Performance study of the simplified theory of plastic zones and the Twice Yield method for the fatigue check

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Summary

As elastic-plastic fatigue analyses are still time consuming the simplified elastic-plastic analysis (e.g. ASME Section III, NB 3228.5, the French RCC-M code, paragraphs B 3234.3, B 3234.5 and B3234.6 and the German KTA rule 3201.2, paragraph 7.8.4) is often applied. Besides linearly elastic analyses and factorial plasticity correction (K_e-factors) direct methods are an option. In fact, calculation effort and accuracy of results are growing in the following graded scheme: a) linearly elastic analysis along with K_e correction, b) direct methods for the determination of stabilized elastic-plastic strain ranges and c) incremental elastic-plastic methods for the determination of stabilized elastic-plastic strain ranges.

The paper concentrates on option b) by substantiating the practical applicability of the simplified theory of plastic zones STPZ (based on Zarka's method) and – for comparison – the established Twice Yield Method. Application relevant aspects are particularly addressed. Furthermore, the applicability of the STPZ for arbitrary load time histories in connection with an appropriate cycle counting method is discussed.

Note, that the STPZ is applicable both for the determination of (fatigue relevant) elastic-plastic strain ranges and (ratcheting relevant) locally accumulated strains. This paper concentrates on the performance of the method in terms of the determination of elastic-plastic strain ranges and fatigue usage factors. The additional performance in terms of locally accumulated strains and ratcheting will be discussed in a future publication.

Keywords

Simplified elastic-plastic fatigue analyses, simplified theory of plastic zones, STPZ, Zarka's method, Twice Yield Method, thermal cyclic loading, load-time histories

1. Introduction

Incremental elastic-plastic code conforming fatigue analyses are the most accurate and least conservative way of considering the cyclic deformation behavior. The major drawback is time and space consuming analyses for complicated 3D structures and complex loading conditions. That's why the simplified elastic-plastic analysis is often the method of choice. It relies on elastic finite element analyses and the application of an appropriate correction procedure for plasticity effects. Nevertheless, the standard correction proposed by the codes is known to be overly conservative in many design situations. In this context, the application of direct methods, such as the STPZ and the established Twice Yield method, appear to be interesting alternatives. This paper covers both the theoretical fundamentals of the STPZ and Twice Yield methods with emphasis on thermal cyclic loading conditions as well as practical application examples.

2. Fundamentals of the simplified theory of plastic zones (STPZ)

An alternative to simplify the calculation of elastic-plastic stress and strain ranges is the Simplified Theory of Plastic Zones (STPZ) as described in more detail in the following section.

2.1 General Outline

The STPZ aims at capturing the elastic-plastic stress and strain ranges due to cyclic loading between two states of loading in the condition of plastic shakedown. The material is assumed to exhibit piecewise linear kinematic hardening. At present, the theory is worked out for a bi-linear or tri-linear stress-strain curve. A von Mises yield surface is adopted along with an associated flow rule. Small deformation and additivity of elastic and plastic strains are assumed. Yield stress may depend on temperature, but the elastic properties and the elastic-plastic tangent moduli must not.

In this theory the state of plastic shakedown is addressed directly, i.e. without going through the entire load history step by step. The basic idea of the present simplification goes back to Zarka's method [e.g. 1,2,3]. A transformed internal variable (TIV) is introduced in this context. Its value can be estimated in the state of plastic shakedown by local consideration. Sequentially, the elastic-plastic response of the structure can be gained by a series of linear elastic analyses. As a consequence, however, the results are only approximations compared to those of an incremental analysis through the entire load history, and no information is obtained about the evolution of stress and strain during cycling and the number of cycles required to achieve shakedown.

For illustration purpose monotonic loading with linear kinematic hardening and temperature independent yield stress is considered first (section 2.2), before cyclic loading with temperature dependent yield stress (section 2.3) and multi-linear kinematic hardening (section 2.4) is addressed.

2.2 Outline for monotonic loading with linear kinematic hardening and temperature independent yield stress

Let σ_{ij} be the stress tensor, $\sigma_{ij}^{f.el}$ the tensor of stress obtained by a fictitious elastic analysis, ξ_{ij} the tensor of the backstress due to linear kinematic hardening, and ρ_{ij} the residual stress. The TIV Y_{ij} introduced by Zarka is then defined by

$$\sigma'_{ij} = \sigma_{ij}^{'f.el} + \rho'_{ij} \quad ; \quad i, j = 1,2,3 \tag{1}$$

$$Y_{ij} = \xi_{ij} - \rho'_{ij} \tag{2}$$

where ()′ indicates the deviatoric portion of a tensor. Note that ρ'_{ij} is the deviatoric portion of the residual stress. The fictitious elastic stresses are always considered to be known, because they can be easily obtained for any state of loading by a linear analysis.

If σ_y is the uniaxial yield stress, the von Mises yield surface can be considered as a (hyper)circle with radius σ_y centered in ξ_{ij} in the deviatoric stress (hyper)space and can be expressed as

$$f\left(\sigma'_{ij} - \xi_{ij}\right) - \sigma_y \le 0 \tag{3}$$

Reformulation leads to

$$f\left(\sigma'^{f.el}_{ij} - Y_{ij}\right) - \sigma_y \le 0 \quad (4)$$

representing a (hyper)circle with radius σ_y centered in $\sigma'^{f.el}_{ij}$ in the (hyper)space of the TIV. Active yielding requires the equal sign (= 0) in equations (3) and (4), σ'_{ij} and Y_{ij} being on the edge of the respective circle, as illustrated in Figure 1. The negative residual stress $-\rho'_{ij}$ is then outside of the circle in the TIV-space. The volume V of a structure can be split up into a portion remaining elastic (Ve) and a portion yielding actively (Vp). It should be noted that equation (3) contains two unknown tensors $\left(\sigma'_{ij} \text{ and } \xi_{ij}\right)$, equation (4) only one $\left(Y_{ij}\right)$.

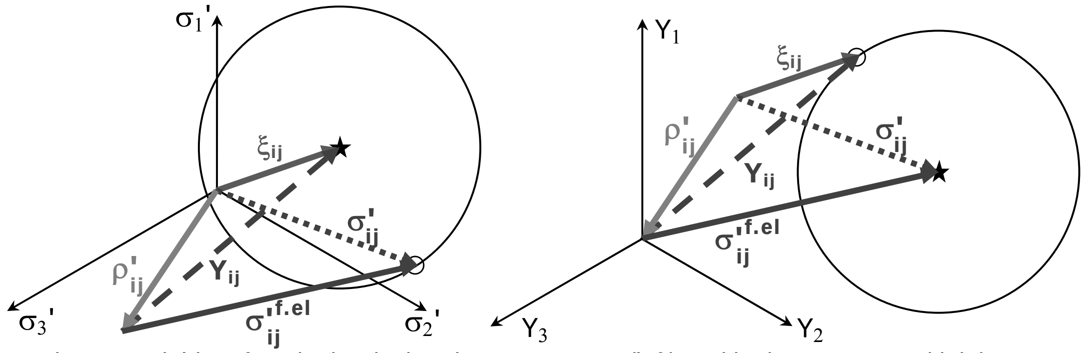

Figure 1: Yield surface in the deviatoric stress space (left) and in the TIV-space (right)

The situation shown in Figure 1 is characterized by a significant amount of directional redistribution of stress as visualized by the large angles between $\sigma'_{ij}, \sigma'^{f.el}_{ij}$ and ξ_{ij}. If directional redistribution is not that large, a good approximation of Y_{ij} can be obtained by projecting the negative of the residual stress to the von Mises circle in the TIV-space:

$$Y_{ij} = \sigma'^{f.el}_{ij} - \sigma'_{ij}\left(\frac{\sigma_y}{\sigma_v}\right) \quad (5)$$

where σ_v is the von Mises effective stress of the stress tensor σ_{ij}:

$$\sigma_v = \sqrt{\frac{3}{2}\sigma'_{ij}\,\sigma'_{ij}} \quad (6).$$

If directional redistribution is not possible, e.g. in case of uniaxial stresses, this projection provides exact results.

Once Y_{ij} is known (either exactly or approximately), the elastic-plastic strains can be reformulated adopting the additivity assumption and Hooke's law as well as the linear kinematic hardening rule

$$\varepsilon^{el-pl}_{ij} = \varepsilon^{el}_{ij} + \varepsilon^{pl}_{ij} \quad (7)$$

$$\varepsilon^{el}_{ij} = E^{-1}_{ijkl}\sigma_{kl} \quad (8)$$

$$\varepsilon^{pl}_{ij} = \frac{3}{2C}\xi_{ij} \quad ; \quad C = \frac{E \cdot E_t}{E - E_t} \quad (9)$$

where E_{ijkl} is the elasticity matrix, E the Young's modulus and E_t the elastic-plastic tangent modulus, Figure 2.

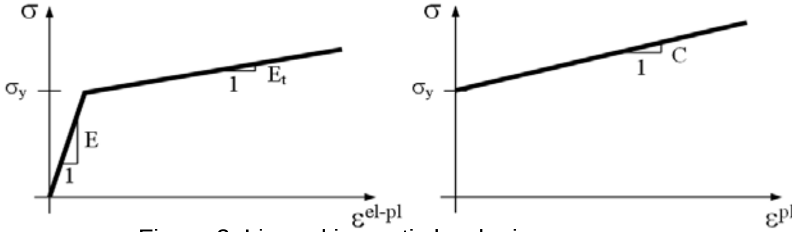

Figure 2: Linear kinematic hardening

By using an Operator L_{ijkl} to convert a tensor into its deviatoric part and introducing the abbreviations

$$\varepsilon_{ij}^{*} = \varepsilon_{ij}^{el-pl} - \varepsilon_{ij}^{f.el} \tag{10}$$

$$\left(E_{ijkl}^{*}\right)^{-1} = E_{ijkl}^{-1} + \frac{3}{2C} L_{ijkl} \tag{11}$$

$$\varepsilon_{ij,0} = \frac{3}{2C} Y_{ij} \tag{12}$$

we get

$$\varepsilon_{ij}^{*} = \begin{cases} \left(E_{ijkl}^{*}\right)^{-1} \rho_{kl} + \varepsilon_{ij,0} & \text{in Vp} \\ E_{ijkl}^{-1} \rho_{kl} & \text{in Ve} \end{cases} \quad (13).$$

Equation (13) is a linear relation between the residual stress ρ_{ij} and the residual strain ε_{ij}^{*}. In case of isotropic material, E_{ijkl} consists only of Young's modulus E and Poisson's ratio ν, while E_{ijkl}^{*} consists of a modified Young's modulus E^{*} and modified Poisson's ratio ν^{*}:

$$E^{*} = E_t \quad ; \quad \nu^{*} = 0{,}5 - (0{,}5 - \nu)\frac{E_t}{E} \quad (14).$$

The residual stresses can thus be determined by a linear elastic analysis of the structure with homogeneous boundary conditions loaded only by the initial strains $\varepsilon_{ij,0}$ in Vp. This is termed "modified elastic analysis" (MEA) because the elastic parameters as well as the loading are modified. Note that in this step ANSYS® is used for the solution of the linearly elastic boundary value problem including the initial strains $\varepsilon_{ij,0}$. The level of $\varepsilon_{ij,0}$ depends on the level of the real loading while the values of E^{*} and ν^{*} do not and are known in advance. However, the geometry of Ve and Vp changes with the load level. Once the modified elastic analysis is performed, i.e. $\rho_{ij}^{'}$ and ε_{ij}^{*} are known, the stress can be determined via equation (1) and the elastic-plastic strain from equation (10). This gives rise to an iterative procedure, since any initial guess about the geometry of Vp and the residual stress field may now be improved.

Thus we get the following workflow:

- *Step 1*: Perform a fictitious elastic analysis → $\sigma_{ij}^{f.el}$
- *Step 2*: Initial estimation of geometry of Vp (all locations of structure where the von Mises equivalent stress exceeds the yield stress: $\sigma_v^{f.el} > \sigma_y$)
- *Step 3*: In each location within Vp: projection of origin (= initial negative deviatoric residual stress) on yield surface in TIV-space → $Y_{ij} = \sigma'^{f.el}_{ij}\left(1 - \frac{\sigma_y}{\sigma_v^{f.el}}\right)$
- *Step 4*: Modify the elastic parameters in Vp (E → E^{*}, ν → ν^{*}), delete all loading, apply initial strains, perform MEA → ρ_{ij} as described above
- *Step 5*: Superposition of the fictitious elastic (Step 1) and modified elastic analysis MEA results (Step 4) → $\sigma_{ij} = \sigma_{ij,Step1}^{f.el} + \rho_{ij,Step4}$
- *Step 6*: Improved estimation of the geometry of Vp (all locations of structure where the von Mises equivalent stress exceeds the yield stress: $\sigma_v > \sigma_y$)

- *Step 7*: In each location within Vp: projection of negative deviatoric residual stress on the yield surface in TIV-space → $Y_{ij} = \sigma'^{f.el}_{ij} - \sigma'_{ij} \left(\frac{\sigma_y}{\sigma_v} \right)$
- *Step 8*: Introduce the modified elastic parameters in Vp (E^*, ν^*), (re)establish the elastic parameters in Ve (E, ν), delete all loading, apply initial strains, perform MEA → ρ_{ij}
- *Step 9*: Superposition of the fictitious elastic and modified elastic analysis results → σ_{ij}
- Repeat from step 6 until differences between two iterations become satisfactorily small.

At last, the elastic-plastic strain is obtained via

$$\varepsilon_{ij}^{el-pl} = \varepsilon_{ij}^{*} + \varepsilon_{ij}^{f.el} \qquad (15).$$

Note that the associated stress state σ_{ij} is already known from Step 5.

2.3 Cyclic loading with temperature dependent yield stress

As mentioned above, the STPZ is a direct method to provide an approximation to the elastic-plastic strain range between two load cases without performing incremental analyses throughout a load histogram. Thus, the two points in time causing the maximum strain range in a structure during a transient thermomechanical load history must be identified in advance. This can mostly be done with sufficient accuracy on the basis of a number of fictitious elastic analyses for many points in time during the load history. The loading is then assumed to vary linearly with time between these two extremes, named "minimum" and "maximum" state of loading. The corresponding fictitious elastic stress states are $\sigma_{ij,min}^{f.el}$ and $\sigma_{ij,max}^{f.el}$.

Once the loading conditions constituting the two extremes of a thermomechanical loading cycle are identified, the range of loading is treated in the same way as a monotonic loading, so that, for example,

$$\Delta\sigma_{ij}^{f.el} = \sigma_{ij,max}^{f.el} - \sigma_{ij,min}^{f.el} \qquad (16)$$

is applied instead of $\sigma_{ij}^{f.el}$ in section 2.2. The range of the TIV according to equation (5) is then different to those obtained by the lower and the upper estimates in Zarka's method [1,2,3].

A number of analyses showed that the effect of temperature dependent elastic-plastic material parameters of a bilinear stress-strain relationship is largely attributed to the temperature dependence of the yield stress, rather than to the temperature dependence of Young's modulus, Poisson's ratio and hardening modulus. This can be accounted for within the framework of the STPZ by substituting σ_y in the previous section by the sum of the two yield stresses according to the temperatures associated with the maximum and minimum states of loading, $\sigma_{y,max} + \sigma_{y,min}$.

Thus, for cyclic loading with temperature dependent yield stress the following modifications are introduced in section 2.2 related to a bilinear stress-strain curve:

$$\sigma_{ij}^{f.el} \quad \rightarrow \quad \sigma_{ij,max}^{f.el} - \sigma_{ij,min}^{f.el}$$

$$\sigma_y \quad \rightarrow \quad \sigma_{y,max} + \sigma_{y,min}$$

which eventually leads to the elastic-plastic strain range

$$\varepsilon_{ij}^{el\text{-}pl} \quad \rightarrow \quad \varepsilon_{ij,max}^{el\text{-}pl} - \varepsilon_{ij,min}^{el\text{-}pl}$$

2.4 *Multi-linear stress-strain curve*

A multi-linear stress-strain curve can be adopted within the framework of Zarka's method and the STPZ based on the idea of overlay models. The basic assumption of overlay models is that a structural volume is considered to be made of a number of different volumes (which may be called layers) with different material behavior, all occupying the same volume and coupled by the same deformations.

The most widely used overlay model is the Besseling model, where each layer exhibits a linear elastic - perfectly plastic behavior. Thus, three layers are required to model a tri-linear stress-strain curve with a tangent modulus > 0 in the third segment. In the present framework, each layer can be and must be modeled by a linear kinematic hardening material so that only two layers are required for a tri-linear stress-strain curve. In each of these layers, the theory as outlined in sections 2.2 and 2.3 for a bilinear stress-strain relation applies. In the following, the individual layers are addressed by roman numbers I and II, while the states representing the entire second and the third segment of the stress-strain curve, i.e. the combined action of two layers, are addressed by Arabic numbers 1 and 2 respectively. The first segment of the tri-linear stress-strain curve is thus defined by Young's modulus E, the second by the first vertex in σ_{y1} and the tangent modulus E_{t1}, and the third segment by the second vertex in σ_{y2} and the tangent modulus E_{t2}. The tri-linear segmentation is shown in Figure 3.

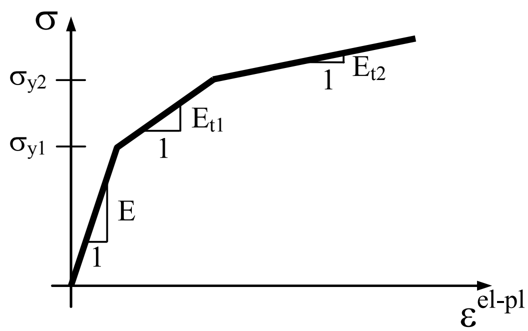

Figure 3: Tri-linear stress-strain-curve

In the third segment of the stress-strain curve, both layers are going plastic simultaneously so that equation (13) becomes for the individual layers I and II

layer I:

$$\varepsilon^*_{ij,I} = \left(E^*_{ijkl}\right)^{-1}_{I} \rho_{kl,I} + \varepsilon_{ij,0I} \quad \text{in } V_{pI} \tag{17}$$

layer II:

$$\varepsilon^*_{ij,II} = \left(E^*_{ijkl}\right)^{-1}_{II} \rho_{kl,II} + \varepsilon_{ij,0II} \quad \text{in } V_{pII} \tag{18}$$

and for the entire material

$$\varepsilon^*_{ij,2} = \left(E^*_{ijkl}\right)^{-1}_{2} \rho_{kl} + \varepsilon_{ij,02} \quad \text{in } V_{p2} \tag{19}$$

In the modified elasticity matrices $\left(E^*_{ijkl}\right)_I, \left(E^*_{ijkl}\right)_{II}, \left(E^*_{ijkl}\right)_2$ the modified elastic parameters are given by

$$E^*_I = E_{tI} \tag{20}$$

$$\nu^*_I = 0{,}5 - (0{,}5 - \nu)\,\frac{E_{tI}}{E_I} \tag{21}$$

$$E^*_{II} = E_{tII} \tag{22}$$

$$\nu^*_{II} = 0{,}5 - (0{,}5 - \nu)\,\frac{E_{tII}}{E_{II}} \tag{23}$$

$$E_2^* = E_{t2} \tag{24}$$

$$\nu_2^* = 0{,}5 - (0{,}5 - \nu)\frac{E_{t2}}{E} \tag{25}$$

where E_I and E_{II} are the Young's moduli in layers I and II, and E_{tI} and E_{tII} the tangent moduli respectively.

The six parameters of the bilinear stress-strain curves in the two layers ($E_I, E_{tI}, E_{II}, E_{tII}, \sigma_{yI}, \sigma_{yII}$) can be identified from the five parameters of the tri-linear stress-strain curve ($E, E_{t1}, E_{t2}, \sigma_{y1}, \sigma_{y2}$) via the field equations (equilibrium and continuity conditions) for an elementary volume of material subjected to a uniaxial stress state. However, these conditions are not sufficient to determine the parameters uniquely, so that one additional assumption can be chosen freely. In the present framework it is convenient to choose that the modified Poisson's ratio is identical in both layers:

$$\nu_{II}^* = \nu_I^* \tag{26}$$

This leads to

$$\frac{E_{tI}}{E_I} = \frac{E_{tII}}{E_{II}} = \frac{E_{t2}}{E} \tag{27}$$

$$\nu_{II}^* = \nu_I^* = \nu_2^* \tag{28}$$

and to the following relations for the material parameters in the two layers

$$E_{II} = \frac{1+\nu}{\frac{3}{2} - \left(\frac{1}{2} - \nu\right)\frac{E_{t1}}{E}} \, \frac{E_{t1} - E_{t2}}{1 - \frac{E_{t2}}{E}} \tag{29}$$

$$E_I = E - E_{II} \tag{30}$$

$$E_{tI} = \frac{E_{t2}}{E} E_I \tag{31}$$

$$E_{tII} = \frac{E_{t2}}{E} E_{II} \tag{32}$$

$$\sigma_{yI} = \sigma_{y1} \frac{E_I}{E} \tag{33}$$

$$\sigma_{yII} = \sigma_{y1} \frac{E_{II}}{E} + \left(\sigma_{y2} - \sigma_{y1}\right) \frac{1 - \frac{E_{t2}}{E_{t1}}}{1 - \frac{E_{t2}}{E}} \tag{34}$$

Consequently, we get

$$E_{ijkl,I}^* = E_{ijkl,2}^* \frac{E_{tI}}{E_{t2}} \tag{35}$$

$$E_{ijkl,II}^* = E_{ijkl,2}^* \frac{E_{tII}}{E_{t2}} \tag{36}$$

Making use of the equilibrium and the continuity conditions in an elementary volume of material for the residual stresses and strains

$$\rho_{ij,2} = \rho_{ij,I} + \rho_{ij,II} \tag{37}$$

$$\varepsilon_{ij,II}^* = \varepsilon_{ij,I}^* = \varepsilon_{ij,2}^* \tag{38}$$

and the initial strains in the two layers with linear kinematic hardening according to equation (12)

$$\varepsilon_{ij,0I} = \frac{3}{2}\frac{1}{E_{tI}}\left(1 - \frac{E_{tI}}{E_I}\right) Y_{ij,I} \qquad (39)$$

$$\varepsilon_{ij,0II} = \frac{3}{2}\frac{1}{E_{tII}}\left(1 - \frac{E_{tII}}{E_{II}}\right) Y_{ij,II} \qquad (40)$$

we get the initial strain for the third segment of the stress-strain curve:

$$\varepsilon_{ij,02} = \frac{3}{2}\frac{1}{E_{t2}}\left(1 - \frac{E_{t2}}{E}\right)\left(Y_{ij,I} + Y_{ij,II}\right) \qquad (41).$$

$Y_{ij,I}$ and $Y_{ij,II}$ can be found by projecting the negative of the residual stresses $-\rho_{ij,I}$ and $-\rho_{ij,II}$ to the von Mises circle in the TIV-space of each of the two layers separately as described in section 2.2. The modified elastic analyses can then be performed with the modified elastic parameters E_2^*, ν_2^* given in equations (24) and (25) and the loading by initial strains $\varepsilon_{ij,02}$ given by equation (41).

2.5 Limits

The following points explain the limits of the STPZ in the currently available development version:

- For every identified cycle the whole process of the STPZ has to be carried out. This may lead to high computing times. E.g. for a transient with a lot of sub cycles, which are identified by a rainflow algorithm [8], the computing time can even be longer than for the incremental elastic-plastic analysis of one cycle (including the sub cycles).
- In some cases the results of the STPZ are sensitive to the estimated strain range. This may yield overestimated stress ranges.
- Manageability: A lot of preparation has to be done by hand (e.g. application of the boundary conditions to the model).

2.6 Remarks

The STPZ is implemented via user subroutines into ANSYS®.

Many practical applications show that usually two to five modified elastic analyses are sufficient to get a close approximation to the elastic-plastic strain range in the state of plastic shakedown of thermomechanical cyclic loading problems compared with incremental analyses.

Within the same framework, assessment of strain accumulation due to a ratcheting mechanism is also possible. This feature is not discussed by way of example in this paper. It requires a more detailed discussion of the general performance of material models for the simulation of local ratcheting.

3. Twice Yield Method

Another alternative for the determination of stabilized elastic-plastic strain ranges is the Twice-Yield method which is based on work by Kalnins (see [4], [5], [6]) and is a proposed method in the ASME Code, Section VIII, Div. 2 [7].

The basis of the method as described in the ASME Code is the estimation of stress and strain ranges of each cycle based on a single elastic-plastic analysis in a quasi-monotonic fashion. Reducing the analysis effort to a simple monotonic loading is a key of this simple method. The method also requires a simplified approach for any transient temperatures involved within a cycle by using a weighted value for transient temperature changes within a cycle, i.e. by the following equation:

$$T = 0.75 \cdot T_{max} + 0.25 \cdot T_{min} \qquad (42)$$

Stress-strain hysteresis loops of cycled material can be obtained by using the cyclic stabilized stress-strain curve based on any nonlinear stress-strain law, i.e. based on a Ramberg-Osgood type of equation or any type of nonlinear or piecewise linear representation.

Starting loading from zero stress and strain, the local stress-strain path will follow the cyclic stress-strain curve. At the reversal of the loading direction, the path follows a stress-strain curve that is doubled both in stress as well as in strain direction (see Figure 4). The doubled stress-strain curve (twice-yield curve) is used to create any hysteresis branches according to Masing's hypothesis (both up- and downward branch for any load range) for any subsequent load ranges.[1]

The Twice-Yield-Method simplifies this approach and does not take into account any memory rules but only the doubled stress-strain curve to obtain stress and strain ranges for each load cycle. Load cycles for variable amplitude loading may be counted by methods as described in the ASME code up front and stored as load ranges vs. number of occurences. The application of the Twice-Yield method for a single load range therefore results in stress and strain ranges of the hysteresis loop but not in information on mean strain or (often more important) mean stress.

The Twice-Yield method was expanded to multiaxial behaviour by using equivalent stress and strain quantities based on von Mises formulations for multiaxial stress and multiaxial plastic strain.

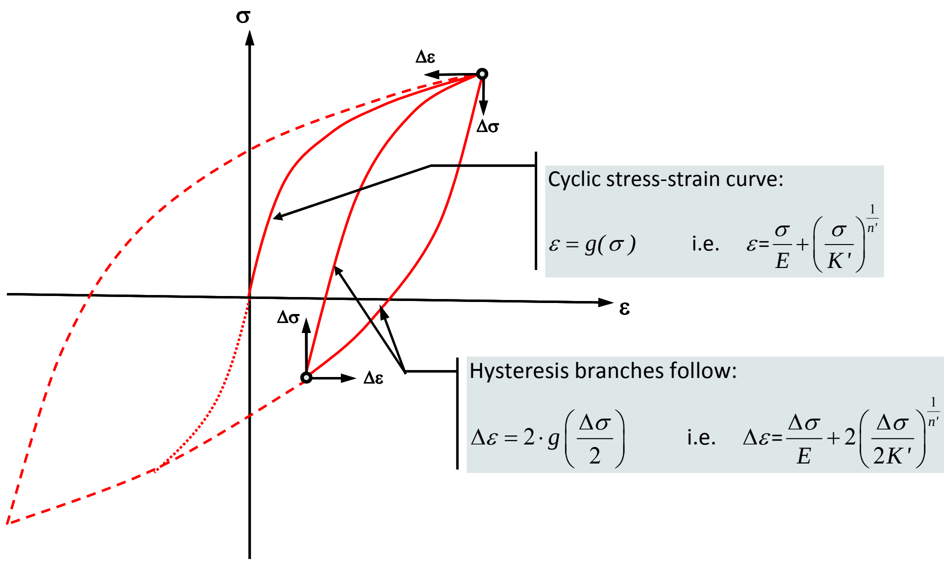

Figure 4: Masing Behavior for uniaxial loading, shown using a Ramberg-Osgood formulation

After cycle counting, e.g. using the min-max method as described in the ASME code, a first reversal point at time t_1 and a second reversal point at time t_2 define each hysteresis loop. The first point of the stress-strain loop at t_1 is taken as a point of reference to calculate ranges of tensor quantities of stress and strain for each hysteresis loop. This means for any hanging hysteresis loops the upper point is taken as reference point and for standing hysteresis loops the lower point is taken as reference point.

The starting point of each hysteresis loop defines a reference point involving stress $\sigma_{ij}^{(t_1)}$ and plastic strain $p_{ij}^{(t_1)}$ to calculate the corresponding ranges for the hysteresis loop. Relative stress and plastic strain values are then calculated for each hysteresis loop:

$$\Delta\sigma_{ij} = \sigma_{ij}^{(t_1)} - \sigma_{ij}^{(t_2)}$$
$$\Delta p_{ij} = p_{ij}^{(t_1)} - p_{ij}^{(t_2)} \quad (43)$$

For every hysteresis loop separately, the following equations supply an *equivalent elastic-plastic stress range* and an *equivalent plastic strain range* by using

[1] In case of variable amplitude loading, a number of memory rules like the rules described by Clormann and Seeger [8] could be applied to design the different stress-strain hysteresis loops. This will not be considered in this paper though.

$$\Delta\sigma_{eq} = \frac{1}{\sqrt{2}}\sqrt{\begin{aligned}&\left(\Delta\sigma_{11}-\Delta\sigma_{22}\right)^2+\left(\Delta\sigma_{22}-\Delta\sigma_{33}\right)^2+\left(\Delta\sigma_{33}-\Delta\sigma_{11}\right)^2+\ldots\\ &\ldots+6\left(\Delta\sigma_{12}{}^2+\Delta\sigma_{23}{}^2+\Delta\sigma_{13}{}^2\right)\end{aligned}} \quad (44)$$

$$\Delta\varepsilon_{peq} = \frac{\sqrt{2}}{3}\sqrt{\begin{aligned}&\left(\Delta p_{11}-\Delta p_{22}\right)^2+\left(\Delta p_{22}-\Delta p_{33}\right)^2+\left(\Delta p_{33}-\Delta p_{11}\right)^2+\ldots\\ &\ldots\frac{3}{2}\left(\Delta p_{12}{}^2+\Delta p_{23}{}^2+\Delta p_{13}{}^2\right)\end{aligned}} \quad (45)$$

The *equivalent stress range* and the *equivalent plastic strain range* is then combined to obtain a damage relevant *equivalent total strain range* of a cycle by [5]

$$\Delta\varepsilon_{eff} = \frac{\Delta\sigma_{eq}}{E} + \Delta\varepsilon_{peq} \quad (46).$$

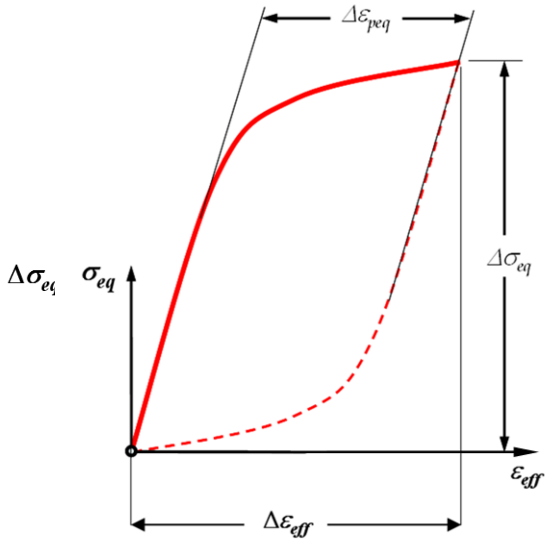

Figure 5: Ranges of equivalent quantities for a single load range applied to the structure

The resulting value of total equivalent strain $\Delta\varepsilon_{eff}$ together with the (uniaxial) strain-life curve will be used to calculate either a partial damage value or the cycles to failure.

The Twice-Yield approach offers some significant advantages.

- Calculation is much simpler and thus faster than incremental elastic-plastic analyses through many load cycles.
- It requires solving a finite element model only for a single monotonic loading up to the largest value of stress differences obtained from the load history. The elastic-plastic stress-strain tensors as obtained for this monotonic loading and stored in the results file then can be taken to efficiently assess fatigue damage of any loading cycle of arbitrary loading histories.
- The multiaxial equivalent values for stress and plastic strain are usually two numbers obtained by postprocessing a finite element analysis. In case of each single hysteresis loop the analysis reduces to store those two values for further processing.
- No plasticity law capable of cyclic and kinematic hardening is required since the analysis uses just a quasi-static solution involving a nonlinear stress strain law based on the stabilized cyclic stress-strain curve. For example Ramberg-Osgood format can be used for a monotonic incremental plasticity analysis.

The method though does not capture the following effects:

- Well-known memory effects from uniaxial strain based fatigue analysis (memory rules to correctly obtain the stress-strain cycles of subsequent cyclic variable amplitude loading).

- Mean stress levels of each computed stress cycle. In case of transient loading for specific repeated load sequences mean stress might not relax and thus be important.
- Transient effects like non-proportional hardening or any other transient memory effects of the material.
- Multiaxial plasticity effects (i.e. kinematic hardening) occurring especially for non-proportional stress response.
- Non-Masing material behaviour, since Masing behaviour is a prerequisite for the simplifications.
- Ratcheting or shakedown under cyclic loading (except the cyclic hardening or softening which is captured in the cyclic material data required for computing) which would require specific kinematic hardening rules and suitable plasticity models.

The described expansion to multiaxial behaviour should be used with caution in case of nonproportional loading. Since pressure vessels often face proportional loading, this restriction usually might be of less importance but this should be mentioned.

4. Performance Study by way of examples

4.1 General Remarks

The typical examples relevant for thermomechanical power plant applications are considered in the following for a discussion of the performance of the STPZ and the Twice Yield method with regard of the determination of fatigue relevant strain ranges and fatigue assessment:

- Nozzle example 1
- Stepped pipe
- Disk
- Nozzle example 2

The examples (see also [9]) are shown in Figure 6 through Figure 9. A detailed respective study has been carried out in [10].

4.2 Flow Chart for using the STPZ

This section describes the necessary steps to apply the STPZ as part of a fatigue analysis.

- An elastic fatigue analysis according to e.g. ASME NB 3200 [11] is done. From this analysis the following information is obtained:
    - Identification of the two load steps which form the stress cycle.
    - The alternating stress amplitude S_{alt}, from which the estimated strain range is calculated.
    - The average cycle temperature.
- The stress-strain curve for the specific given average cycle temperature is created by interpolating between the given stress-strain curves.
- From this newly created stress-strain curve the tri-linear stress-strain curve is calculated from the estimated strain according to:

$$\varepsilon_{est} = \min\left(\frac{S_{alt}}{E}\,;0.01\right)$$

- All this information is gathered and prepared in order to create an ANSYS® input file for the STPZ.
- Besides the ANSYS® input file, the model database and the structural result file of the elastic calculation is needed.
- According to the STPZ the elastic-plastic strain range is calculated. It is transferred to the stress range and in the end the usage factor is calculated.

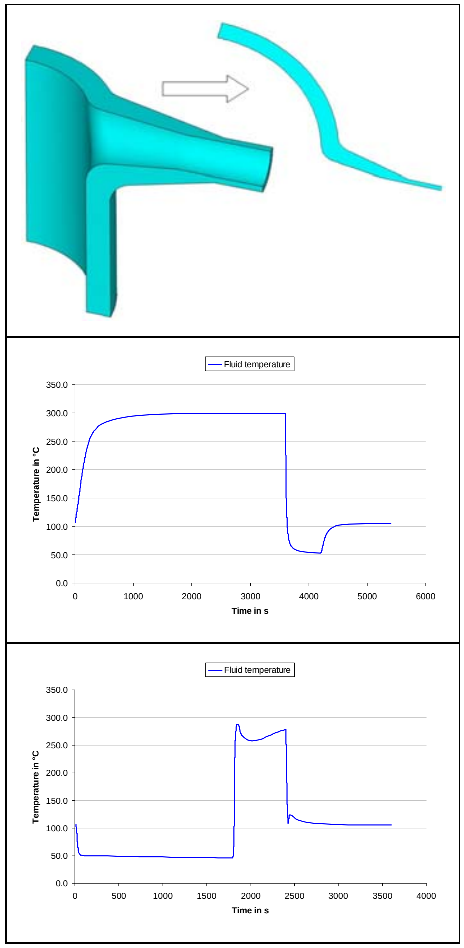

Figure 6: Geometry and thermal loading for nozzle example 1

4.3 Results

The results of both the STPZ and the Twice Yield Method in comparison with incremental elastic-plastic analysis as well as different K_e procedures (see [9] and [11] to [13]) are shown in Figure 10.

All results are within 30% deviation with respect to the fatigue usage factor in comparison with the incremental elastic-plastic results. Regarding the STPZ, all of the results are conservative (with regard to the incremental elastic-plastic reference analysis) and deliver a lower usage factor than the simplified elastic-plastic analysis according to different applicable design codes. The conclusion is drawn from Figure 10 that both the STPZ and the Twice Yield Method outperform all comparable design code based plasticity correction methods (K_e factors) by reducing overly conservative results. Note that ASME III K_e values (Figure 10) are from NB3200 and not from CC N-779.

Thus, the STPZ and the Twice Yield Method can be seen as an intermediate between simplified elastic-plastic analysis (K_e factors) and incremental elastic-plastic analysis. For practical engineering application, a case dependent graded approach as shown in Figure 11 is proposed.

Note, that the cladding example studied in [9] (as well as one complex 3D nozzle example) has consciously been exempted from this comparison. The application of both the simplified elastic-plastic analysis (K_e factors) and the STPZ is critical in this case. For the time being, incremental elastic-plastic analyses are the method of choice in the case of cladding.

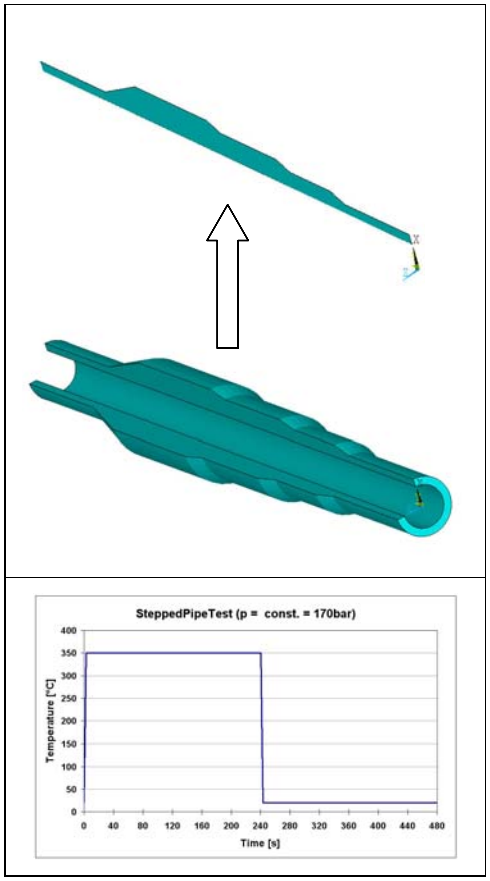

Figure 7: Geometry and thermal loading for the stepped pipe

5. Application to arbitrary load time histories

Both the STPZ and the Twice Yield Method can be applied to arbitrary load time histories. In this case e.g. a rainflow algorithm (e.g. [8]) can be used to identify the cycles. For the identified cycles the procedures according to section 3 respectively 4.2 are applied afterwards in a straight forward way.

In general, the counting algorithm respectively the load time history has only an influence on the identification of cycles. The STPZ is using this cycle information as essential input. This means that the initial cycle may be the result of a design transient, measured transients or arbitrary load (temperature) time histories.

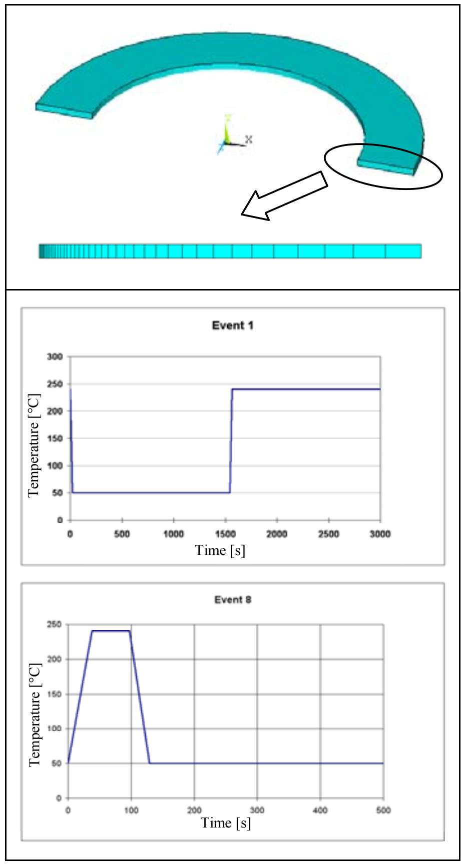

Figure 8: Geometry and thermal loading for the disk example

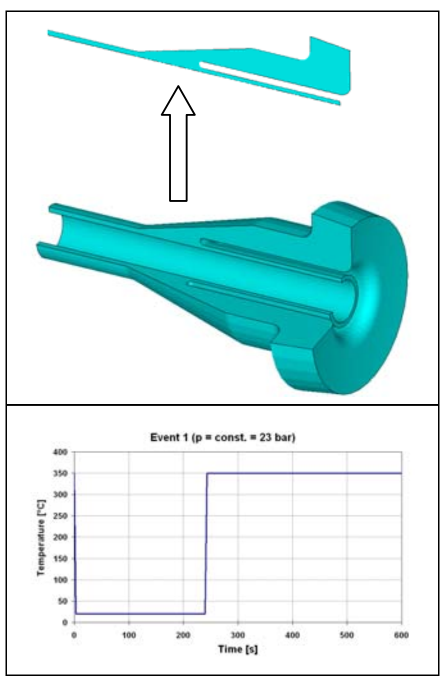

Figure 9: Geometry and thermal loading for nozzle example 2

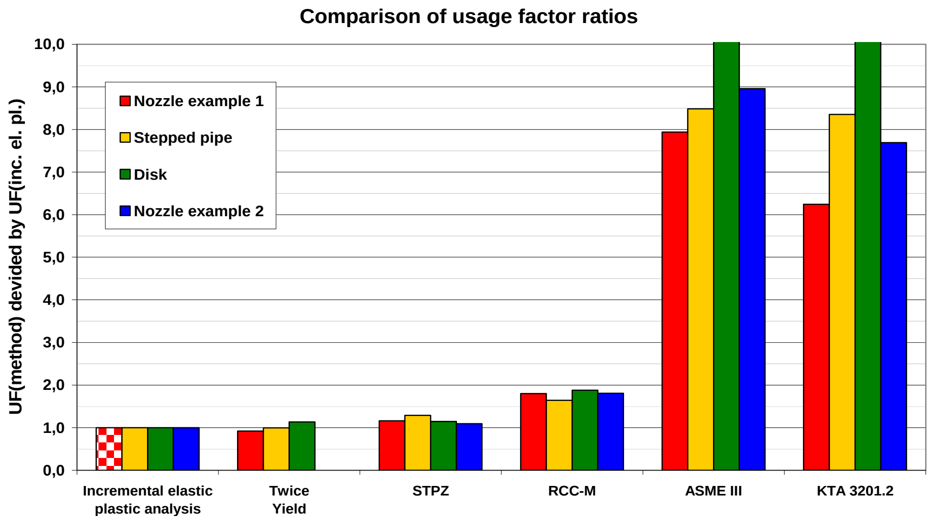

Figure 10: Analysis results of different methods compared to incremental elastic-plastic reference analyses

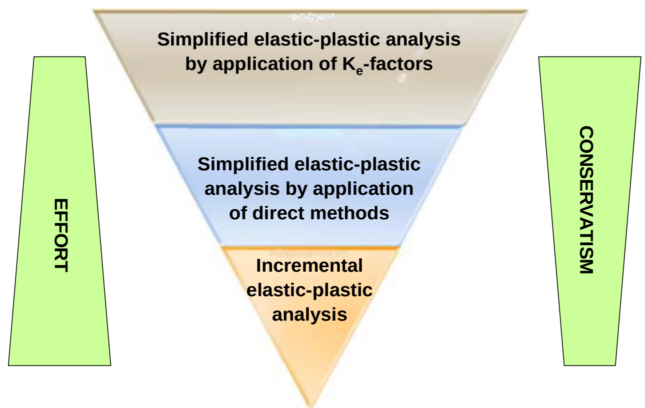

Figure 11: Staged approach to elastic-plastic analyses

6. Conclusions

Low cycle fatigue analyses of power plant components require an appropriate consideration of cyclic plasticity for the predominant thermomechanical loading conditions. The applicable design codes such as [11] to [13] primarily differentiate between a simplified elastic-plastic (K_e factors, see [9]) and an incremental elastic-plastic analysis (i.e. incremental elastic-plastic FEA of one loading cycle based on the cyclic stress-strain curve). In the case of thermal cyclic loading conditions typical for power plant operation the plasticity correction factors K_e typically show shortcomings by considerably overestimating the real elastic-plastic strain ranges and thus the corresponding fatigue usage factors. The alternative of an incremental elastic-plastic analysis is often time consuming and exhaustive for complicated geometry and loading conditions. The application of the STPZ as well as the Twice-Yield-Method offers an intermediate option and a compromise between accuracy, conservatism and calculation effort. Thus, the presented methods become a valuable part of the toolbox in a graded and case dependent application scheme. The applicability for typical power plant components has been shown by way of example in this paper. Note, that a special numerical implementation for the STPZ is required. The method is not part of standard distributions of finite element software. The theoretical basis of this implementation is explained in the first part of the paper. Furthermore, the fundamentals of the Twice Yield method and its performance by way of example in comparison to the incremental elastic-plastic analysis and the STPZ were shown. The application of this method does not require a special numerical implementation and the procedure is an integrated part of ASME Code, Section VIII, Division 2 [7]. Since the results obtained for a limited number of test cases led usage factors close to the values obtained using an incremental plasticity model, further validation is recommended though.

7. References

[1] Zarka,J.; Frelat,J.; Inglebert,G.; Kasami-Navidi,P.: A New Approach in Inelastic Analysis of Structures, Martinus Nijhoff Publishers, Dordrecht, 1988 and 1990

[2] Zarka,J.; Engel,J.J.; Inglebert,G.: On a Simplified Inelastic Analysis of Structures, Nuclear Engineering and Design 57 (1980), pp 333-368

[3] Zarka,J.: On Simplified Mathematical Modelling of Cyclic Behaviour, Res Mechanica 4 (1982), pp 203-223

[4] Kalnins, A.: Fatigue Analysis of Pressure Vessels with Twice-Yield Plastic FEA. ASME 2001 PVP Conference, Atlanta, Georgia, July 23-26. ASME Bound Vol. 419 43-52.

[5] Kalnins, A.: Fatigue Analysis in Pressure Vessel Desing by Local Strain Approach: Methods and Software Requirements. J. of Press. Vess. Tech., 2006, Vol. 128, 2-7

[6] Kalnins, A.: Twice-Yield Method for Assessment of Fatigue Caused by Fast Thermal Transient According to 2007 Section VIII-Division 2 of AME B&PV Code. Proc. of the 2008 ASME Pressure Vessel and Piping Division Conference, July 27-31, 2008, Chicago, Ill. USA, PVP2008-61397.

[7] ASME Boiler & Pressure Vessel Code. 2010. Section VIII, Division 2 – Alternative Rules

[8] Clormann, U.H.; Seeger, T.: Rainflow – HCM. Ein Zählverfahren für Betriebsfestigkeitsnachweise auf werkstoffmechanischer Grundlage. Stahlbau 55 (1986), No. 3, pp. 65/71

[9] Lang, H.; Rudolph, J.; Ziegler, R.: Performance study of K_e-factors in simplified elastic plastic fatigue analyses with emphasis on thermal cyclic loading. International Journal of Pressure Vessels and Piping 88 (2011), pp. 330/347

[10] Deller, S.: Untersuchungen zur Vereinfachung elasto-plastischer Ermüdungsanalysen thermozyklisch beanspruchter nuklearer Kraftwerkskomponenten. Diplomarbeit, FH München, 2011

[11] ASME Boiler & Pressure Vessel Code. 2010. Section III, Division 1 – Subsection NB: Class 1 Components. Rules for Construction of Nuclear Power Plant Components

[12] RCC-M Edition 2007, Section I, Subsection B: Class 1 components. Design and Construction Rules for Mechanical Components of PWR Nuclear Islands

[13] KTA 3201.2 (06/96). 1996. Components of the Reactor Coolant Pressure Boundary of Light Water Reactors. Part 2: Design and Analysis