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HIGH FREQUENCY BOUNDARY INTEGRAL METHOD AS AN ALTERNATIVE TO STATISTICAL ENERGY ANALYSIS

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Abstract

Statistical energy analysis (SEA) is the most established method for the prediction of sound and vibration transmission in complex structures today. However, SEA has some shortcomings. It is not an easy task to gain all necessary input information such as coupling loss factors nor do the SEA results always contain all the desired information. To solve this problem, several alternative methods have been proposed. On of these methods, introduced by A. Le Bot, generalises the power balance used in SEA for one subsystem and applies it to differential volumes. Starting from this point, an integral equation similar to the Kirchhoff-Helmholtz-equation may be set up that uses energy variables. This equation may then be solved using a boundary element technique. Very few information regarding the method and only some results on practical structures are available, so here some results are added from a practical point of view. The high frequency or energy boundary element method has been implemented. Simple test structures have been defined. Results on the test structures are compared to SEA and EFEM results. The method may be used as an alternative to SEA. The results are more detailed and are sufficient at least for flat isotropic structures.

I. INTRODUCTION

Common deterministic numerical methods like the finite element method (FEM) or the boundary element method (BEM) implicate a high computational effort when used for high frequency calculation. Besides other reasons, this circumstance usually forbids their application in such case. The method established for such problems is the statistical energy analysis (SEA). But in many cases SEA calculation gains results which are not differentiated enough. Therefore, SEA is not always applicable in an optimal manner. To solve this problem, several alternative methods were already proposed. Here, one of these methods shall be examined. The method was originally developed by Le Bot [1] and is a boundary integral method that uses energy variables. It is applicable to one-, two- and three-dimensional problems. In what follows, first an overview about the underlying theory is given. Then, the model of a simple structure is presented and the results of the calculation are discussed.

II. THEORY OVERVIEW

The starting point is the principle of energy conservation. While in SEA this principle is applied on a more global subsystem level, here an infinitesimal volume shall be considered. The energy content W of this volume may be altered only by:

1. "loss" of energy within the volume (due to dissipation),
2. injection of energy within the volume (due to excitation from outside the structure),
3. transport of energy through the boundary of the volume.

The transport of energy occurs always strictly with the group velocity $\mathbf{c_g}$ and therefore $\mathbf{c_g}W$ is the energy flow per area. For the energy a continuity relation may be set up and power loss P_{diss} and power input P_{in} may be added:

$$\frac{\partial}{\partial t}W = -\nabla \cdot (\mathbf{c_g}W) + P_{in} - P_{diss}. \tag{1}$$

If this equation is divided by the volume, a relation for the energy density w arises. The product of energy density and group velocity is the acoustical Poynting vector $\mathbf{I}$. Thus,

$$\frac{\partial}{\partial t}w = -\nabla \cdot \mathbf{I} + p_{in} - p_{diss}, \tag{2}$$

where p denotes power densities.

The power loss density may be related to energy density by use of the attenuation coefficient m:

$$p_{diss} = mc_g w. \tag{3}$$

The attenuation coefficient gives the attenuation of a plane propagative wave after a certain distance. It is related to the loss factor η by $m = \eta\omega/c_g$.

For steady state conditions, (2) becomes:

$$p_{in} = \nabla \cdot \mathbf{I} + mc_g w. \tag{4}$$

Now, **I** is time averaged too and may be called *sound intensity* as commonly done.

As long as no boundary conditions are formulated the application of this equation is limited to unbounded domains. In what follows, an indirect approach to solve this problem shall be used. It starts with the solution for the field of a point source at location S. This solution must be satisfy

$$\nabla \cdot (\mathbf{c_g} w(M)) + m c_g w(M) = 0, \qquad M \neq S. \tag{5}$$

If space is isotropic, the field is symmetric around S. Equation (5) reduces to

$$\frac{1}{r^{n-1}} \frac{\partial}{\partial r}(r^{n-1} w(M)) + m w(M) = 0, \qquad M \neq S. \tag{6}$$

According to problem dimensionality, $n = 1,2$ or 3. A possible solution for w and **I** respectively are:

$$A\,G(S,M) = \frac{\mathrm{e}^{-mr}}{r^{n-1}} \text{ and } A\,\mathbf{H}(S,M) = \mathbf{c_g} \frac{\mathrm{e}^{-mr}}{r^{n-1}} \tag{7}$$

with $r = |M - S|$. Because of the isotropy the group velocity vector is oriented in the direction of the wave propagation, so $\mathbf{c_g} = c_g \mathbf{u}_{SM}$, where $\mathbf{u}_{SM}$ is the unit vector from S to M. To determine the unknown factor A, mathematical distribution theory is used elsewhere [1]. Instead, the following simple consideration should be used here. Without damping ($m = 0$) the power flowing through a surface ∂S in a constant distance R from S must match the input power. This power may be estimated by integration of intensity over the surface elements $\mathrm{d}\mathbf{A}(M)$:

$$P_{in} = \int_{\partial S} \mathbf{H}(S,M) \mathrm{d}\mathbf{A}(M) = A \mathbf{c_g} \frac{1}{R^{n-1}} \int_{\partial S} \mathrm{d}\mathbf{A}(M). \tag{8}$$

With the dimension dependent full space angle $\gamma_n = 2, 2\pi, 4\pi$ for $n = 1,2,3$ the factor A is then:

$$A = \frac{P_{in}}{\gamma_n c_g}. \tag{9}$$

To introduce boundary conditions it is premised that the sound field within a domain Ω may be synthesised by superposition of the fields of one or more primary sources within the domain and the fields of secondary virtual sources existing on the boundary of the domain (Figure 1). This is an adoption of Huygens principle for energy variables and implies that energy densities and intensities caused by the sources may simply be added. The energy density in the field of a primary source at S with a uniform directivity and a source strength P_{in} is $w(M) = P_{in}/(\gamma_n c_g) G(S,M)$, see (9). For sources that are not point sources a source density $\rho(S)$ (source strength per volume) is more adequate. For the secondary sources at the boundary $\partial\Omega$ a directivity symmetric to the boundary

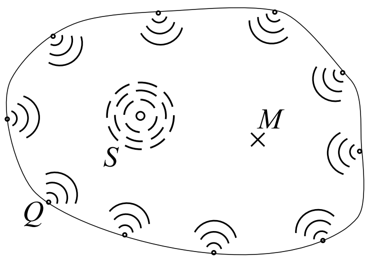

FIG. 1: Superposition at field point M of fields of primary (at S) and secondary sources at boundary points P

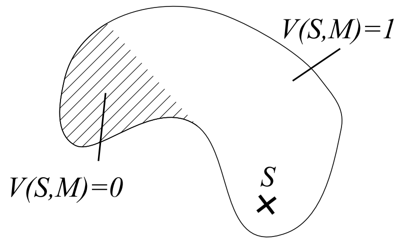

FIG. 2: Visibility at non-concave domain

normal may be accounted for. This may be done by using a function f of the angle between the normal $\mathbf{n}_Q$ at boundary point Q and the emission direction $\mathbf{u}_{MQ}$.

With the above assumptions energy density and intensity at field point M are given by

$$w(M) = \int_\Omega \rho(S)G(S,M)dS + \int_{\partial\Omega} \sigma(Q) f(\mathbf{u}_{MQ}, \mathbf{n}_Q) G(Q,M)dQ, \tag{10}$$

$$\mathbf{I}(M) = \int_\Omega \rho(S)\mathbf{H}(S,M)dS + \int_{\partial\Omega} \sigma(Q) f(\mathbf{u}_{MQ}, \mathbf{n}_Q) \mathbf{H}(Q,M)dQ. \tag{11}$$

At any point M within a non-concave shaped domain only those sources do contribute that are visible from M (Figure 2). A visibility function $V(S,M)$ that equals 1 if S is visible from M and equals 0 otherwise is multiplied to G and $\mathbf{H}$ to account for this:

$$G(S,M) = \frac{\mathrm{e}^{-mr}}{r^{n-1}} V(S,M) \qquad \text{and} \qquad \mathbf{H}(S,M) = \mathbf{c_g} \frac{\mathrm{e}^{-mr}}{r^{n-1}} V(S,M). \tag{12}$$

The source density per boundary area (or line, or point) $\sigma(Q)$ characterises the strength of a secondary source at Q. It depends on the power incident on Q. Moreover, a part of these power may be absorbed at Q (consider a porous absorber lining) or transmitted into another domain connected at the boundary. If the boundary-normal components of the intensity are considered, a simple boundary condition may be formulated:

$$I_{n,out}(Q) = \rho(Q) I_{n,in}(Q), \tag{13}$$

where ρ is the reflection efficiency. The normal component of a intensity radiated in a solid angle $\mathrm{d}\theta$ in the direction of another boundary point Q'' is:

$$\mathrm{d}I_{n,out}(Q) = c_g \sigma(Q) f(\mathbf{u}_{Q''Q}, \mathbf{n}_Q) \mathrm{d}\theta. \tag{14}$$

As space is considered isotropic, c_g is independent of θ and the total intensity integrated over the half space visible from Q is then

$$I_{n,out}(Q) = c_g \sigma(Q) \gamma \qquad \text{where} \quad \gamma = \int_{HR} f(\mathbf{u}_\theta, \mathbf{n}) d\theta. \tag{15}$$

Incident waves may come from secondary as well as from primary sources within the domain. The normal component of the intensity of a wave originating from boundary point Q' may be calculated from

$$\mathrm{d}I_{n,in,2} = \mathrm{d}\mathbf{I}_{n,in,2} \cdot \mathbf{n}_Q = \sigma(Q')\mathbf{H}(Q',Q) f(\mathbf{u}_{Q,Q'}, \mathbf{n}_{Q'}) \mathrm{d}Q' \cdot \mathbf{n}_Q, \tag{16}$$

where $\mathrm{d}Q'$ is the boundary section covered by the source. Integration over the whole boundary yields the total intensity $I_{n,in,2}$.

The intensity of a primary source of extension $\mathrm{d}S$ at S is

$$\mathrm{d}I_{n,in,1} = \mathrm{d}\mathbf{I}_{n,in,1} \cdot \mathbf{n}_Q = \rho(S)\mathbf{H}(S,Q)\mathrm{d}S \cdot \mathbf{n}_Q. \tag{17}$$

Integration over the domain together with (15) and (16) results in an integral equation:

$$\sigma(Q) = \frac{\rho}{\gamma c_g} \left(\int_\Omega \rho(S)\mathbf{H}(S,Q) dS + \int_{\partial\Omega} \sigma(Q') f(\mathbf{u}_{QQ'}, \mathbf{n}_{Q'}) \mathbf{H}(Q',Q) dQ' \right) \cdot \mathbf{n}_Q. \tag{18}$$

This equation may be solved numerically using a boundary element technique. The boundary is then partitioned into N flat boundary elements $\partial\Omega_i$. Integration over $\partial\Omega$ is now equivalent to the summation over integrals over the N elements. If the integrand is assumed to be constant over the element the integral may be approximated by simple multiplication of the integrand at element mid point Q_i and element area. The integration over Ω may be substituted by a summation over a finite number of sources N_S. Thus, the integral equation becomes a system of linear equations:

$$\sigma_i = \frac{\rho}{\gamma c_g} \left(\sum_{j=1}^{N_S} \frac{P_{in,j}}{\gamma_n c_g} \mathbf{H}(S_j, Q_i) \cdot \mathbf{n}_i + \sum_{\substack{j=1 \\ j \neq i}}^{N} \sigma_j f(\mathbf{u}_{Q_i Q_j}, \mathbf{n}_j) \mathbf{H}(Q_j, Q_i) \cdot \mathbf{n}_i \right), \qquad i = 1 \ldots N. \tag{19}$$

Solution of this system yields the source strengths σ_i and enables the calculation of energy density and intensity using (10) and (11) at any point M within the domain. Necessary integrations are again carried out numerically:

$$w(M) = \sum_{j=1}^{N_S} \frac{P_{in,j}}{\gamma_n c_g} G(S_j, Q_i) + \sum_{j=1}^{N} \sigma_j f(\mathbf{u}_{MQ_j}, \mathbf{n}_j) G(Q_j, M), \tag{20}$$

$$\mathbf{I}(M) = \sum_{j=1}^{N_S} \frac{P_{in,j}}{\gamma_n c_g} \mathbf{H}(S_j, Q_i) + \sum_{j=1}^{N} \sigma_j f(\mathbf{u}_{MQ_j}, \mathbf{n}_j) \mathbf{H}(Q_j, M). \tag{21}$$

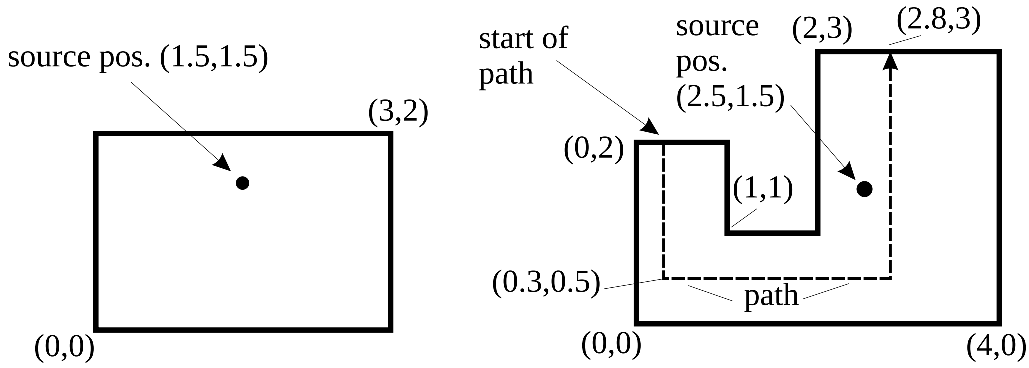

FIG. 3: Test structures: simple rectangular plate (left), non concave plate (right)

It should be noted in passing that the method described above can easily be extended to handle several interconnected domains [1].

III. IMPLEMENTATION

To test the method, it was implemented using the Python programming language. The code allows the computation of problems in one single two dimensional domain. Possible directivity functions are 1 and $\mathbf{u} \cdot \mathbf{n}$. Visibility tests are implemented in a simple exact, but computationally expensive manner. As the calculation of the visibility function get even more expensive with the number of boundary elements, the code becomes ineffective if the number of elements is too high.

IV. TEST STRUCTURES AND MODELS

Two test structures were examined. The first structure is a 1 mm thick rectangular flat steel plate ($E = 2.1 \cdot 10^{11}$ Pa, $\rho = 7800$ kg/m^3) of dimensions 2 m$\times$3 m that is exited with P_{in} =1 W at (1.5 m,1.5 m) from the corner.

The second test structure is somewhat more complex. It is again a flat steel plate with the same thickness and material parameters as above, but it is not concave shaped and its overall dimensions are 4 m$\times$3 m.

For both structures boundary element meshes with variable element size and number were used. An additional mesh of triangular elements enabled the visualisation of the energy density on the plates. The energy finite element results reported below were based on a mesh of elements of size 0.2 m.

V. RESULTS

The first objective was to examine the reliability of the method. To this end, the simple rectangular plate was calculated at different frequencies and different damping values. Numerical integration over the elements in the domain yields the total energy. This result was compared to the result of an SEA model. The SEA model used in this case was very simple and consisted of only one subsystem. The energy stored in the plate may be calculated from $W = P_{in}/\omega\eta$, one of the basic SEA relations.

Calculation were carried out for the frequencies 1 kHz and 10 kHz and damping values η =0.1%, 1% and 10% and for both possible directivity functions. Boundary meshes with an element size of 1 m, 0.5 m, 0.25 m, 0.1 m and 0.05 m were used subsequently.

The relative error made in the boundary element calculation is characterised by $err = (W_{BE} - W_{SEA})/W_{SEA}$. This error is plotted in Figure 4 against the size of the boundary element used. Only for a constant or uniform directivity the error becomes acceptable small with increasing number of elements, i.e. decreasing element size. This suggests that at least for two dimensional domains and bending waves the method does not yield meaningful results if $\mathbf{u} \cdot \mathbf{n}$ is used as directivity function but provide plausible results if a constant directivity is applied.

A second objective was to compare the method with a rival method, the energy finite element method (EFEM), e.g. [2]. This method starts at the same differential equation (4), but states then:

$$\mathbf{I} = -\frac{c_g^2}{\eta\omega}\nabla w \qquad \text{and consequently} \qquad p_{in} = -\frac{c_g^2}{\eta\omega}\nabla w + \eta\omega w \tag{22}$$

in analogy to the static heat conduction equation. This analogy is not exact [3] in the case considered here, but it is sometimes argued that the error introduced will not be too large for practical structures.

The results from the two methods were compared with each other and with the results from a simple one-subsystem SEA model. Calculations were made for three different frequency / loss factor combinations. In Figure 5 the energy density is plotted along a 6.25 m long path depicted in Figure 3. It is obvious that both EFEM and the HF-BE method yield more detailed results compared to SEA. Moreover, especially in that region of the path which is not visible from the source, EFEM predicts much higher energy levels than the HF-BE method as it neglects any shadowing effects. In addition, EFEM does not handle the direct field in a correct way. It was shown elsewhere that it leads to an $1/\sqrt{r}$ dependence of energy density rather than to the correct $1/r$ dependence [1]. This behaviour may lead to over-prediction in the case with stronger damping. On the other hand side, though the results of HF-BE method seem to be more plausible, a proof of exactness cannot be given here.

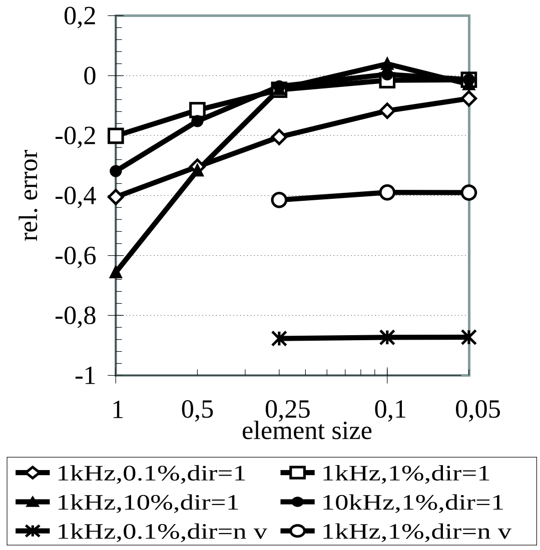

FIG. 4: Relative error versus element length for the rectangular plate

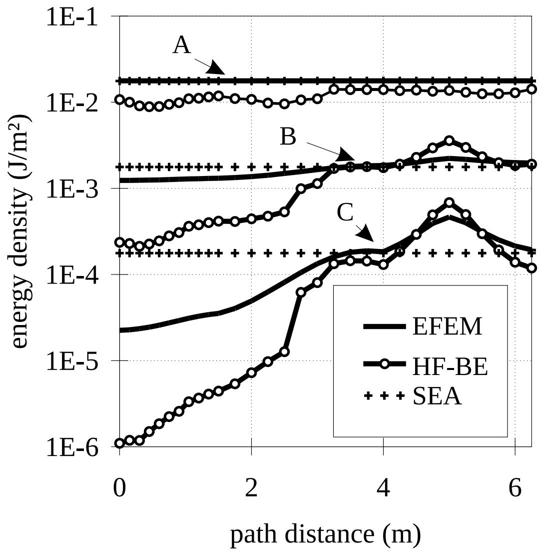

FIG. 5: Plot of energy density along the path (A: f=1kHz, η=0.1%, B: f=1kHz, η=1%, C: f=10kHz, η=1%)

VI. CONCLUSION

A power flow method for the prediction of high frequency vibration was examined that bases on an integral formulation for the energy density in a bounded domain. Numerical solution is provided by means of a boundary element technique. The results for two simple test structures were examined with the objective to examine the reliability and exactness of the method. If was found that the method yields more plausible results than the energy finite element method and therefore may serve as an alternative to SEA. Is especially useful when the effect of the direct field is dominating.

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