Brief Comparison of Kirchhoff-Helmholtz-BEM and Acoustical Energy BEM for Exterior Domains

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Introduction

The classical boundary element method (BEM) in acoustics is based on the Kirchhoff-Helmholtz integral equation and is especially well suited for the treatment of exterior domain radiation and scattering problems. However, the computational effort required depends on the Helmholtz number, i.e. the ratio of the typical dimension of the radiating or scattering structure and the wavelength that governs the problem. Thus, despite the fast solvers and techniques available, for large structures and high frequencies the calculation may require considerable resources. In such cases, the results are often unnecessary detailed and need further reduction during the post processing, e.g. spatial or frequency-band averaging.

The acoustical energy boundary element method (EBEM)[1, 2, 3] is an approach that uses an integral equation in terms of sound intensity and sound energy density. Inherent statistical assumptions limit its use to medium and high frequencies, but the computational effort needed is much less than for classical BEM.

In this paper, both methods are compared using simple example problems for radiation and scattering in the exterior domain. First, the theory for both methods is briefly reviewed. Then, two example problems are presented and the results as well as the computational cost are considered.

Theory

The acoustic wave equation is usually formulated in terms of the sound pressure $\underline{p}$. The Helmholtz form of this equation is

$$(\nabla^2 + k^2)\underline{p} = 0. \tag{1}$$

The Green's function for the wave operator in three dimensions is

$$\underline{G}(\mathbf{x}, \mathbf{y}) = \frac{1}{4\pi r}\,\mathrm{e}^{-jkr},\ r = |\mathbf{x} - \mathbf{y}| \tag{2}$$

and together with Green's second identity an integral equation may be set up. This Kirchhoff-Helmholtz integral equation

$$\int_S \underline{p}(\mathbf{x})\frac{\partial \underline{G}(\mathbf{x},\mathbf{y})}{\partial \mathbf{n_x}} - \underline{G}(\mathbf{x},\mathbf{y})\frac{\partial \underline{p}(\mathbf{x})}{\partial \mathbf{n_x}}\,\mathrm{d}\,S(\mathbf{x}) = \underline{p}(\mathbf{y}) \cdot \begin{cases} 1, & \mathbf{y} \in V, \mathbf{y} \notin S \\ 1/2, & \mathbf{y} \in S \\ 0, & \mathbf{y} \notin V \end{cases} \tag{3}$$

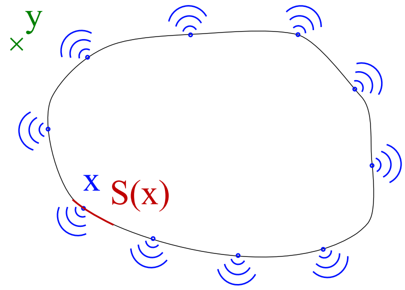

Figure 1: Sound radiating or scattering body in exterior domain

characterizes the sound field in terms of the sound pressure and its normal derivative on the boundary of the domain containing the sound field. It gives the sound pressure at a certain location $\mathbf{y}$ by integration of function of these two quantities over the boundary. Fig. 1 shows the location $\mathbf{y}$ and the location $\mathbf{x}$ at the boundary. If $\mathbf{y}$ is also placed at the boundary, the integral equation can be solved by a collocation approach to get $\underline{p}(\mathbf{x})$ or $\frac{\partial \underline{p}(\mathbf{x})}{\partial \mathbf{n_x}}$.

The EBEM is formulated in terms of energy quantities, namely the sound intensity $\mathbf{I}$ and the sound energy density w. It bases on the energy continuity equation, which reads

$$\nabla \cdot \mathbf{I} = 0 \tag{4}$$

in case of no dissipation and for stationary sound fields. This equation is applicable to any local region and assumes ensemble or frequency averaging. Hence, any method based on it is a statistical method.

For purely propagating waves the sound intensity is related to the energy density by the group velocity $\mathbf{I} = \mathbf{c}_g w$. The appropriate Green's function for (4) is:

$$\mathbf{H}(\mathbf{x}, \mathbf{y}) = \frac{1}{4\pi r^2}\frac{\mathbf{c}_g}{c_g},\ r = |\mathbf{x} - \mathbf{y}|\,. \tag{5}$$

Using Huygens principle integrals may be set up that characterize the sound field in terms of an "energy source strength" σ at the boundary:

$$\int_S \sigma(\mathbf{x})\mathbf{H}(\mathbf{x},\mathbf{y})\,\mathrm{d}\,S(\mathbf{x}) = \mathbf{I}(\mathbf{y}), \tag{6}$$

$$\int_S \sigma(\mathbf{x})G_w(\mathbf{x},\mathbf{y})\,\mathrm{d}\,S(\mathbf{x}) = w(\mathbf{y}). \tag{7}$$

If $\mathbf{y}$ is located at the boundary, $\mathbf{I}(\mathbf{y})$ can be related to σ using the absorption coefficient α. Thus, an integral equation may be constructed from (6):

$$\int_S \sigma(\mathbf{x})\mathbf{H}(\mathbf{x},\mathbf{y})\,\mathrm{d}\,S(\mathbf{x})\cdot\mathbf{n}_{\mathbf{y}} = -\frac{1}{2\,(1-\alpha(\mathbf{y}))}\sigma(\mathbf{y}),\ \mathbf{y}\in S. \tag{8}$$

This equation may be solved by a collocation approach to get $\sigma(\mathbf{x})$. From this, both $\mathbf{I}$ and w in the sound field may be calculated.

Generally, both (3) and (8) can be solved using a numerical boundary element technique. To this end, the boundary must be discretized into elements with a total of N nodes.

In case of the classical BEM, the sound pressure and its normal derivative are then interpolated by shape functions and the N nodal values of both quantities. Consequently, the integral equation (3) is transformed into a system of N equations with $2N$ unknowns. Thus N boundary conditions either for the sound pressure or its normal derivative are necessary to solve the problem. It can be noted in passing that the complex-valued coefficient matrix of the system strongly depends on the frequency and at certain frequencies a possibility for irregular solutions exists for exterior domain problems. Moreover, a certain frequency dependent mesh density is necessary for a satisfactory interpolation of the quantities on the boundary.

The EBEM requires only the quantity σ to be interpolated. A system of N equations is formed for the N nodal values of σ. No further boundary conditions are necessary. In contrast to the classical BEM, the real-valued coefficient matrix does not depend on frequency if c_g is not frequency dependent. No irregular solutions exist. The boundary mesh density has only minor influence on the result.

Examples

The first example is focused on sound scattering. A sphere is radiating sound that is scattered by a large cylinder shaped structure. The setup is shown in Fig. 2. For the classical BEM a fine mesh with 3266 nodes was used that allows for the calculation up to 370 Hz. The calculation was performed for 10 frequencies in the 250 Hz octave band using LMS VirtualLab software. The computing time amounted to approximately 15 min on a workstation. The results for the RMS sound pressure were averaged over the frequency to get a result for the 250 Hz octave band.

The EBEM calculation was performed for 250 Hz only using a coarse mesh with 797 nodes and Python-based software. The computing time was approximately 1.5 s.

The results for both methods are shown in Fig. 4. For direct comparison the sound pressure level normalized to the sound power level is plotted on a halfsphere with 10 m radius. While for the classical BEM distinct scattering patterns are visible, the EBEM result shows only a 'diffuse' scattering. The result could be improved using visibility tests[1], but this would have a considerable effect on the computing time.

In the second example sound radiation is considered. The setup consists of three spheres of slightly different diameter as shown in Fig. 3. A sector on one of the spheres acts as a source. Again, the BEM model uses a fine mesh (2663 nodes), while a coarse mesh (761 nodes) is used for the EBEM. The computing times are similar to that of the first example.

The results for the far field are compared in Fig. 5. While both results are similar, the sound pressure level on the surface of spheres is different. In particular, for the zones distant from the source sector a unrealistically low sound pressure level is predicted by the EBEM. This effect can also be seen in the vicinity of the spheres (Fig. 6). While the classical BEM predicts the scattered field in the shadow zone in detail, the EBEM result gives only a rough approximation.

Conclusion

A brief comparison of the classical boundary element method as a deterministic method and the energy boundary element method that uses some statistical assumptions was given. The results of two example calculations have shown that the precision of the classical BEM comes at higher computational cost than EBEM, while EBEM yields only approximate results. In case of the examples, the approximation was good only for the far field.

References

[1] Le Bot, A.: A vibroacoustic model for high frequency analysis. *Journal of Sound and Vibration* 211 (1998), 537–554

[2] Franzoni, L. ; Bliss, D. ; Rouse, J.: An acoustic boundary element method based on energy and intensity variables for prediction of high-frequency broadband sound fields. *The Journal of the Acoustical Society of America* 110 (2001), 3071–3080

[3] Wang, A. ; Vlahopoulos, N. ; Wu, K.: Development of an energy boundary element formulation for computing high-frequency sound radiation from incoherent intensity boundary conditions. *Journal of Sound and Vibration* 278 (2004), 413 – 436

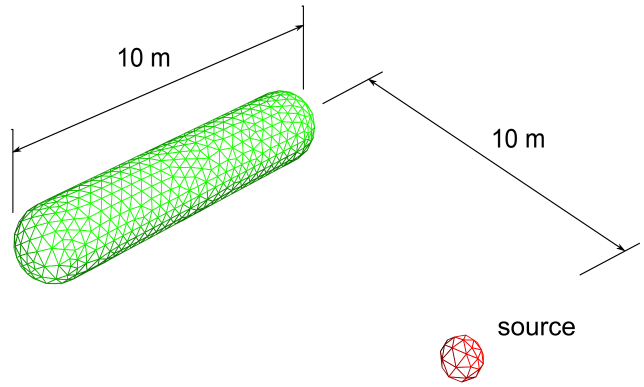

Figure 2: Example 1: scattering by a cylinder-shaped structure

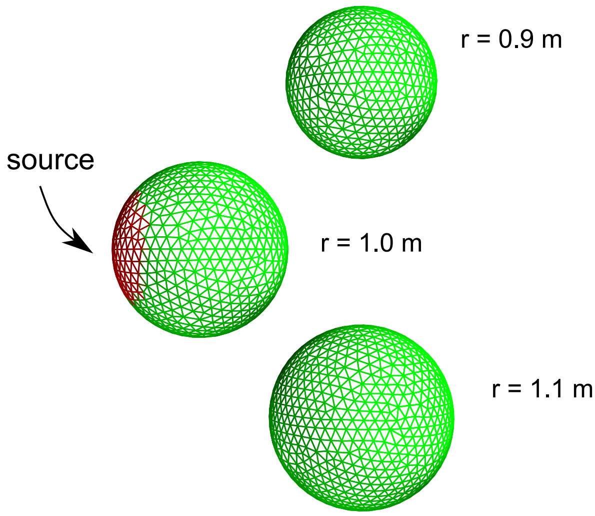

Figure 3: Example 2: three spheres with radiation from a sector on a sphere

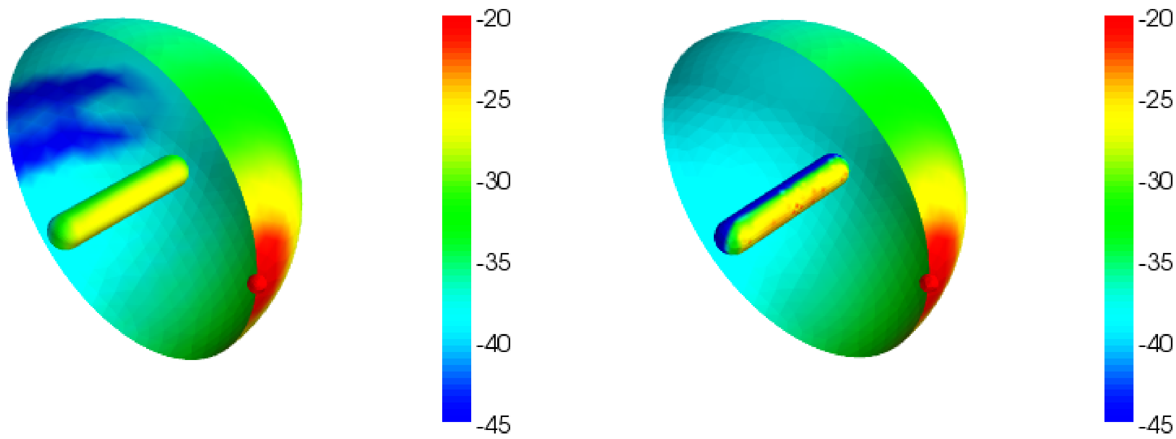

Figure 4: Results for the cylinder-shaped structure

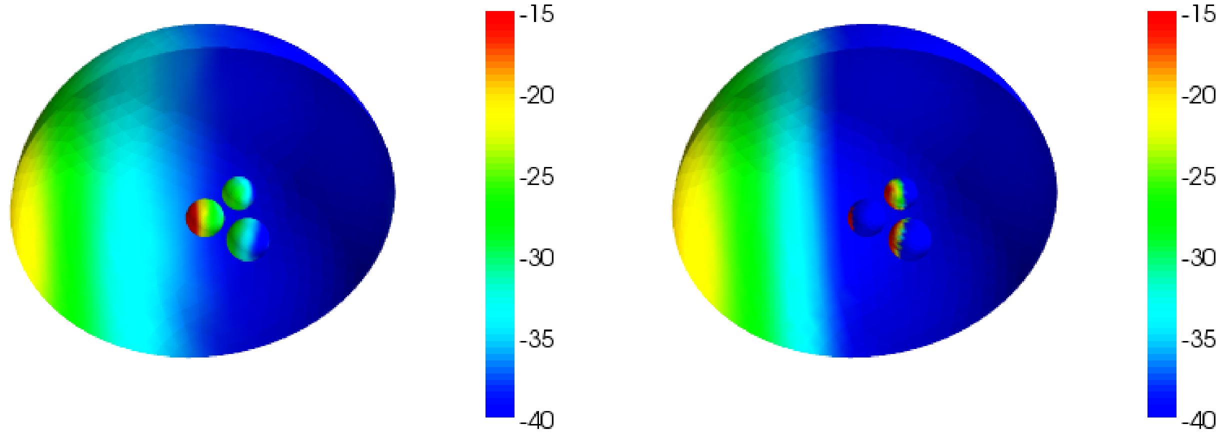

Figure 5: Far field results for the three spheres

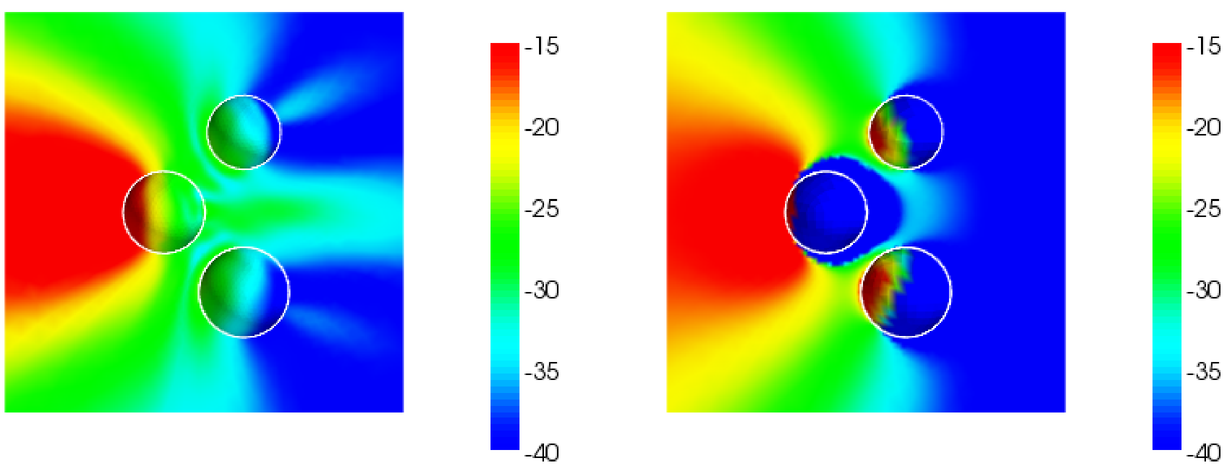

Figure 6: Results for the sound field in vicinity of the three spheres