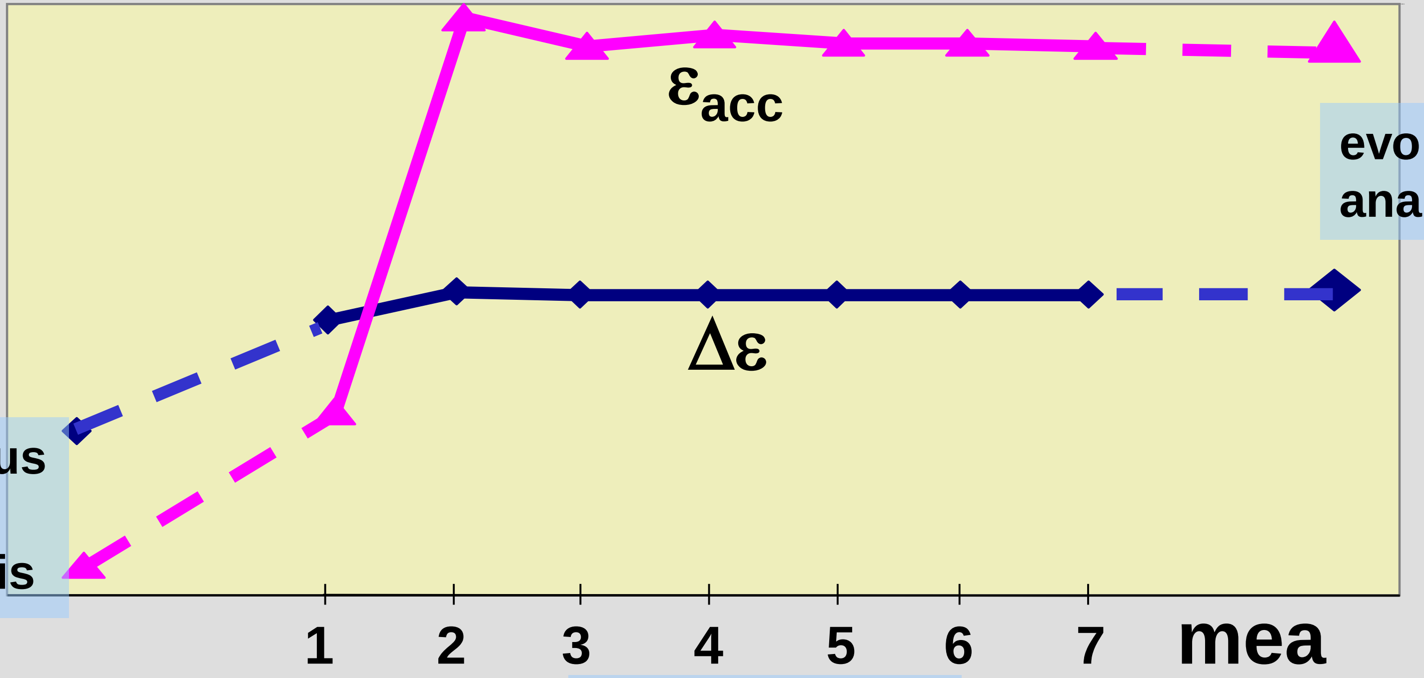


Example 7: PS: Beam under N, Δw_0







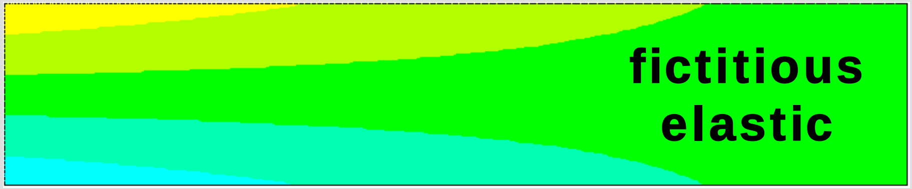


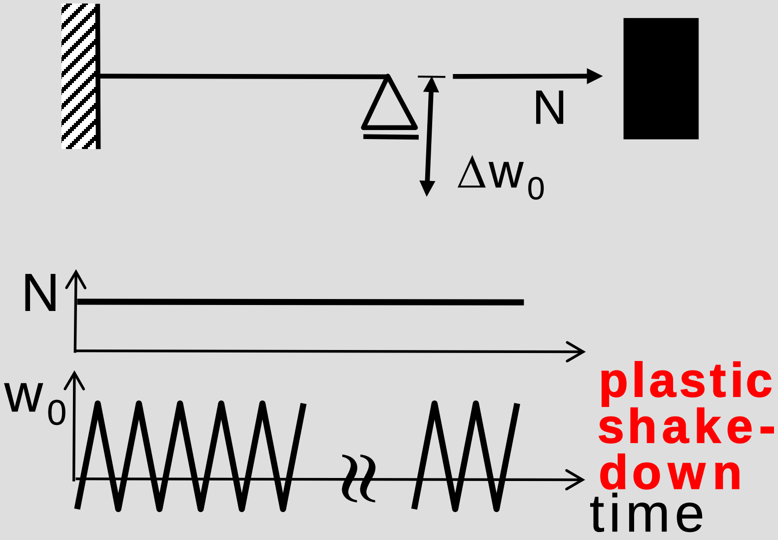
evolutive analysis

fictitious elastic analysis

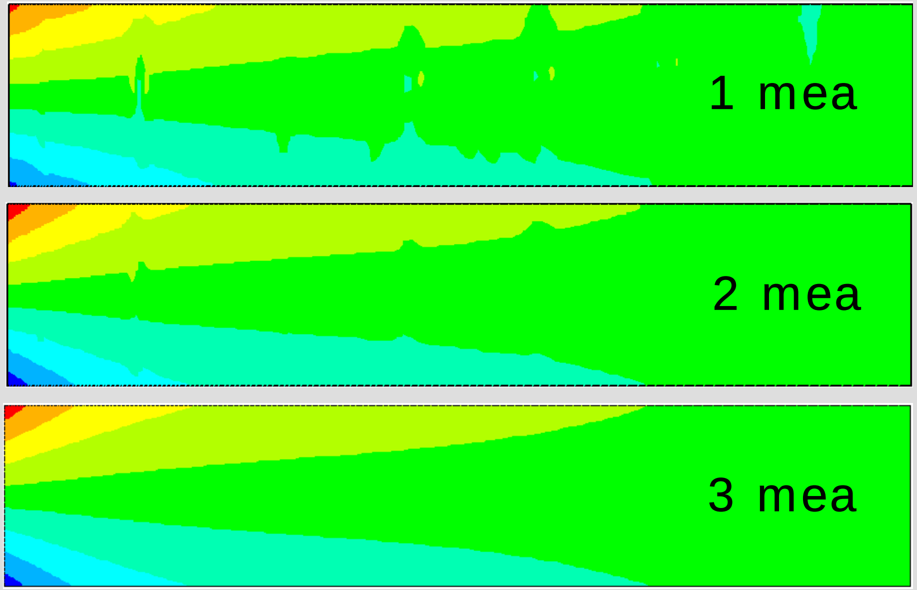
VFZT

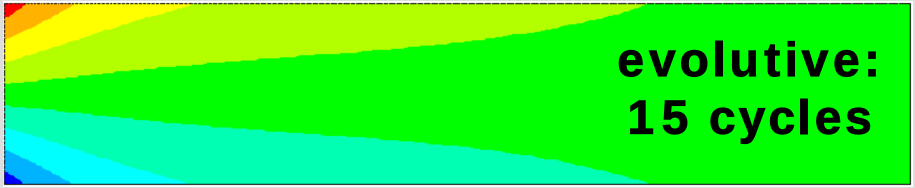
Example 7 (ctd.): PS: Beam: Δε



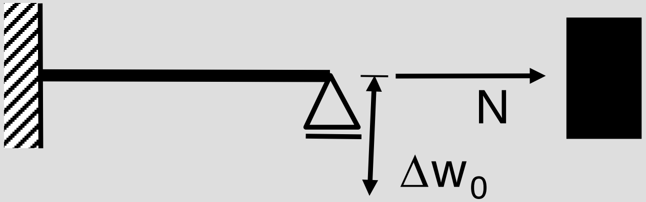


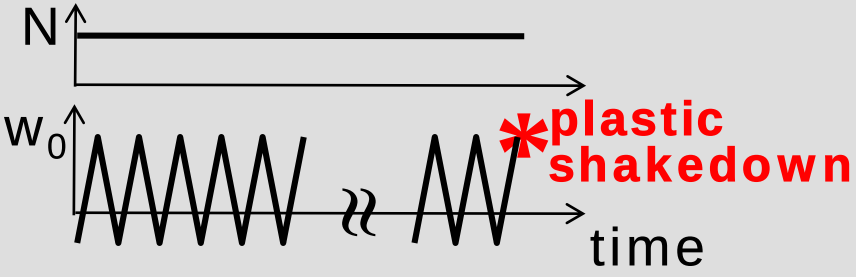
VFZT:

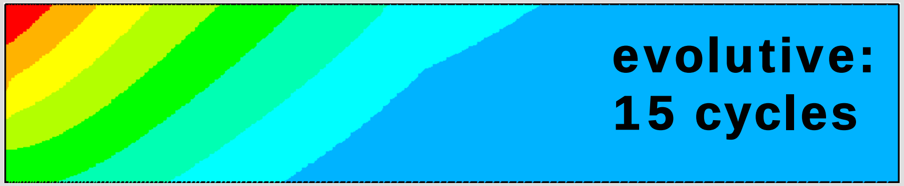




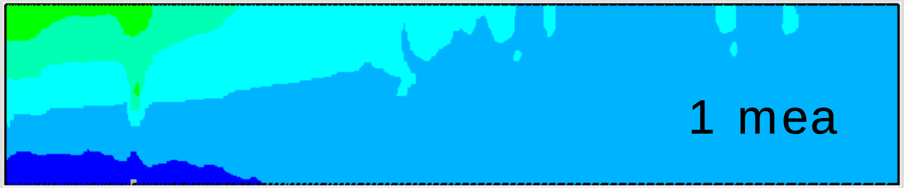
Example 7 (ctd.): PS: Beam: ε_{acc}

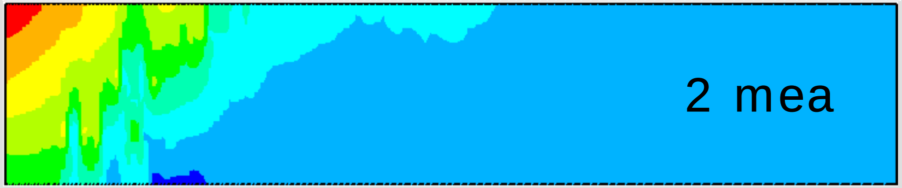


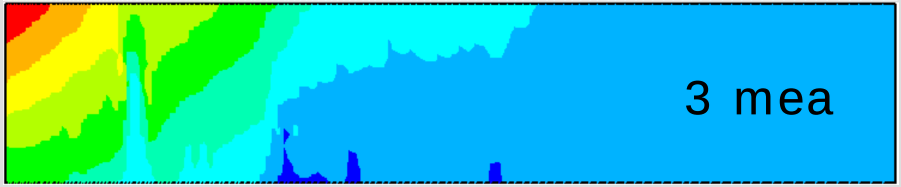


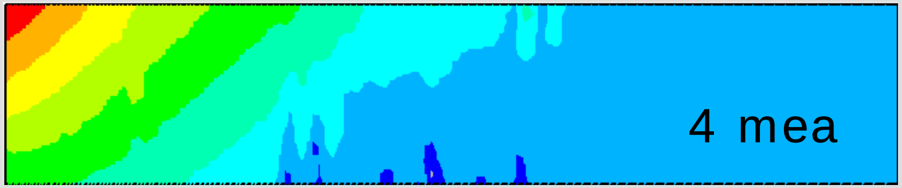


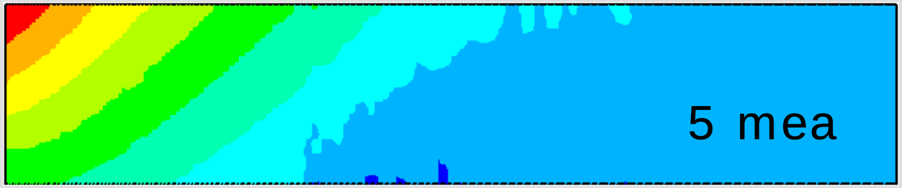
VFZT:



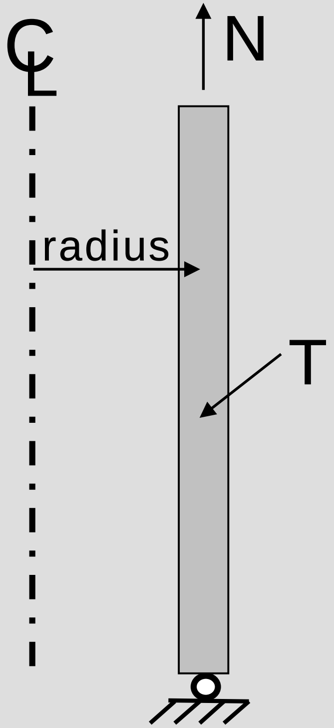


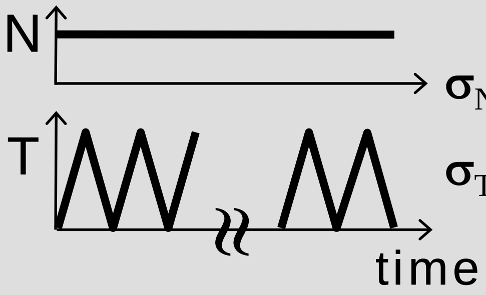






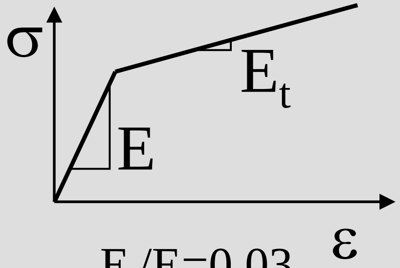
Example 8: PS: Cylindrical Shell under Axial Force and Temperature





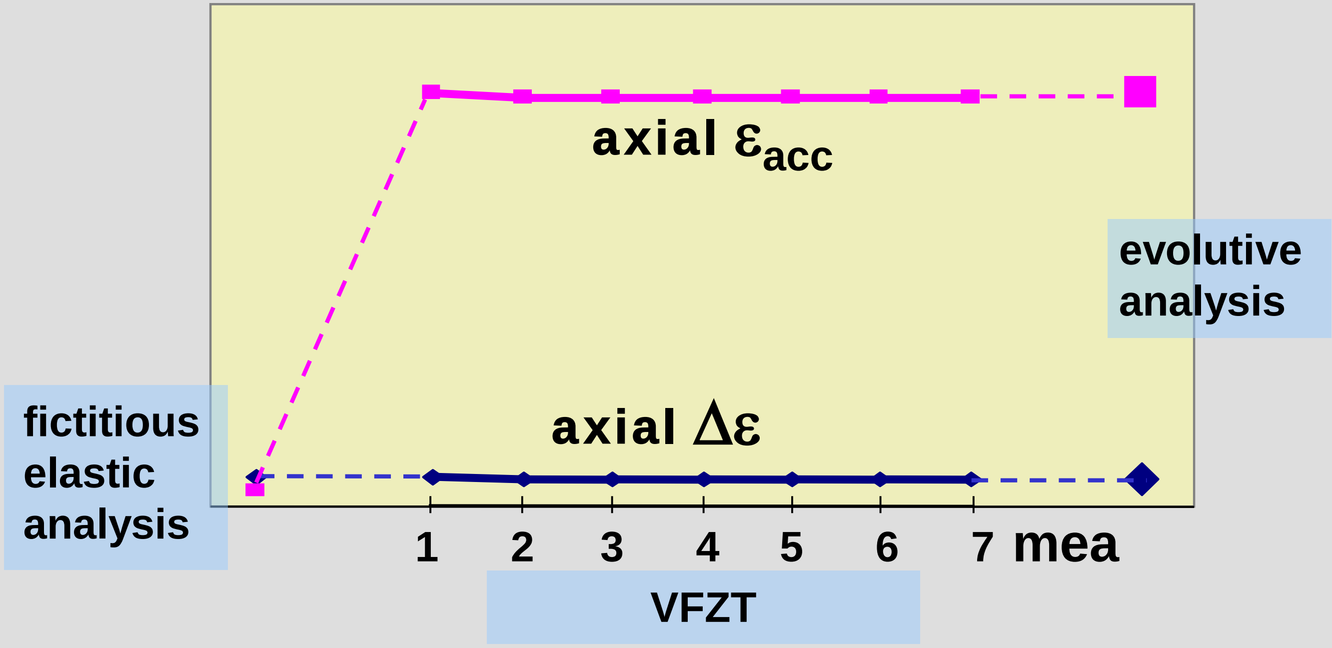
$\sigma_N^{f.el}/\sigma_y=0.8$

$\sigma_T^{f.el}/\sigma_y=2.2$ (→PS)

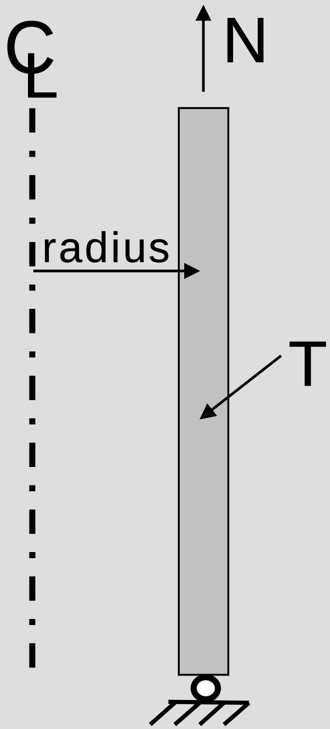


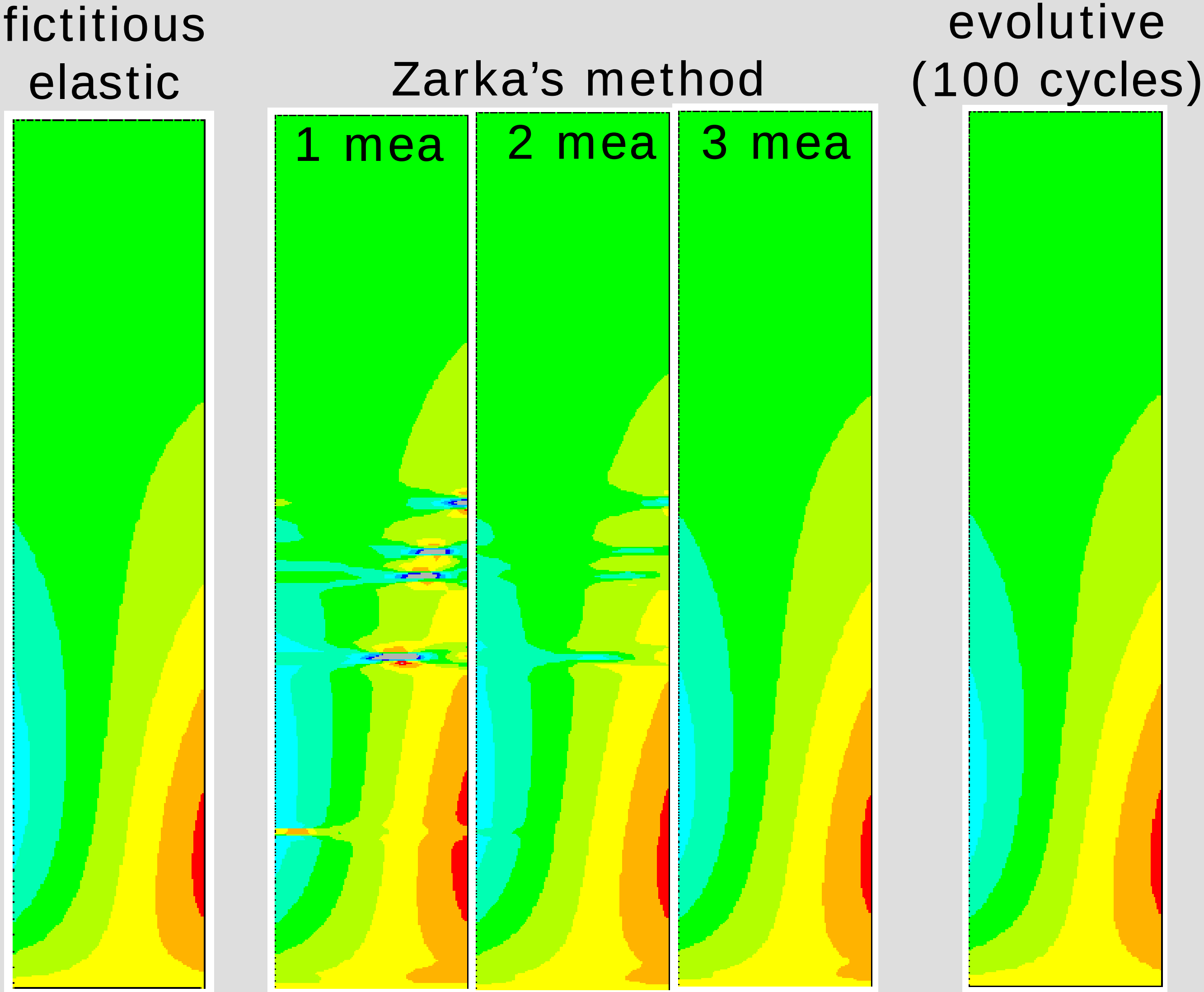
$E_t/E=0.03$

$\nu=0.3$



Example 8 (ctd.): PS: Cylindrical Shell: axial Δε




fictitious elastic

Zarka's method

evolutive (100 cycles)

1 mea

2 mea

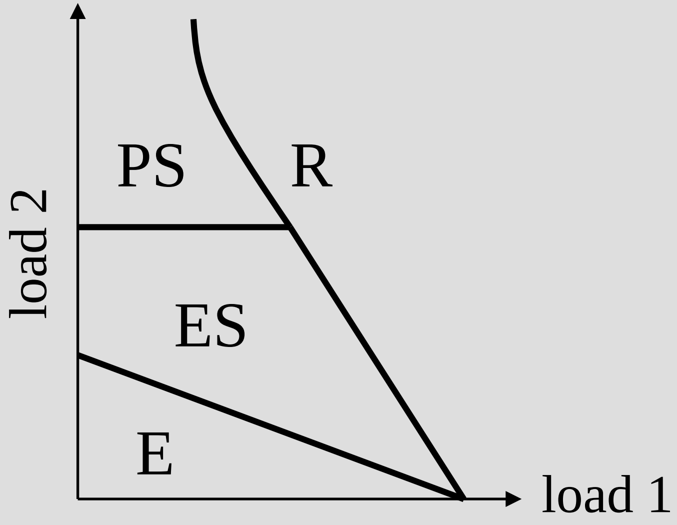
3 mea

Application: Ratcheting-Interaction Diagrams

which combination of load parameters yields ES, PS, R for specific configuration of structure and loading?

→ by shakedown theorems

→ no post-shakedown quantities

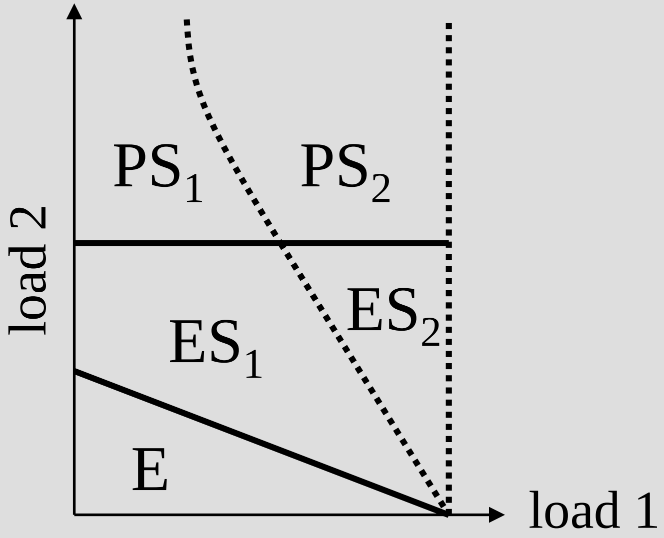


Zarka's method:

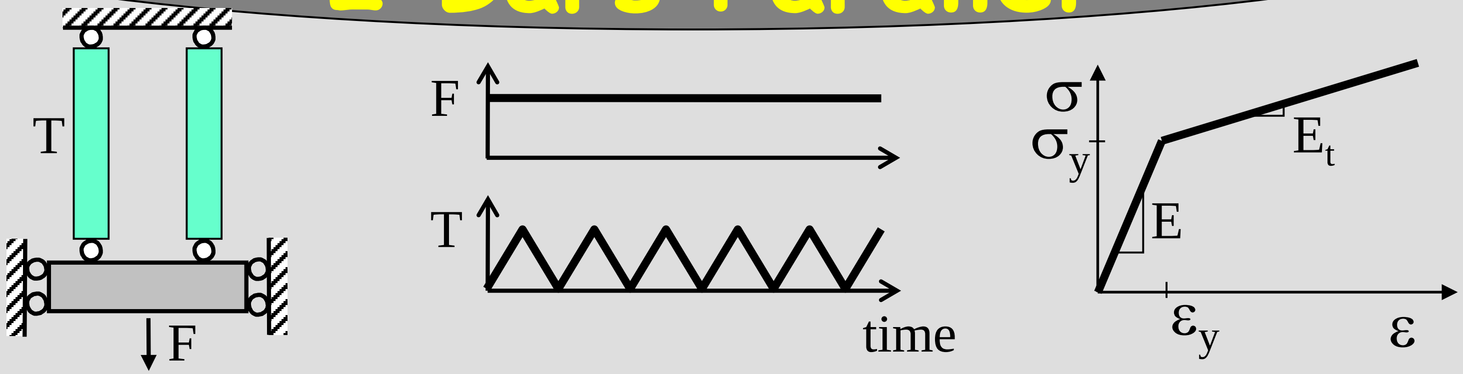
→ ES, PS known a priori

→ R not possible: subregions ES_2, PS_2

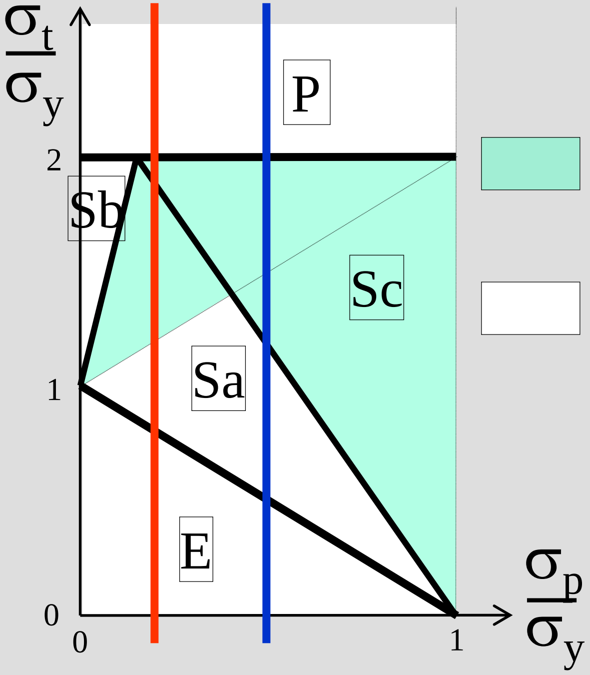
→ post-shakedown quantities ($\Delta\varepsilon$, ε_{acc})



Example 9: Ratch-Inter: 2 Bars Parallel

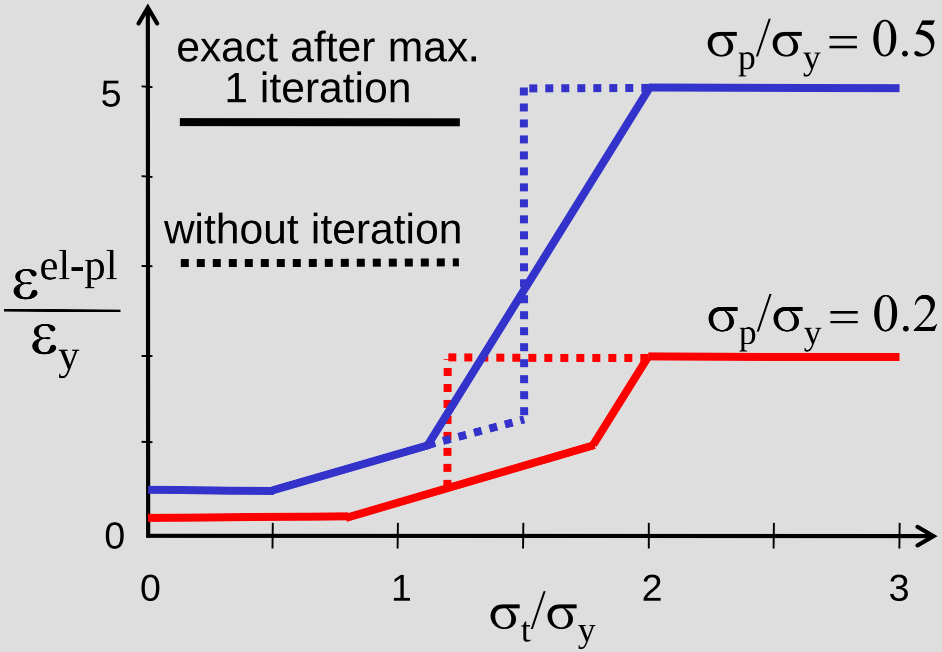


hand calculation for any configuration of loading (T, F) and hard. param. (E_t/E):



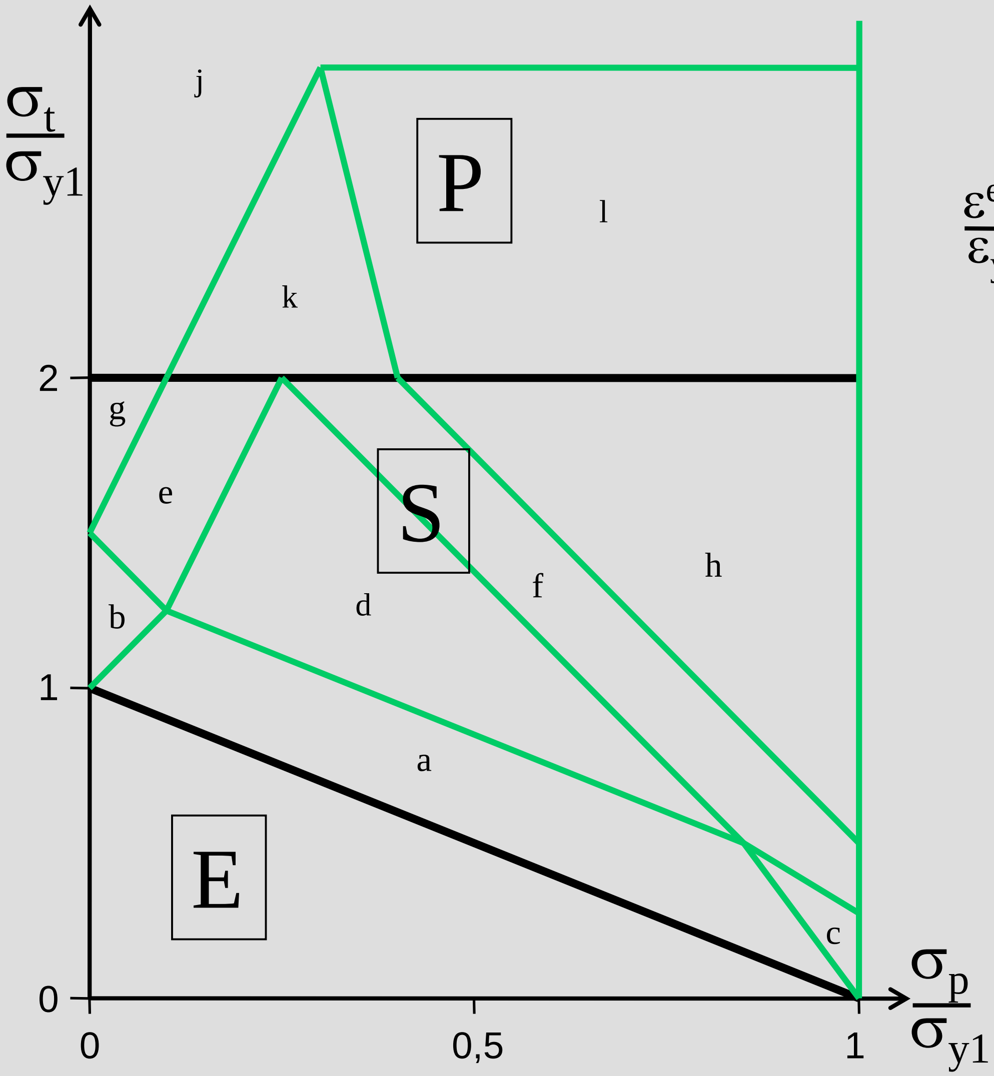
🟩 exact with 1 iteration

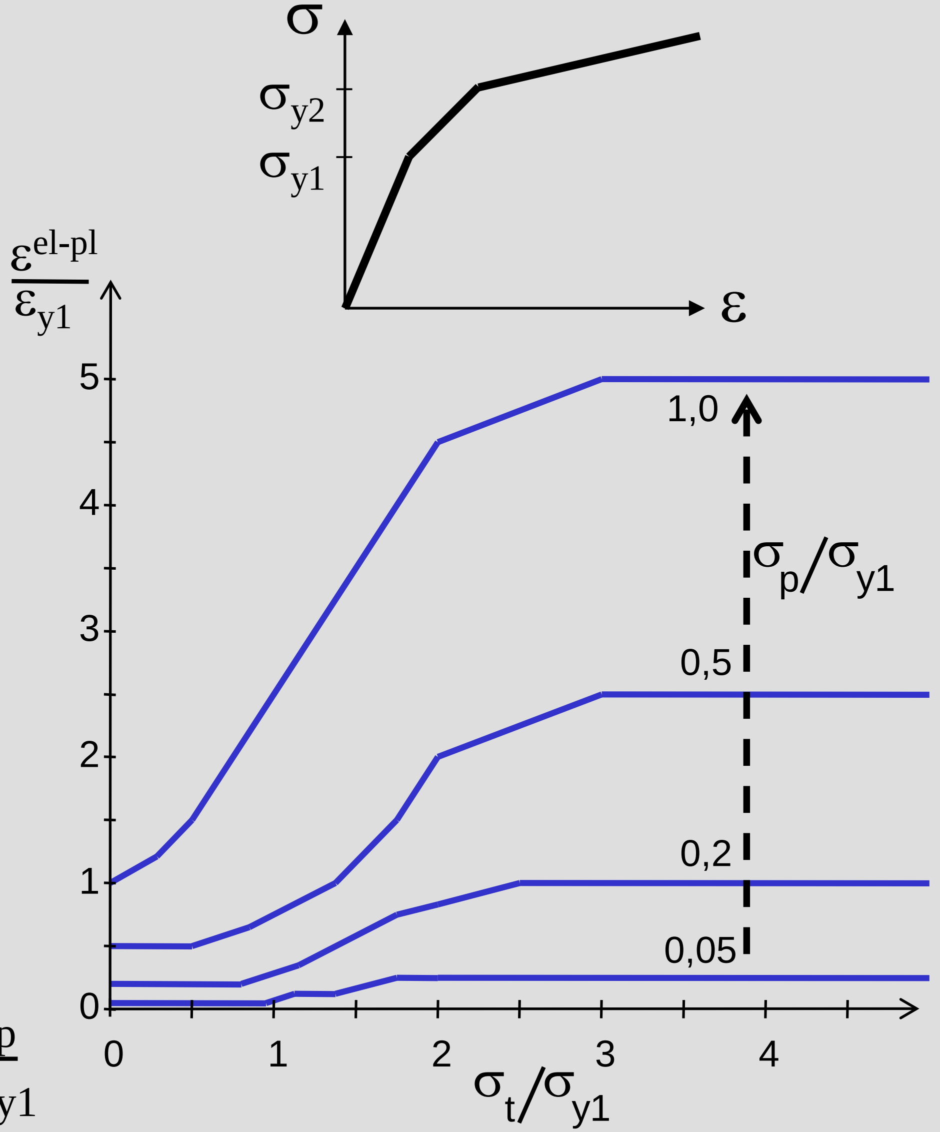
⬜ exact without iteration



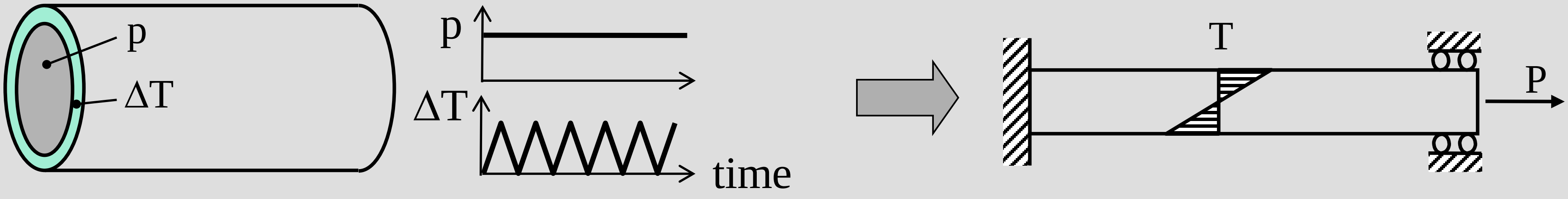
→ exact after maximum 1 iteration

Example 10: Ratch-Inter: 2 Bars Parallel (Trilinear)

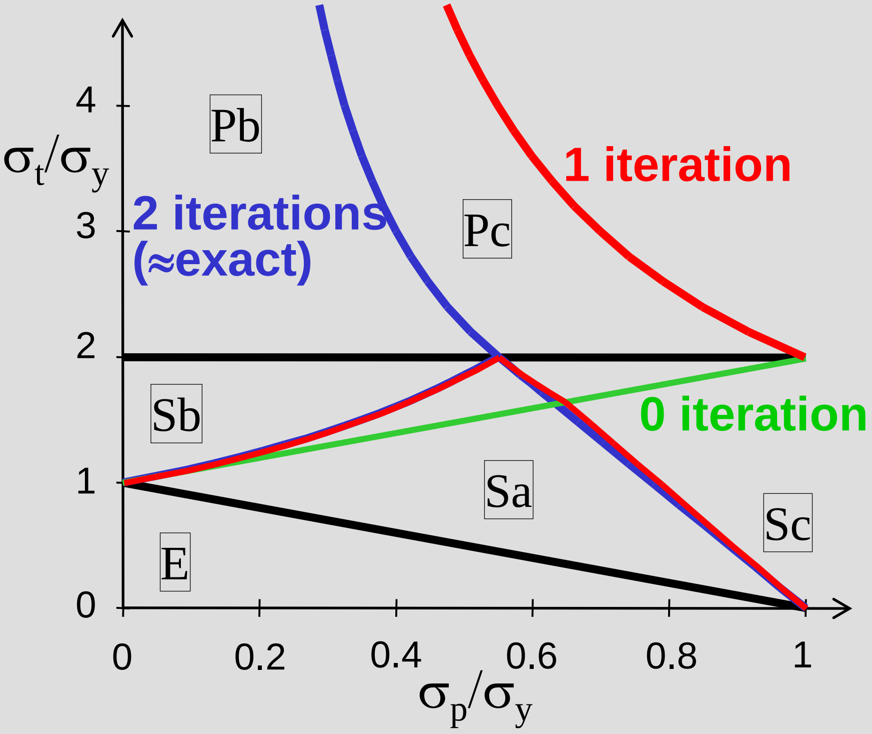


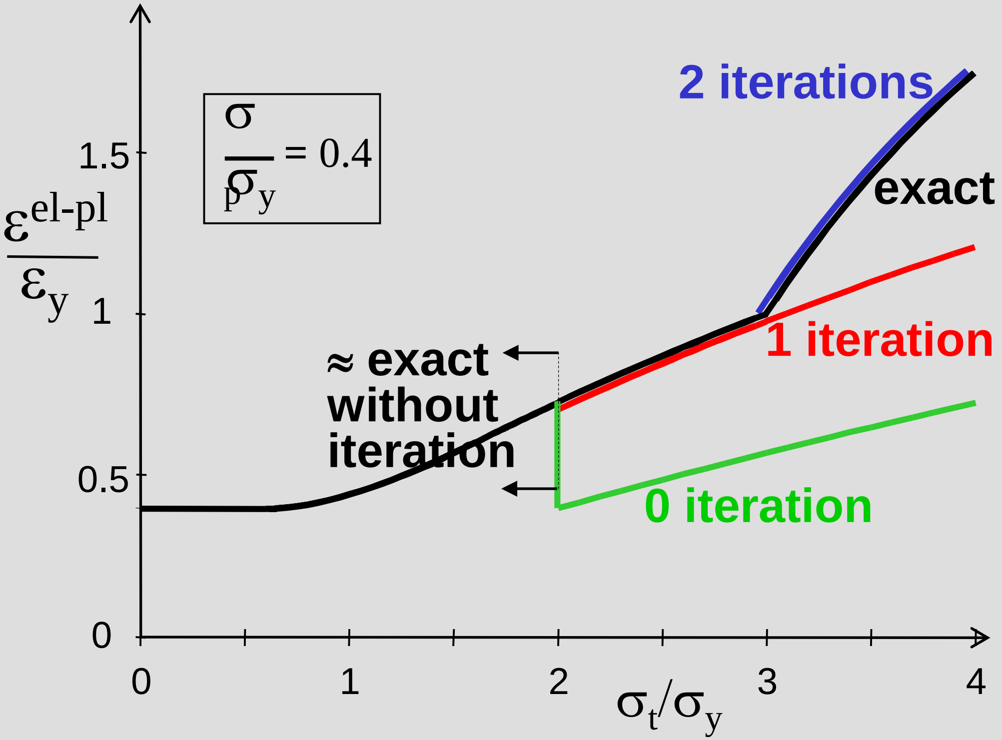


Example 11: Ratch-Inter: Bree Problem



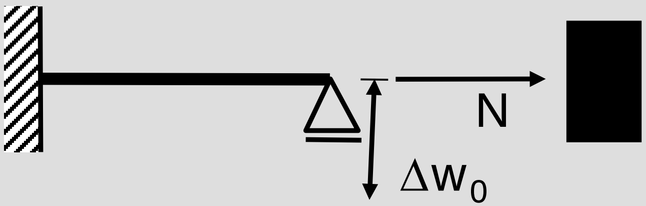
hand calculation for any configuration of loading (T, P) and hard. param. (E_t/E):

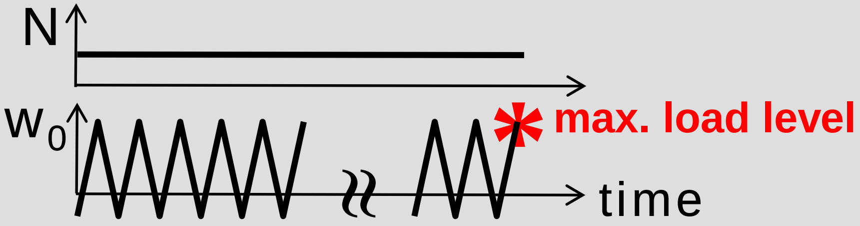




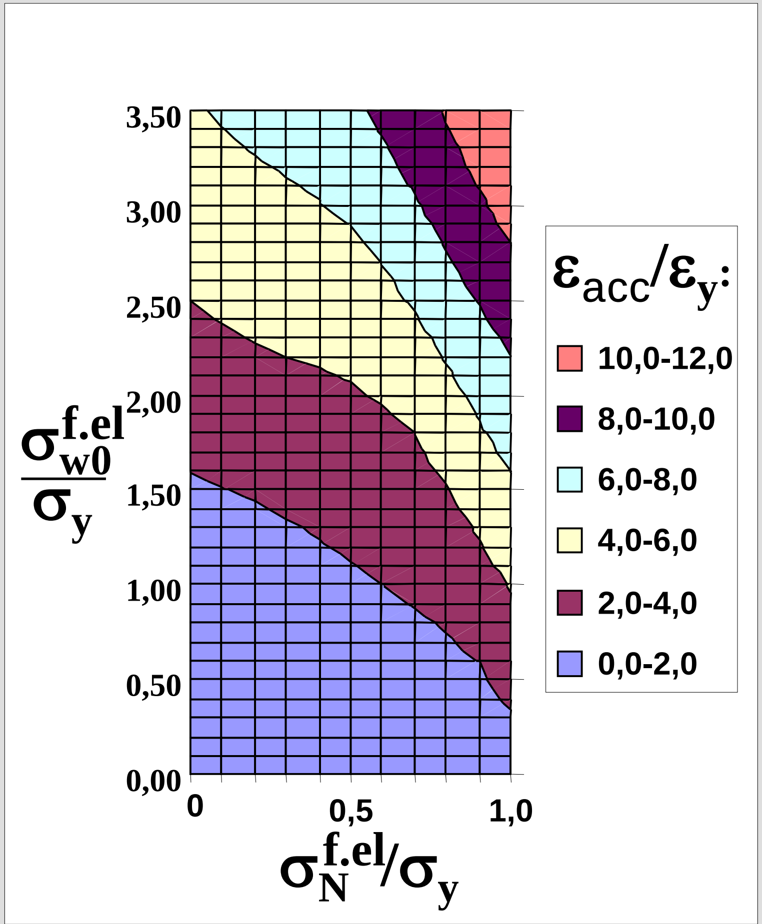
→ close to exact after 2 iterations

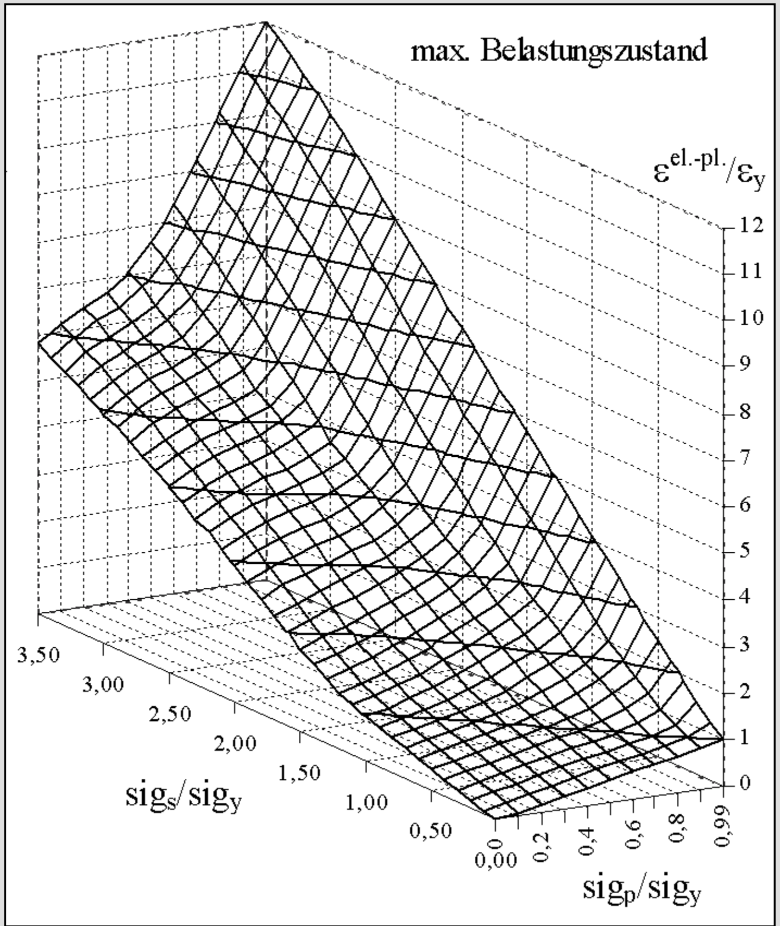
Example 12: Ratch-Inter: Beam under N, Δw_0





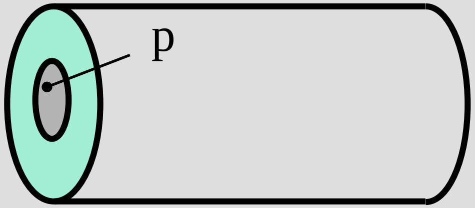
accumulated strain:



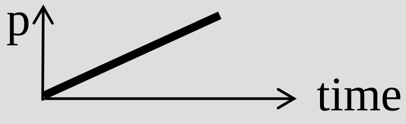


1. Benchmark CUT-FHL: Thick-walled Cylinder

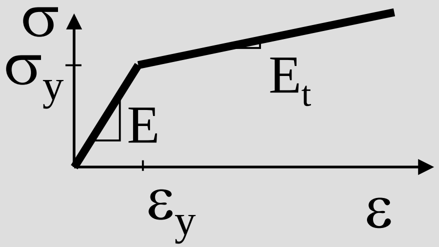
thick walled cylinder
under internal pressure



mon. loading,
load level $\sigma_v^{f.el}/\sigma_y$=1.665



material: lin. kin. hardening



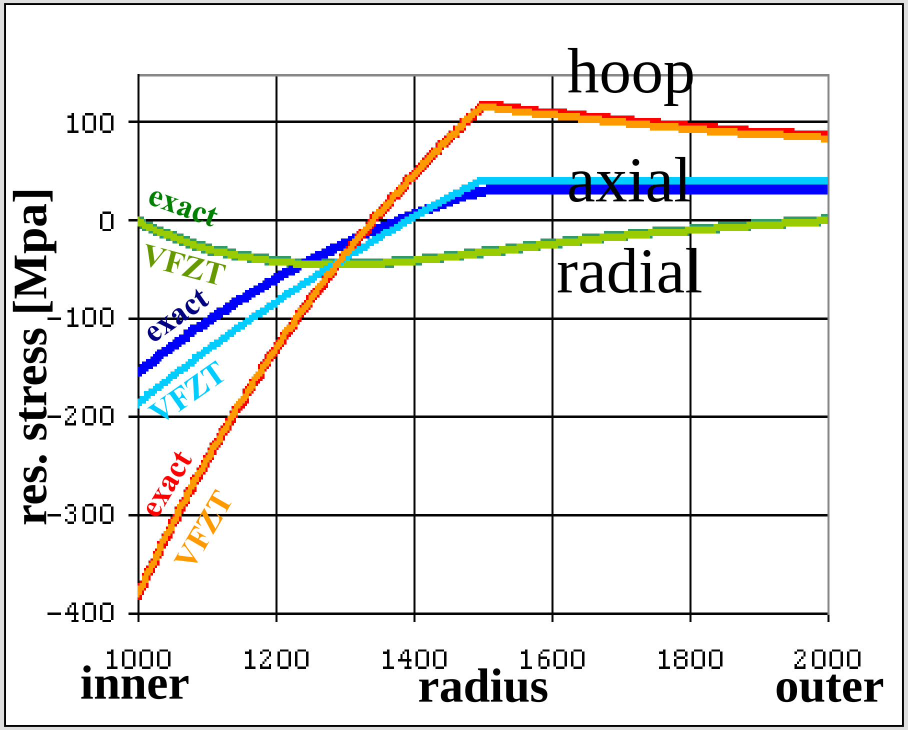
combination	ν	E_t/E
A	0.49	0.001
B	0.49	0.1
C	0.3	0.001
D	0.3	0.1

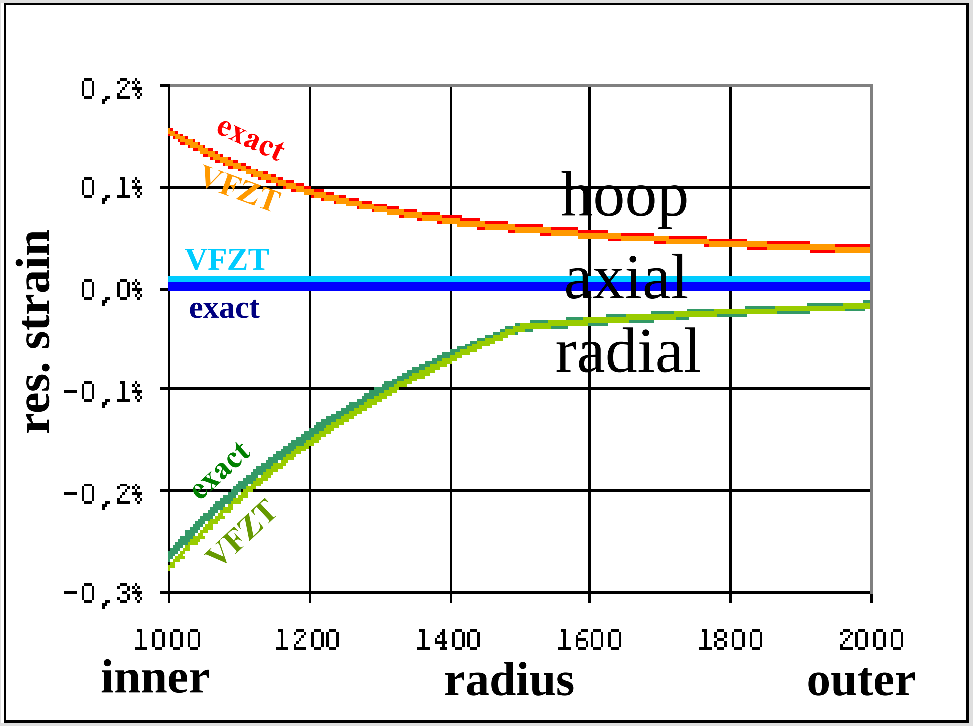
1.Benchmark CUT-FHL (ctd.): Thick-walled Cylinder

results:

combinations A, B, D: all residual stress- and strain-components after 6 mea close to exact values (difference <1%);

combination C ($E_t/E=0.001$, $\nu=0.3$):

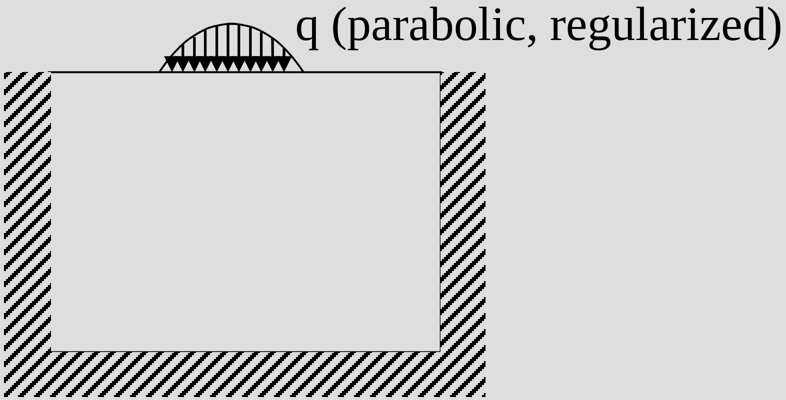




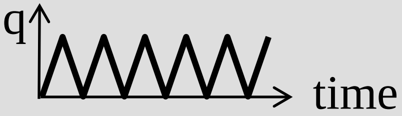
no improvement with more mea → loss in accuracy: **why?**

2.Benchmark CUT-FHL: Hertz Contact

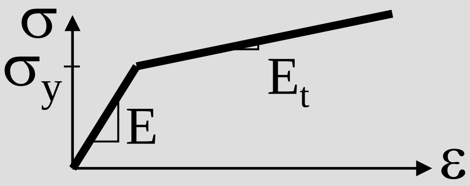
Hertz contact
(plane stress)



stationary cycl. loading (ES);
load level $\sigma_v^{f.el}/\sigma_y$=1.388



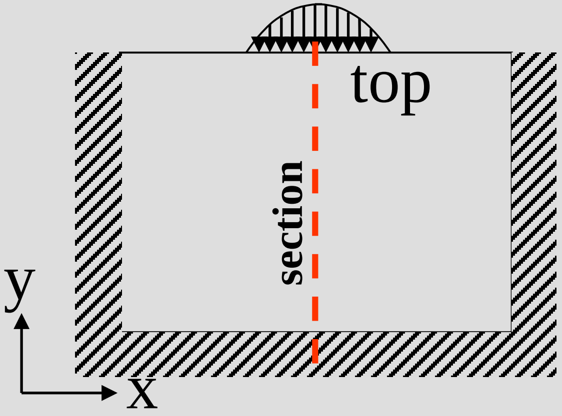
lin. kin. hardening
E_t/E = 0.1

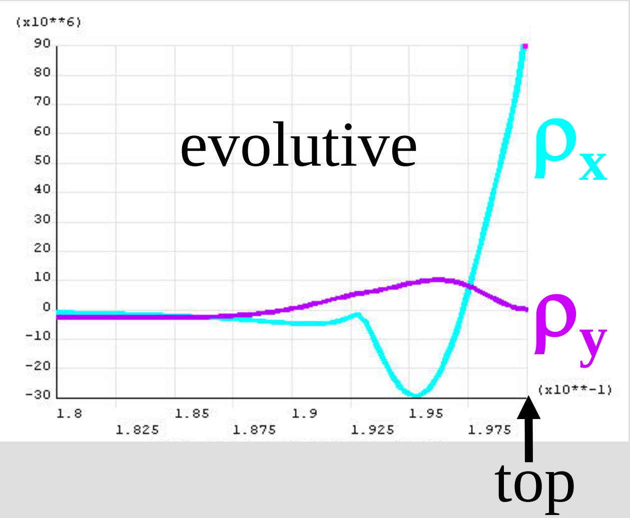


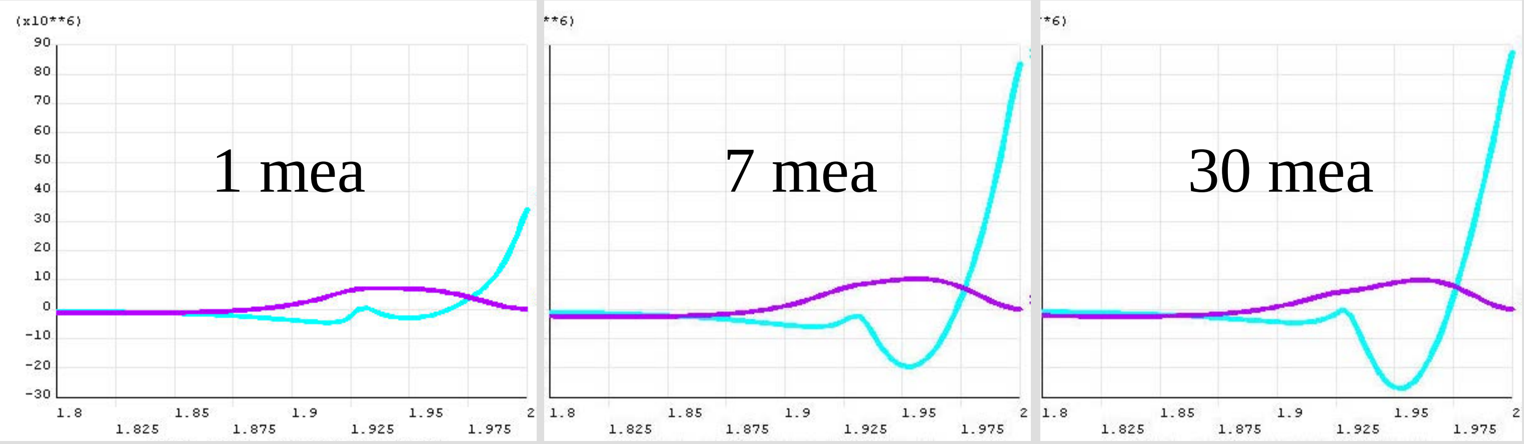
2.Benchmark CUT-FHL (ctd.): Hertz Contact

results:

residual stresses along vertical section:





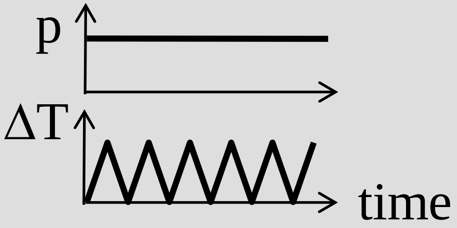


→ slow approximation to exact solution: **why ?**

3. Benchmark CUT-FHL: Bree Problem

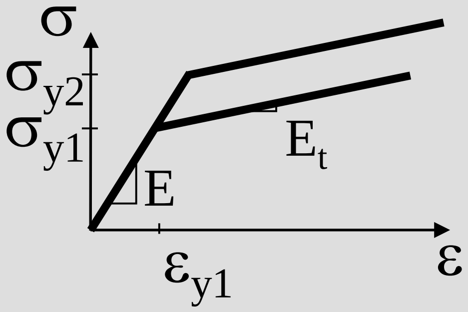
thin-walled cylinder
under internal pressure and
radial temperature gradient

cycl. loading (ES, PS)



combi-nation	Sa	Sb	Sc	Pb	Pc
$\sigma_p^{f.el}/\sigma_y$	0.7	0.2	0.8	0.2	0.8
$\sigma_t^{f.el}/\sigma_y$	1.2	1.9	1.9	4	4

lin. kin. hardening:
$E_t/E = 0.1$,
T-dependent yield stress



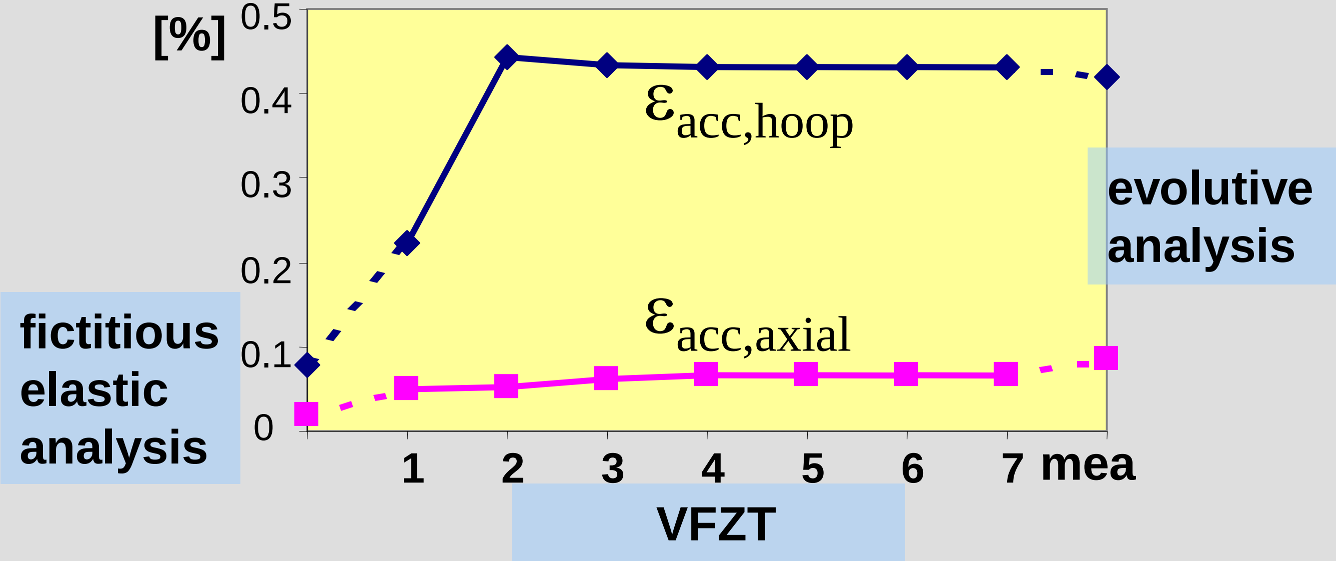
3.Benchmark CUT-FHL (ctd.): Bree Problem

σ_y= 200 at inner and outer surface for min. load level (no temperature),

σ_y= 161.1 at inner surface and 238.9 at outer surface for max. load level (fully developed T-gradient)

results for combination Pc:

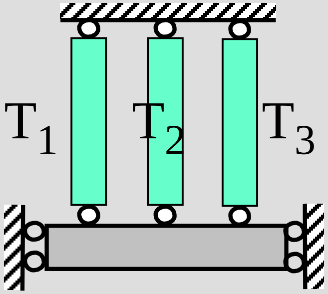
- range values = exact after 2 mea
- accum. state at minimum load level:

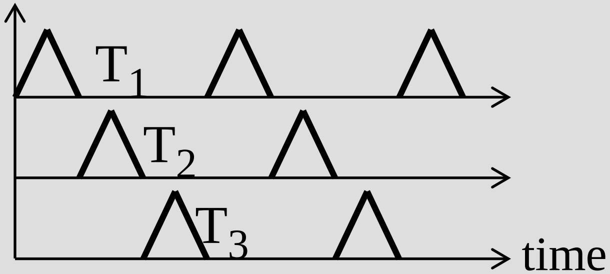


→ ε_{acc} reasonable, but not superior: **why?**

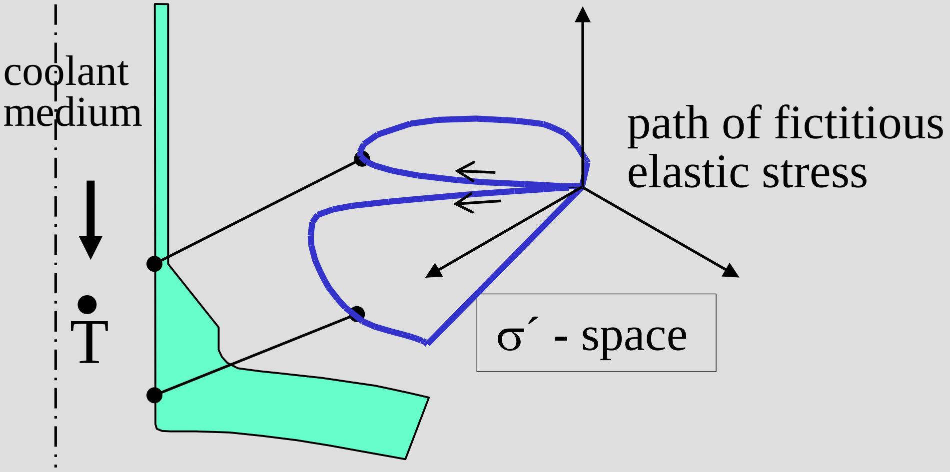
Further Development: Non-radial Loading

3-Bar Problem:

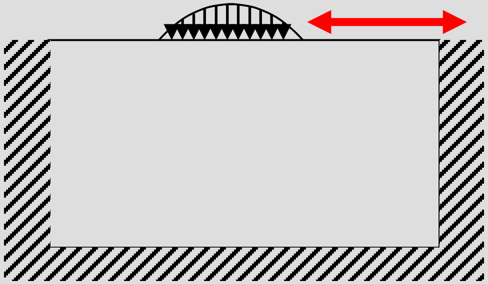




Nozzle:



Moving Load:



Summary

Zarka's Method (and it's modifications by FHL) is a simplified method to provide post-shakedown quantities in ES and PS by direct means ($\Delta\varepsilon$ for fatigue, ε_{acc} for ratcheting-assessment, displacements)

is implemented in commercial FE-program ANSYS for monotonic and cyclic loading

examples show in case of radial loading:

- good approximation to exact solution
- at small numerical effort (few linear elastic analyses)
- in particular if directional redistribution is not strongly developed
- range-values (PS) usually better than mean values

benchmarks between CUT and FHL raised some questions