# Low-Fidelity Stochastic Approach for Airfoil-Turbulence Interaction Noise

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## Introduction

The noise due to the interaction of turbulent flow with the leading edge of an airfoil is a primary source of aerodynamic noise. The problem is illustrated in Fig. 1. Turbulence is, by its very nature, stochastic and therefore has a broad frequency spectrum which makes it more difficult to calculate the effect induced by it. Acoustic radiation from an airfoil subjected to a real flow is one such effect induced by turbulence and demands high-performance computing for its prediction numerically. The methods which are available now range back from analytical ones, which are fast but not precise enough to capture nonlinear effects, to the numerical one, which heavily depends on the computing resources [1].

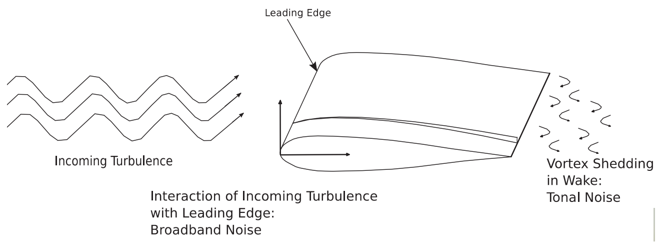


**Figure 1:** Incidence turbulence interacting with the leading edge of an airfoil

The equations that are used in the solution of the problems relevant to unsteady aerodynamics are a simplified version of the governing equations of the fluid dynamics. The CFD method has a high fidelity in predicting the Airfoil-Turbulence Interaction Noise (ATIN), and its computational cost limits its applications at conceptual and preliminary design stages of an aircraft or a turbomachine. Besides, using only an experimental method to address the effect of the interaction is also not a viable option because of its high cost. To overcome the above issues, methods and tools capable of rapidly and accurately predicting the sound radiation due to the airfoil-turbulence interaction are needed.

Boundary Element Methods (BEMs) are one such class of numerical methods which have been in use to deal with the boundary value problems throughout the physics from electromagnetics to fluid mechanics at both low and high Reynolds numbers. Unsteady BEM solutions are still productive with flow physics and give accurate solutions at computational times that are several orders of magnitude faster than Navier-Stokes solvers. BEMs assume that fluid flow is incompressible, irrotational (except at *singular* points) and inviscid, that is, a potential flow. This leads to simplified forms of the continuity and momentum equations that govern the fluid flow. To calculate the sound radiation from a body, it is vitally important to estimate the unsteady loading due to turbulence accurately. Most authors have opted to use a frequency domain analysis in which the incident disturbance is specified by a harmonic gust [2-4]. Grace [5] concluded that time-domain analyses have a number of advantages over the harmonic gust approaches, yet seldom used for leading-edge noise because it is difficult to model the turbulent inflow using a set of discrete vortices.

The present paper shows the development of a turbulent inflow using a set of discrete vortices. Primarily, the singularity of a potential vortex is discussed with a possible substitution of viscous-core vortex instead, then the application of synthetic eddy modelling in generating a turbulent inflow signal using these discrete vortices. The then generated spectra are optimised to the von Karman energy spectra. This paper presents the first part of the research work on ATIN. The entire study focuses on the development, validation, and demonstration of a transient, two-dimensional stochastic boundary element solver for the acoustic radiation from the airfoil-turbulence interaction. The proceeding sections discuss the efficiency as well as the drawbacks of BEM, and the implementation of hybrid panel method using the vortex particle with finite viscous core vortex method.

## Numerical Method: Boundary Element Method for Unsteady Flow

In this section, the foundations of the potential flow for external flow aerodynamics are presented. Perhaps the oldest and the most extensively used computer method for simulating aerodynamic flows is the panel method (boundary element method applied to fluid dynamics).

## Governing Equations

Inviscid flows are described on the bases of the continuity equation and the inviscid momentum equations. An arbitrary solid body immersed in an incompressible and irrotational fluid, defining a velocity potential $\Phi$ and combining it with the continuity equation gives rise to a second order partial differential equation,

$$\nabla^2\Phi = 0. \tag{1}$$

The above equation, commonly known as Laplace equation, belongs to elliptic partial differential equation class, and any elliptic PDE will have characteristics which occur in a pair, each a complex conjugate of other. Since this class of problems have the highest derivative given by $\nabla^{2n}$, it requires $n$ boundary conditions to solve it. For Equation (1), the required boundary condition prescribed on all the segment of the boundary is,

$$\nabla\Phi \cdot \hat{n} = 0, \tag{2}$$

where $\hat{n}$ is the normal unit vector to the surface. The above equation is usually known as the no-penetration condition for

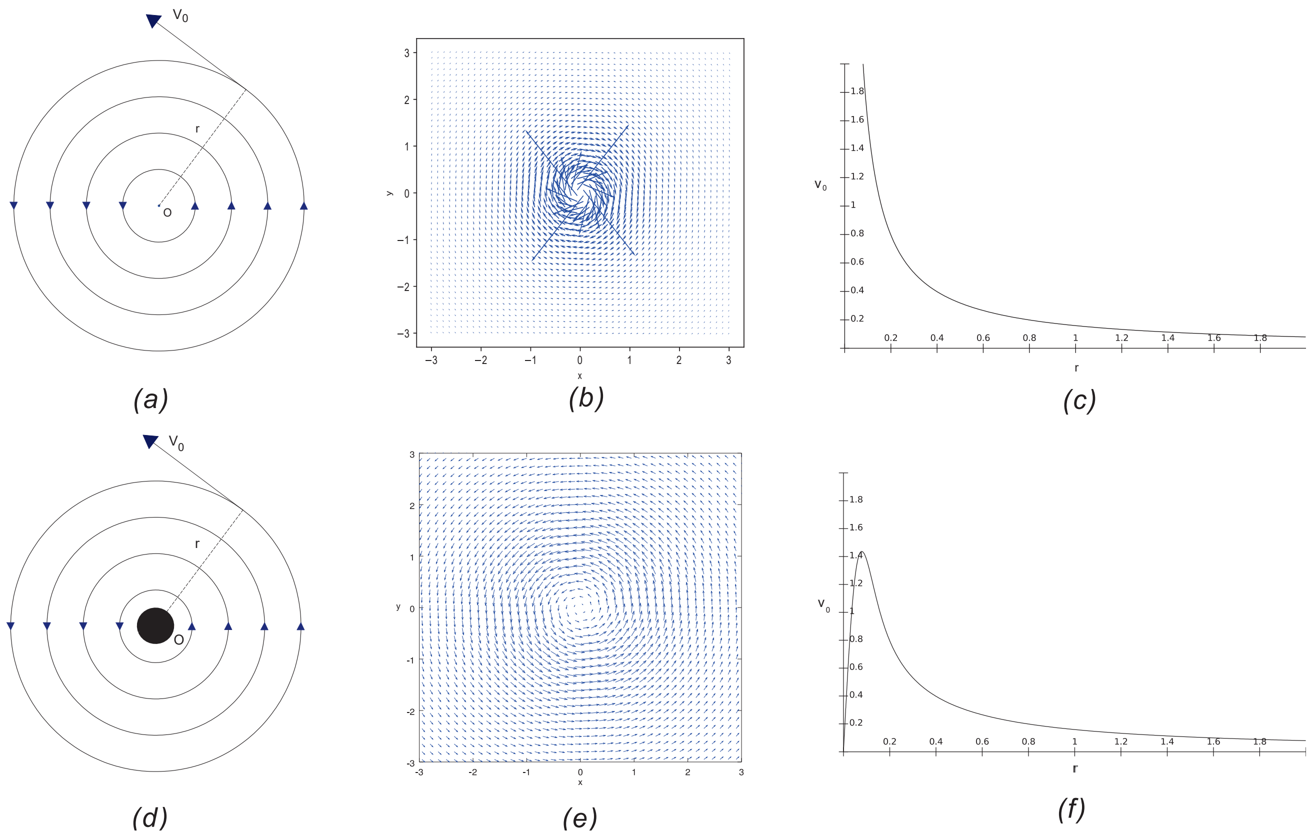


**Figure 2: (a)** conventional point vortex flow, **(b)** velocity quiver plot for the vortex flow, and **(c)** tangential velocity $\boldsymbol{v_\theta}$ with respect to the radial distance *r*; **(d)** vortex with viscous-core, **(e)** velocity quiver plot for the viscous-core vortex, and **(f)** tangential velocity $\boldsymbol{v_\theta}$ with respect to the radial distance *r* for the viscous-core vortex

the normal velocity component. In addition, there's one more condition which states that the disturbance created by the solid body should become zero as the position of influence tends to infinity,

$$\nabla\Phi \to U_\infty \; for \; \vec{r} \to \infty, \tag{3}$$

where $U_\infty$ is the freestream velocity.

## Solution of the Laplace Equation

The basic concept of the linear potential flow is to find the flow functions that satisfy the Equation (1), and because it is a linear partial differential equation, the individual solutions can be added further to create more complicated flow fields (superposition principle). The four fundamental potential functions that are used to create arbitrary flow fields are *uniform flow, source/sink, vortex, doublet*.

## Challenges with BEM

There has been a regular development in panel methods, yet the most modern panel methods still have severe drawbacks. The main drawbacks are the inviscid assumption and the singularity problem which occurs when the point vortex solution is used with infinite vorticity at the core. The infinite vorticity at the centre makes it difficult to use a set of discrete vortices to represent a real atmospheric disturbance. The point vortex with a finite circulation Γ induces a tangential velocity $v_\theta$ at a radial distance of *r* as,

$$v_\theta = \frac{\Gamma}{2\pi r}, \tag{4}$$

$$v_\theta \to \infty \; as \; r \to 0. \tag{5}$$

The induced velocity term has a singularity when the point of evaluation for induced velocity is located at the centre of the vortex. Likewise, when the evaluation point is very near to the vortex core, there is an unphysically large induced velocity at that point. However, in real vortices, there is always a core region surrounding the axis where the particle velocity stops increasing and then decreases to zero as r goes to zero. Within that region, the flow is no longer irrotational: the vorticity $\vec{\omega}$ becomes non-zero, with direction roughly parallel to the vortex axis. The possible solution is either to use a cut-off radius, $\delta$ [5], or to use a viscous vortex model with a finite core size by multiplying a factor to remove the singularity [7].

## Improved Vortex Modelling

As discussed in preceding section, the irrotational point vortex imposes severe limitations to its applicability in modelling a turbulent inflow signal. In order to remove the singularities, lamb vortices are substituted in place of the potential vortex. The tangential velocity $v_\theta$ in the circumferential direction is given by,

$$v_\theta = \frac{\Gamma}{2\pi r}\left(1 - \exp\left(-\frac{r^2}{r_c^2}\right)\right), \tag{6}$$

where Γ is the circulation, $r$ is the radial distance of the point of influence from the centre, and $r_c$ is the core radius of the vortex. Fig. 2 shows the difference between the potential vortex and the improved vortex with a viscous core.

## Linear Turbulent Velocity Modelling

To develop the inflow turbulent for the computation of unsteady loading, it is of utmost importance to define a term which depends on time and position and is a function of the turbulent velocity fluctuations for the turbulence related sound. One such method to generate inflow turbulence is Synthetic Eddy Modelling. Various techniques to generate synthetic turbulence (briefly defined as an artificial turbulent flow signal, not involving the solution to fluid flow equations) have been designed over the years. These techniques have a different level of accuracy and depend upon the problem being solved. The current work uses the synthetic eddy modelling technique to generate the inflow turbulence for the prediction of leading-edge noise. The pioneering work of Kraichnan [8] and Sescu & Hixon [9] laid the foundation for synthetic turbulence, where the artificial velocity field was constructed using the superposition of Fourier modes with random frequencies, wave number, and amplitudes. A synthetic eddy method assumes that turbulence can be considered as a superposition of eddies [10]. The technique developed here uses the singularity-free vortices (discussed in preceding sections) and derive divergence-free inflow turbulence. The fluctuating term is defined as the scalar streamfunction $\Psi(\boldsymbol{x}, t)$, from which the velocity components are deduced subsequently. Thus, the 2D fluctuating velocities are

$$u'(\mathbf{x}, t) = \epsilon_{ij} \frac{\partial \Psi(\mathbf{x}, \mathrm{t})}{\partial y}, \quad (7)$$

where $\epsilon_{ij}$ denotes the 2D $\epsilon$-tensor. The synthetic turbulence field with realistic velocity spectra can be constructed by imposing certain regularisation and limitation on the sizes, shapes and directional strengths of the randomly distributed vortices in the 2D space. The scalar streamfunction constituting the influence of all the vortices in the space is written as:

$$\Psi(\mathbf{x}, \mathrm{t}) = \Delta_e \sum_{i=1}^{N_e} \left[\psi_{x,i}(\mathbf{x}, t)\mathbf{e}_x + \psi_{y,i}(\mathbf{x}, t)\mathbf{e}_y\right], \quad (8)$$

where $\Delta_e$ indicates the average distance between two adjacent vortices. This distance is a function of the size of the vortex window $A_e$, and the number of vortices in it. $\mathbf{e}_x$ and $\mathbf{e}_y$ are the directional unit vectors in the Cartesian coordinate system. $\psi_{\xi,i}$ is a dimensionless shape function using the Gaussian profiles. These shape functions are the functions of the length scales $\Lambda$ and the turbulent intensity $Tu$,

$$\psi_{\xi,i}(\mathbf{x}, t) = \mathrm{f}(\Lambda, \mathrm{Tu}). \quad (9)$$

For the shape function of the eddies, the Gaussian profile is chosen which is defined as:

$$\psi_{\xi,i}(\mathbf{x}, t) = \sigma_{\xi,i}^{-1/2} \exp\left[-\left(3\sigma_{\xi,i} r_i\right)^2\right], \quad (10)$$

$$\forall \xi \in \{x, y\} \;\&\; i \in \{1,2, \dots, N_e\},$$

where $\sigma_x$, $\sigma_y$ is a set of constants that determines the size of each vortex component, $r_i$ is the distance of the point of influence from the centre location of the vortex.

## Generating a realistic 2D turbulence

The primary goal of this paper is to create a realistic inflow turbulent velocity field that can be used to predict the leading-edge noise. Since the atmospheric turbulence is stochastic in nature, there must be specific parameters which introduce this randomness in such a way the statistics of the synthetic turbulence bears a resemblance to that of the natural one. Equations 8 to 11 will construct the artificial turbulent field, and from them, a total of 6 parameters are chosen to control the distribution of the vortices in the vortex window to reproduce the statistics of von Karman energy spectra for homogeneous isotropic turbulence. These parameters are listed in Table.1

**Table 1:** Parameters controlling the turbulence statistics

| Parameters | Definitions |
|---|---|
| $N_e$ | Number of the vortices in the vortex window |
| $A_e$ | Area of the vortex window |
| $R_G^{max}$ | Upper limits for the vortex size (radius) |
| $R_G^{min}$ | Lower limits for the vortex size (radius) |
| $\epsilon_{x,upper}$ | Upper limits in eddy strength in the x-direction |
| $\epsilon_{x,lower}$ | Lower limits in eddy strength in the x-direction |

The target energy spectra, Von Kármán, is identified and then an error function $\varepsilon(k)$ is further defined to optimise the newly developed spectra based on the selected parameters which are influencing the shape functions and thus the artificial turbulence,

$$\varepsilon(k) = \begin{cases} \left|\log_{10} \frac{\langle E_{new}(k)\rangle}{E_{tar}(k)}\right| \\ \left|E_{tar}(k) - E_{new}(k)\right| \end{cases}. \quad (11)$$

The error function $\varepsilon(k)$ gives the largest deviation of the spanwise-averaged numerical spectra from the desired von Kármán ones within the specified frequency range.

The optimisation is done using the Genetic algorithm[1] [11]. The algorithm is inspired by the mechanism of natural selection, a biological process in which stronger individuals are likely be the winners in a competing environment. It presumes that the potential solution of a problem is an individual and can be represented by a set of parameters.

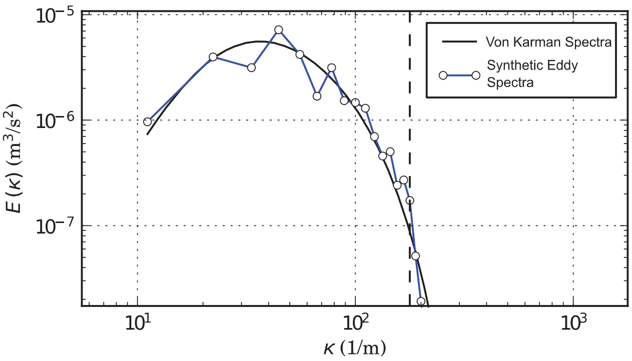


**Figure 4:** Comparison of the newly generated synthetic turbulence spectra with the von Karman energy spectra

It is shown in the Fig. 4 that after the optimisation, the generated synthetic turbulence with the improved vortices has a remarkable resemblance to the von Karman energy spectra. With more values of wavenumber *k*, the comparison can be made smooth.

## Literatur


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[1] The depth discussion of the Genetic Algorithm is beyond the scope of current paper. The interested readers may refer to Ref.11