Optimal planar microphone array arrangements

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Introduction

The usefulness of a microphone array for the purpose of acoustical measurements depends on a number of factors. One of them is the arrangement of the microphones. It is well known that the arrangement of sensors in an array determines the overall properties of the array and it has been shown that different arrangements lead to different properties, see e.g. [1]. Because the number of microphones in an array determines also the cost of the measurement equipment and its operation, there have been a number of efforts to optimize the arrangements that are used to get the best results from a given number of microphones, see e.g. [2, 3, 4, 5]. Most of these approaches make use of a parametrized arrangement (concentric circles, possibly multi-armed spirals etc.), were the properties of the microphone array can be tuned or optimized by alteration of the parameters. In what follows an attempt is made to synthesize optimal microphone arrangements without the need to apply possibly cumbersome numerical optimization. The analysis is restricted to planar arrangements.

Method

Properties of microphone arrays

In general, any planar microphone array may be described by the number of microphones N, the aperture D, which is the overall dimension of the arrangement, and the form and pattern of the arrangement. If a beamforming method is used for the processing of the microphone signals, the array forms a directional sound receiver that can be steered to different locations in order to provide simultaneous characterization of multiple sound sources. The directional characteristics of this array receiver may be given by its point spread function (PSF). The PSF is the output from the array in the presence of a point source.

Fig. 1 shows a cut plane of an example of a two-dimensional PSF together with the color-coded map of such a PSF. Two properties of the PSF are of interest. First, the width of the beam b determines how good two sources can be separated with the array and should be small. The second property is the minimum level difference ΔL_S between the main lobe at the source position and the unavoidable side lobes. It determines the maximum level difference between a major and a minor source that can be allowed if the source should be identified using the array. ΔL_S should preferably be large.

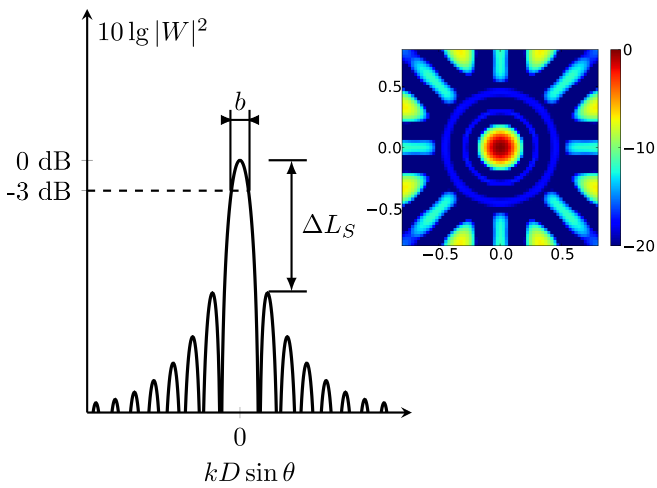


Figure 1: PSF Example with beam width b and minimum sidelobe level ΔL_S. θ gives the look direction from the array and k is the wave number. Inset shows a two-dimensional map of a similar PSF.

Continuous aperture

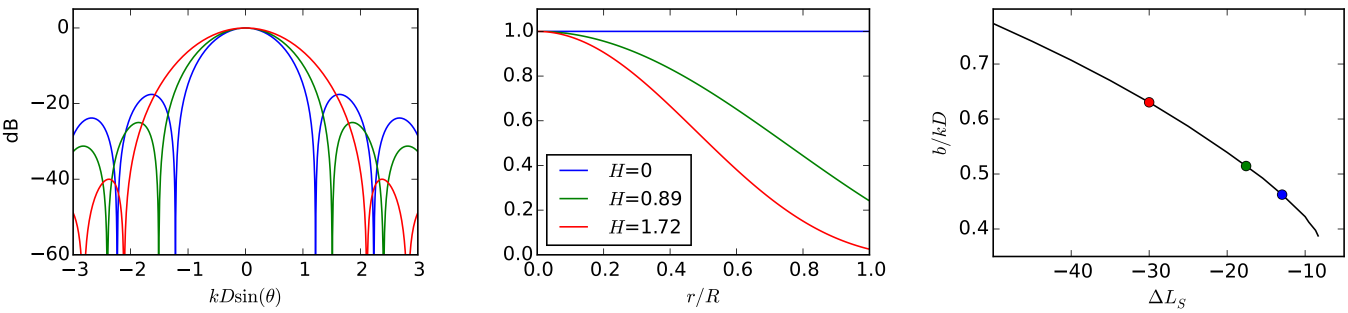
While the PSF of an array can only be calculated numerically, the PSF of a sound receiver that is continuously distributed over a plane may be estimated from analytical calculation. One case is a circular continuous aperture, where the PSF is given [1] by

$$W(k_r) = 2\frac{\mathrm{J}_1(kR\sin\theta)}{kR\sin\theta} \quad (1)$$

and J_1 is the first order Bessel function and $R = \frac{D}{2}$. It turns out that in this case $\Delta L_S = 17.57$ dB. In order to increase this value, a weighting may be introduced where the contribution from certain regions within the circle is attenuated. Different concepts for this weighting exist [6]. If only monotonic functions in the radius coordinate r are considered, one option is the weighting proposed by Hansen [7]

$$f_H(H,r) = \mathrm{I}_0\left(\pi H\sqrt{1-\left(\frac{r}{R}\right)^2}\right), \quad H \geq 0, \quad (2)$$

that depends on the parameter H and uses the modified zeroth order Bessel function I_0. For $H = 0$ a uniform weighting (equivalent to no weighting) results. Larger values of H lead to larger values of ΔL_S, but produce also wider main lobes, see Fig. 2. While any weighting will have its influence on both ΔL_S and b, (2) appears to produce optimal results in the Pareto sense, i.e. the largest ΔL_S for a given b and the smallest b for a given ΔL_S. The weighting given in (2) can be generalized when $\mathrm{I}_0\left(\pi H\frac{r}{R}\right)$ is used as the weighting function for $H < 0$.



(a) PSF **(b)** weighting function **(c)** beam width and sidelobe level

Figure 2: Weighting for a continous aperture proposed by Hansen [7]

Spatially sampled aperture

In order to make the transition from a continuous aperture to an array of microphones, the latter may be considered as a form of the first that is spatially sampled. A first approach for this sampling is to distribute the microphone positions uniformly over the aperture so that each microphone stands for an area of the same size. Another consideration is that beamforming makes use of the phase (or signal time) difference for each possible microphone pair. Thus, in order to get a maximum of information and the to treat all azimuthal directions of incidence the same, the microphones should be uniformly distributed over all angles ϕ seen from the center of the circle.

It is relatively easy to produce a microphone arrangement with uniform distribution of microphones over the circle in form of a regular square or triangular lattice. However, in these lattices some directional differences between microphone pairs will occur much more often than others. A number of different approaches (e.g. [1, 4, 5]) take this into account and produce arrangements that have a uniform distribution over all angles (spirals, circles, etc.). However, they do not produce a uniform distribution over the circle area in a controlled way.

Both requirements – uniform distribution over the circle and over all directions – are realized in some disc phyllotaxis. One example is the flower head of a sunflower, where each floret (and later each seed) occupies the same area and the florets are evenly distributed over all directions. The arrangement can described by Vogel's [8] spiral

$$r = R\sqrt{\frac{n}{N}}, \qquad n = 1, 2, \ldots, N \tag{3}$$

$$\phi = 2\pi n \frac{(1+\sqrt{V})}{2} \tag{4}$$

if the parameter is chosen to be $V = 5$. Interestingly, modifying the parameter V in this model results in a great variety of different arrangements (see Fig. 3).

The weighting from (2) can be applied by simply attenuating the microphone output signals with appropriate factors. However, this will not make an efficient use of the microphones, because some of the microphones will then provide only a minor contribution to the array output. Another approach to apply the weighting is to modify the arrangement of the microphones, so that a high density of microphones per area corresponds to a high weighting factor and a low density corresponds to a low weighting factor. An appropriate arrangement is found starting at Vogel's spiral.

Equation (3) can be rewritten as

$$r_n = R\sqrt{\sum_{m=1}^{n}\frac{1}{N}} = \sqrt{\frac{1}{\pi}\sum_{m=1}^{n}\frac{\pi R^2}{N}}, \; n = 1, 2, \ldots, N, \tag{5}$$

which shows that the overall area πR^2 is partitioned to associate the same $\frac{1}{N}$ of the total area to each of the microphones. To associate the individual microphones with different areas, the weighting function is introduced in the equation:

$$r_n = R\sqrt{\sum_{m=1}^{n}\frac{\int_0^R f_H(H,r)dr}{N f_H(H,r_m)}}, \qquad n = 1, 2, \ldots, N. \tag{6}$$

This is a system of equations which can readily be solved using a nonlinear least squares method. Together with (4) the solution gives the arrangement of microphones.

Fig. 4 shows some examples for different values of H. The overlaid Voronoi diagrams demonstrate the different area sizes associated with the microphones. The different arrangements lead to different point spread functions (see Fig. 5) and thus also to different array properties.

Results and discussion

For a given microphone array, the beam width and the side lobe level depend on the Helmholtz number $\frac{D}{\lambda}$ and thus on the frequency. While the influence on the beamwidth can be removed using the Helmholtz number as a scaling factor, the frequency dependency on the side lobe level cannot be removed easily. However, the ranking of different array arrangements with regard to the side lobe level does not strongly depend on frequency. Thus, the results are reported here for only one frequency. A similar reasoning holds for the number of microphones in the

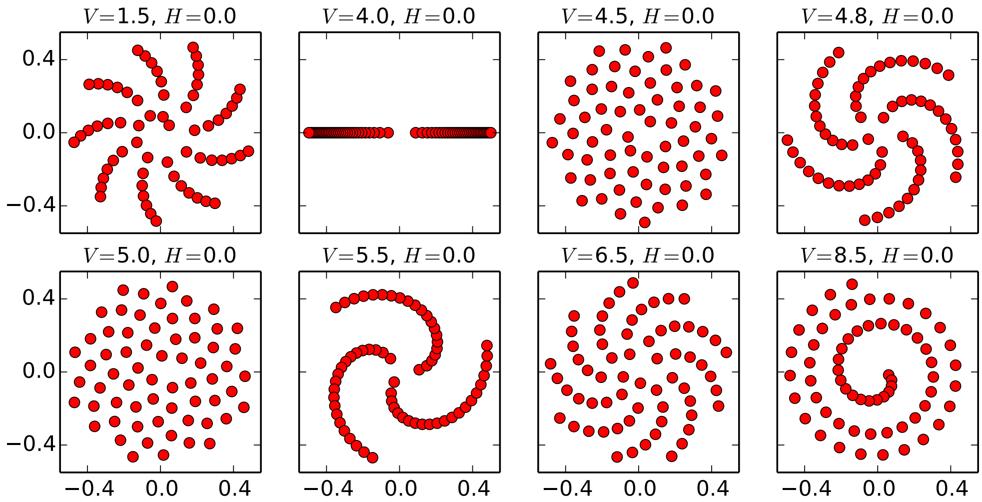


Figure 3: Examples for different arrangements with $N = 64$ produced from (3) and (4)

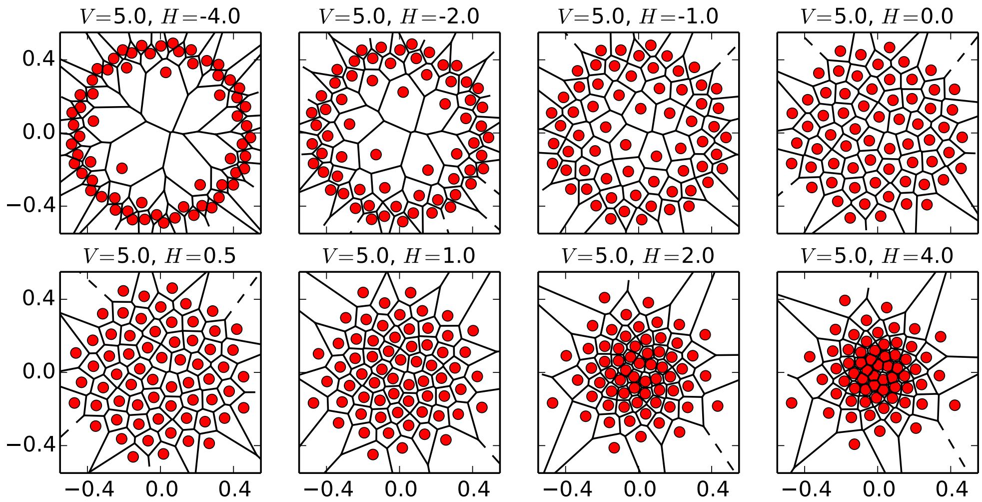


Figure 4: Examples for different arrangements with $N = 64$ produced from (4) and (6) with Voronoi diagrams showing approximately the area per microphone

arrangement, were the results reported here are restricted to the case of $N = 64$.

The microphone array properties for arrangements according to (4) and (6) depend on both the "angle" parameter V and the "radius" parameter H. Fig. 6 shows b and ΔL_S for 940 different parameter sets for $3 \leq V \leq 7$ and $-4 \leq H \leq 4$. It turns out that for a given H, $V = 5.0$ produces the best results. It can therefore be argued that when setting $V = 5.0$ (Vogel's spiral), Pareto-optimal results are produced. Moreover, the position on the Pareto front can be controlled with the parameter H. A negative value produces a more narrow beam and results in a lower ΔL_S, while a positive value widens the beam but gives a better ΔL_S. The analysis for different N and different $\frac{D}{\lambda}$ leads to the similar results not reported here.

Fig. 6 also shows results for other classes of microphone array arrangements. The circle geometry yields the most narrow beam width, but has also a low side lobe level. When comparing to Fig. 4 it becomes obvious that the circle arrangement will also be produced when $H \to -\infty$. It is therefore a special case of the proposed approach. Another class of arrangements that is known [9] to yield good results is the multi-armed logarithmic spiral proposed by Underbrink [4]. Fig. 6 contains results for 45 of these spirals with varying parameters. While the properties of all of these arrangements are very close to each other, they do not reach the Pareto front of the proposed arrangements.

Conclusion

It is shown that an approach that combines the phyllotaxis modeled by Vogel's spiral with a modified Bessel function weighting proposed by Hansen leads to microphone arrangements that have Pareto-optimal properties. The approach does not require numerical optimization and the properties can be systematically adjusted by one parameter H used for the synthesis of a microphone arrangement.

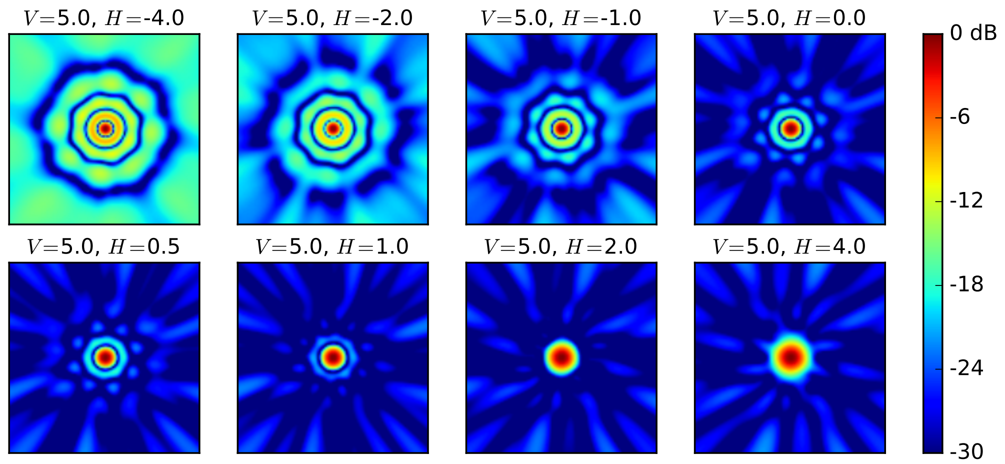


Figure 5: Point spread functions for the example arrangements shown in Fig. 4 for $\frac{D}{\lambda} = 10$ and $|\theta| < \frac{\pi}{4}$

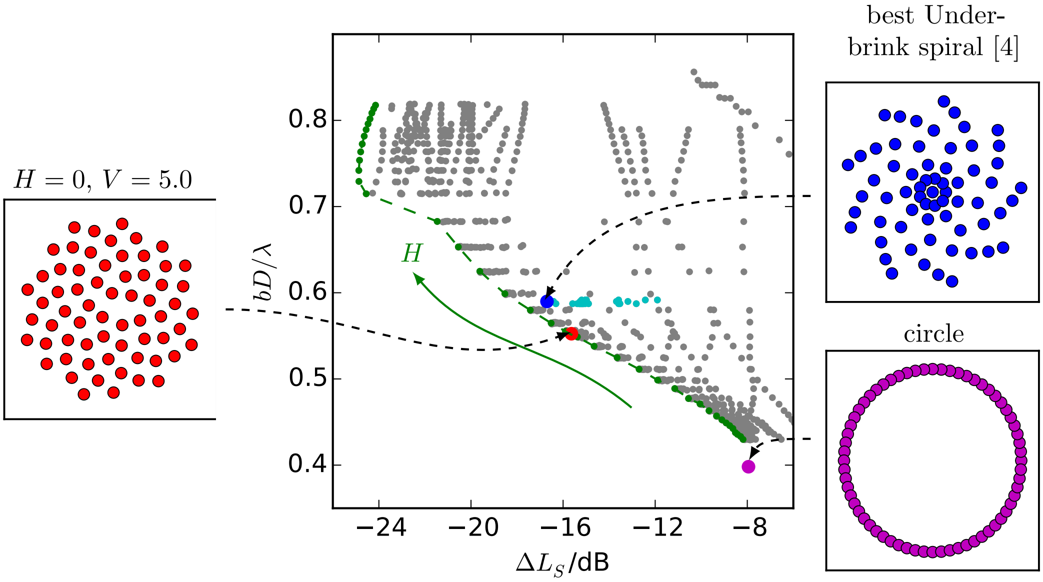


Figure 6: Beam width and sidelobe level for $\frac{D}{\lambda} = 10$ and $|\theta| < \frac{\pi}{4}$ for the proposed arrangements using (6) and (4) and different H and V (gray), for $V = 5.0$ (green), for multi-armed logarithmic spirals after [4] (cyan) and a circle arrangement (magenta)

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