

Time Domain Boundary Element Method for the Leading-Edge Noise subjected to Linear Vorticity

Sparsh Sharma^{a)} Thomas Geyer^{b)} Chair of Technical Acoustics, Brandenburg University of Technology Cottbus-Senftenberg, 03046 Cottbus, Germany

Ennes Sarradj^{c)} Chair of Technical Acoustics, Technical University Berlin, 10587 Berlin, Germany

ABSTRACT

Time-domain boundary element methods (BEM) have been in use to calculate the unsteady loading on the airfoil for quite a long time now. The unsteady loading caused by the response of the body to the turbulent inflow is the leading cause of broadband noise from the turbomachinery. Hence, it is vitally important to calculate the unsteady loading due to turbulence accurately. The present paper shows the development of a turbulent inflow for the time-domain BEM using a set of discrete vortices utilising the principle of synthetic eddy modelling. The generated inflow turbulent signal spectra are optimised to the von Kármán energy spectra.

1 INTRODUCTION

The early research in the field of unsteady aerodynamics [1-3] has been an excellent source of motivation for the discussion of acoustic radiation from a body immersed in a turbulent flow or a body making a nonuniform motion in the flow field. The aerodynamic noise arises because of two different phenomena. The first one is the impulsive noise, which is a result of moving surface or surfaces in nonuniform flow conditions. The other noise mechanism is the result of turbulence and therefore arises in nearly every engineering application. The equations that are used in the solution of such problems are a simplified version of the governing equations of the fluid dynamics. The advent of High-performance computing platforms has enabled the engineers to

^{a)} email: sparsh.sharma@b-tu.de

^{b)} email: thomas.geyer@b-tu.de

c) email: ennes.sarradj@tu-berlin.de

operate massive supercomputers rather than finding a simple analytical model or a low-fidelity numerical method to solve a problem. The CFD method has a high fidelity in predicting the airfoilturbulence interaction (ATIN, shown in Fig.1), and its computational cost limits its applications at conceptual and preliminary design stages of an aircraft or a turbomachine. Besides, using only an experimental method to address the effect of the interaction is also not a viable option because of its high cost. To overcome the above issues, methods and tools capable of rapidly and accurately predicting the sound radiation due to the airfoil-turbulence interaction are needed.



Fig. 1- Incidence turbulence interacting with the leading edge of an airfoil

Time-domain boundary element methods have been in use to calculate the unsteady loading on the airfoil for quite a long time now. Unsteady BEM solutions are still productive with flow physics and give accurate solutions at computational times that are several orders of magnitude shorter than Navier-Stokes solvers. BEMs assume that fluid flow is incompressible, irrotational (except at singular points) and inviscid, that is, a potential flow. This leads to simplified forms of the continuity and momentum equations that govern the fluid flow. The unsteady loading caused by the response of the body to the turbulent inflow is the leading cause of broadband noise from the turbomachinery. So, to calculate the sound radiation from a body, it is vitally important to calculate the unsteady loading due to turbulence accurately. Most authors have opted to use a frequency domain analysis in which the incident disturbance is specified by a harmonic gust [5-7]. Grace [8] concluded that time-domain analyses have a number of advantages over the harmonic gust approaches, yet seldom used for leading-edge noise because it is difficult to model the turbulent inflow using a set of discrete vortices.

The present paper shows the development of a turbulent inflow using a set of discrete vortices. Primarily, the singularity of a potential vortex is discussed with a possible substitution of viscouscore vortex instead, then the application of synthetic eddy modelling in generating a turbulent inflow signal using these discrete vortices. The then generated spectra are optimised to the von Kármán energy spectra. This paper presents the first part of the research work on ATIN. The entire study focuses on the development, validation, and demonstration of a transient, two-dimensional stochastic boundary element solver for the acoustic radiation from the airfoil-turbulence interaction. Undeniably, the sound radiation due to turbulence and turbulence itself is very complicated, and to model, all of its properties in the presented approach without assumptions is not possible. The proceeding sections discuss the efficiency as well as the drawbacks of BEM, and the implementation of a hybrid panel method using the vortex particle with finite viscous core vortex method.

2 NUMERICAL METHOD: BOUNDARY ELEMENT METHOD FOR UNSTEADY FLOW

In this section, the foundations of the potential flow for external flow aerodynamics are presented. Perhaps the oldest and the most extended used computer method for simulating aerodynamic flows is the panel method (boundary element method applied to fluid dynamics).

2.1 Governing Equations

Inviscid flows are described on the bases of the continuity equation and the inviscid momentum equations. An arbitrary solid body immersed in an incompressible and irrotational fluid, defining a velocity potential Φ and combining it with the continuity equation gives rise to a second order partial differential equation,

$$\nabla^2 \Phi = 0. \tag{1}$$

The above equation, commonly known as Laplace equation, belongs to elliptic partial differential equation (PDE) class, and any elliptic PDE will have characteristics which occur in a pair, each a complex conjugate of the other. Since this class of problems have the highest derivative given by ∇^{2n} , it requires n boundary conditions to solve it. For Eq. (1), the required boundary condition prescribed on all the segment of the boundary is

$$\nabla \Phi \cdot \hat{n} = 0, \tag{2}$$

where \hat{n} is the normal unit vector to the surface. The above equation is usually known as the nopenetration condition for the normal velocity component. In addition, there's one more condition which states that the disturbance created by the solid body should become zero as the position of influence tends to infinity,

$$\nabla \Phi \to U_{\infty} \text{ for } \vec{r} \to \infty,$$
 (3)

where U_{∞} is the freestream velocity.

2.2 Solution of the Laplace Equation

The basic concept of the linear potential flow is to find the flow functions that satisfy Eq. (1), and because it is a linear partial differential equation, the individual solutions can be added further to create more complicated flow fields (superposition principle). The four fundamental potential functions that are used to create arbitrary flow fields are *uniform flow, source/sink, vortex, doublet*.

3 CHALLENGES WITH BEM

There has been a regular development in panel methods, yet the most modern panel methods still have severe drawbacks. The main drawbacks are the inviscid assumption and the singularity problem which occurs when the point vortex solution is used with infinite vorticity at the core. The infinite vorticity at the centre makes it difficult to use a set of discrete vortices to represent a real atmospheric disturbance. The point vortex with a finite circulation Γ induces a tangential velocity v_{θ} at a radial distance of *r* as

$$v_{\theta} = \frac{\Gamma}{2\pi r},\tag{4}$$

$$v_{\theta} \to \infty \text{ as } r \to 0. \tag{5}$$

The induced velocity term has a singularity when the point of evaluation for induced velocity is located at the centre of the vortex. Likewise, when the evaluation point is very near to the vortex core, there is an unphysically large induced velocity at that point. However, in real vortices, there is always a core region surrounding the axis where the particle velocity stops increasing and then decreases to zero as r goes to zero. Within that region, the flow is no longer irrotational: the vorticity $\vec{\omega}$ becomes non-zero, with direction roughly parallel to the vortex axis. The possible solution is either to use a cut-off radius, $\delta[8]$, or to use a viscous vortex model with a finite core size by multiplying a factor to remove the singularity [9].

4 IMPROVED VORTEX MODELING

As discussed in the preceding section, the irrotational point vortex imposes severe limitations to its applicability in modelling a turbulent inflow signal. In order to remove the singularities, lamb vortices are substituted in place of the potential vortex. The tangential velocity v_{θ} in the circumferential direction is given by

$$v_{\theta} = \frac{\Gamma}{2\pi r} \left(1 - \exp\left(-\frac{r^2}{r_c^2}\right) \right). \tag{6}$$

where r is the radial distance of the point of influence from the centre, and r_c is the core radius of the vortex. Fig. 2 shows the difference between the potential vortex and the improved vortex with a viscous core.



Fig. 2 - (a) conventional point vortex flow, (b) velocity quiver plot for the vortex flow, and (c) tangential velocity v_{θ} with respect to the radial distance r; (d) vortex with viscous-core, (e) velocity quiver plot for the viscous-core vortex, and (f) tangential velocity v_{θ} with respect to the radial distance r for the viscous-core vortex

4.1 Linear Turbulent Velocity Modelling

To develop the inflow turbulence for the computation of unsteady loading, it is of utmost importance to define a term which depends on time and position and is a function of the turbulent velocity fluctuations for the turbulence related sound. One such method to generate inflow turbulence is Synthetic Eddy Modelling. Various techniques to generate synthetic turbulence (briefly defined as an artificial turbulent flow signal, not involving the solution to fluid flow equations) have been designed over the years. These techniques have a different level of accuracy and depend upon the problem being solved. The current work uses the synthetic eddy modelling technique to generate the inflow turbulence for the prediction of leading-edge noise. The pioneering work of Kraichnan [10] and Sescu and Hixon [11] laid the foundation for synthetic turbulence, where the artificial velocity field was constructed using the superposition of Fourier modes with random frequencies, wave number, and amplitudes. A synthetic eddy method assumes that turbulence can be considered as a superposition of eddies [12]. The technique developed here uses the singularity-free vortices (discussed in the preceding sections) and derives divergence-free inflow turbulence. The fluctuating term is defined as the scalar streamfunction $\Psi(\mathbf{x}, t)$, from which the velocity components are deduced subsequently. Thus, the 2D fluctuating velocity is

$$u'(\mathbf{x},t) = \epsilon_{ij} \frac{\partial \Psi(\mathbf{x},t)}{\partial y},\tag{7}$$



where ϵ_{ij} denotes the 2D ϵ -tensor. The velocity field from a single vortex is shown in Fig. 3.

Fig. 3 - (from upper left to lower right) x-component of the velocity u'induced by a potential vortex, y-component of the velocity v'induced by a potential vortex; x-component of the velocity u'induced by an improved vortex, y-component of the velocity v'induced by an improved vortex

The synthetic turbulence field with realistic velocity spectra can be constructed by imposing certain regularisation and limitation on the sizes, shapes and directional strengths of the randomly distributed vortices in the 2D space. The scalar streamfunction constituting the influence of all the vortices in the space is written as

$$\Psi(\mathbf{x}, \mathbf{t}) = \Delta_e \sum_{i=1}^{N_e} [\psi_{x,i}(\mathbf{x}, t) \mathbf{e}_x + \psi_{y,i}(\mathbf{x}, t) \mathbf{e}_y],$$
(8)

where Δ_e indicates the average distance between two adjacent vortices. This distance is a function of the size of the vortex window A_e , and the number of vortices in it. e_x and e_y are the directional unit vectors in the Cartesian coordinate system. $\psi_{\xi,i}$ is a dimensionless shape function using the Gaussian profile. These shape functions are the functions of the length scales, Λ , and the turbulence intensity, Tu,

$$\psi_{\xi,i}(\mathbf{x},t) = f(\Lambda, Tu). \tag{9}$$

For the shape function of the eddies, the Gaussian profile is chosen which is defined as:

$$\psi_{\xi,i}(\mathbf{x},t) = \sigma_{\xi,i}^{-1/2} \exp\left[-\left(3\sigma_{\xi,i}r_i\right)^2\right],$$
(10)

 $\forall \xi \in \{x, y\} \& i,$

where σ_x , σ_y is a set of constants that determines the size of each vortex component, r_i is the distance of the point of influence from the centre location of the vortex. Fig. 4 shows a schematic representation of the superimposed vortices. The ability of the method to realise the desired statistical properties relies on two parameters: maximum eddy radius, r, and the eddy spacing, Δ_e , which may be expressed as a function of the integral length scale.



Fig. 4 - Implementation of the numerical method (schematically shown on the left side is a set of artificial vortices that are about to interact with an airfoil, shown on the right side, flow from left to right)

4.2 Generating a realistic 2D turbulent signal

The primary goal of this paper is to generate a realistic inflow turbulent velocity field that can be used to predict the leading-edge noise. Since the atmospheric turbulence is stochastic in nature, there must be specific parameters which introduce this randomness in such a way that the statistics of the synthetic turbulence bears a resemblance to that of the natural one. Eqs. (8) to (11) will construct the synthetic turbulent field, and from them, a total of 6 parameters are chosen to control the distribution of the vortices in the vortex window to reproduce the statistics of von Kármán energy spectra [13] for homogeneous isotropic turbulence. These parameters are listed in Table.1

Turbulence Statistics	
Parameters	Definitions
N _e	Number of the vortices in the vortex window
A_e	Area of the vortex window
R_G^{max}	Upper limits for the vortex size (radius)
R_{G}^{min}	Lower limits for the vortex size (radius)
$\epsilon_{x,upper}$	Upper limits in eddy strength in the x-direction
$\epsilon_{x,lower}$	Lower limits in eddy strength in the x-direction

Table 1 - Parameters controlling the turbulence statistics.

The target energy spectra, $E_{tar}(k)$, is identified and then an error function $\varepsilon(k)$ is further defined, Eq. (11), to optimise the newly developed spectra, $E_{new}(k)$, based on the selected parameters which are influencing the shape functions and thus the artificial turbulence.

$$\varepsilon(k) = \begin{cases} \left| \log_{10} \frac{\langle E_{new}(k) \rangle}{E_{tar}(k)} \right| \\ |E_{tar}(k) - E_{new}(k)| \end{cases}$$
(11)

The optimisation is done using the Genetic algorithm¹ [14]. The algorithm is inspired by the mechanism of natural selection, a biological process in which stronger individuals are likely to be the winners in a competing environment. It presumes that the potential solution of a problem is an individual and can be represented by a set of parameters.



Fig. 5 - Comparison of the newly generated synthetic turbulence spectra with the von Kármán energy spectra

¹ The depth discussion of the Genetic Algorithm is beyond the scope of the current paper. The interested readers may refer to Ref.14

It is shown in the Fig. 5 that after the optimisation, the generated synthetic turbulence with the improved vortices has a remarkable resemblance to the von Kármán energy spectra. The spectrum is optimised for Ne=400. With more values of k, the comparison can be made smooth.

5 CONCLUSIONS

The numerical method presented in the paper, to calculate the noise due to airfoil-turbulence interaction, is based on an unbounded domain. Time-domain BEM assumes the fluid flow domain to be infinite contrary to the synthetic eddy modelling which requires the definition of a precise volume (area for 2D) containing all the turbulent structures. Thus, to use eddy modelling with the potential flow method, a finite domain of interest is defined. The anticipated fluctuating velocity field induced by the vortices can then be applied inside this domain with an airfoil. The pressure fluctuations over the body immersed in the turbulent flow field can be calculated using suitable boundary conditions.

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