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# AEROACOUSTIC SOURCE CHARACTERISATION USING SUBSPACE AND DECONVOLUTION TECHNIQUES

Ennes Sarradj, Thomas Geyer, Jens Giesler

Brandenburgische Technische Universität Cottbus
Siemens-Halske-Ring 14, 03046 Cottbus, GERMANY

## ABSTRACT

*The use of phased microphone arrays has become a standard technique for aeroacoustic testing. The ultimate goal is to characterise aeroacoustic source mechanisms with respect to their location, their strength and their spectrum. A number of different analysing methods are available, ranging from classic delay-and-sum beamforming to more sophisticated methods that involve deconvolution of the beamforming map. Practical results from wind tunnel measurements using different processing techniques are presented. Both aerofoil trailing edge noise and noise from a rod-aerofoil configuration serve as examples. The processing techniques used include classic methods as well as such using the signal subspace, such using the noise subspace, the deconvolution approach for the mapping of acoustic sources (DAMAS) and a deconvolution method based on spatial source coherence (CLEAN-SC). The comparison from the results of different techniques shows significant discrepancies. This leads to the conclusion that not a single technique should be used for source characterisation, but the right one should be chosen depending on whether the location or the source strength is of primary interest.*

## 1 INTRODUCTION

The characterisation of any source of air-borne noise requires the knowledge of its location, its strength, its spectrum and its directivity. A number of techniques for the experimental determination of these source characteristics are available. In the case of aeroacoustic sources, whose source mechanisms are connected to an air flow, methods are preferred that allow for a measurement with microphones or other types of sensors out of the flow. Since the simultaneous application of many sensors yields more information than a single microphone, the use of an array of microphones has become a standard technique for aeroacoustic testing.

The output of a microphone array is processed using dedicated signal processing algorithms. The original idea behind these algorithms is to compensate for the different phase or delay of individual microphones. Such *phased* arrays are directional sound receivers with steerable directional characteristics. The underlying beamforming signal processing enables the basic characterisation of a source in terms of its location and strength. Depending on the array, the source and the testing environment, the performance of the source characterisation by means of a simple directional receiver is limited. However, with the increased popularity of microphone arrays, a number of more sophisticated signal processing methods are in use. Some of these algorithms were originally developed for array applications in RADAR, SONAR, telecommunications or radio-astronomy. The general approach is to utilise more information from the array measurement in order to improve the quality of the source characterisation. While some of the methods enable a better source localisation, others focus on the improved determination of source strength and spectrum.

The present paper aims to give a short overview of some analysing methods for microphone array measurements used in aeroacoustic testing. First, the theory for each method is presented briefly. Secondly, two experimental set-ups in an aeroacoustic wind tunnel are introduced. Finally, the respective advantages and shortcomings of the methods are discussed by means of some practical results from tests using the experimental set-ups.

## 2 METHODS

The basis for all signal processing methods is a model of the sound field and the array. Since all of the methods to be outlined in the following sections work in the frequency domain, the model is presented here using complex notation.

A sound source at a location $\mathbf{x}_0$ with a strength $q$ will cause certain sound pressures at the microphones. The vector $\mathbf{p}$ of these sound pressures is thus given by

$$\mathbf{p} = \mathbf{a}(\mathbf{x}_0)q, \tag{1}$$

where $\mathbf{a}$ denotes the respective set of transfer functions from the source to the individual microphone locations. The basic idea of beamforming is to compute the sum of the microphone signals weighted by a complex factor. With the vector of weights $\mathbf{h}$, the beamformer output signal is

$$p_F(\mathbf{x}_t) = \mathbf{h}^H(\mathbf{x}_t)\mathbf{p}. \tag{2}$$

In a practical measurement it is sensible to make use of averaging techniques. Thus, the computation of the beamformer output power is more appropriate:

$$B(\mathbf{x}_t) = \overline{p_F(\mathbf{x}_t)p_F^*(\mathbf{x}_t)} = \mathbf{h}^H(\mathbf{x}_t)\overline{\mathbf{p}\mathbf{p}^H}\mathbf{h}(\mathbf{x}_t) = \mathbf{h}^H(\mathbf{x}_t)\overline{\mathbf{G}}\mathbf{h}(\mathbf{x}_t). \tag{3}$$

The cross spectral matrix $\mathbf{G}$ is estimated using ensemble averaging. Depending on the weights, Equation (3) constitutes a spatial filter.

### 2.1 Delay and Sum

One possible design goal for such spatial filter is to let pass all sound from the location $\mathbf{x}_0$ unattenuated, but to attenuate sound from all other possible directions as much as possible. It can be shown [1], that this leads to

$$\mathbf{h}(\mathbf{x}_t) = \frac{\mathbf{a}(\mathbf{x}_t)}{\mathbf{a}^H(\mathbf{x}_t)\mathbf{a}(\mathbf{x}_t)} \tag{4}$$

and thus to

$$B(\mathbf{x}_t) = \frac{\mathbf{a}^H(\mathbf{x}_t)\overline{\mathbf{G}}\mathbf{a}(\mathbf{x}_t)}{\left(\mathbf{a}^H(\mathbf{x}_t)\mathbf{a}(\mathbf{x}_t)\right)^2}. \tag{5}$$

In time domain this reduces to a simple delay of each signal to compensate for the different times of flight from the source to the microphones and to subsequent summation. Thus, this algorithm is often termed "delay and sum" (DS). A refinement often used in the context of aeroacoustic testing is to delete the main diagonal of $\mathbf{G}$ prior to calculation. In the presence of noise, the localisation is improved at the cost of a somewhat erroneous estimation of source strength. Any results given below were computed using this diagonal removal technique.

## 2.2 Capon

Another possibility to design the spatial filter is to let pass all sound from the source location while attenuating as much as possible all signals from other directions that are actually present. Because the characteristics of this filter depend themselves on the input signals, it is an adaptive filter. The weight vector is given by

$$\mathbf{h}(\mathbf{x}_t) = \frac{\mathbf{G}^{-1}\mathbf{a}(\mathbf{x}_t)}{\mathbf{a}^H(\mathbf{x}_t)\mathbf{G}^{-1}\mathbf{a}(\mathbf{x}_t)} \tag{6}$$

and thus the beamformer output power is

$$B(\mathbf{x}_t) = \frac{1}{\mathbf{a}^H(\mathbf{x}_t)\mathbf{G}^{-1}\mathbf{a}(\mathbf{x}_t)}.$$

This approach was introduced by Capon [2] and is also referred to as "minimum variance" beamformer.

## 2.3 MUSIC

The model (1) does not account for any noise that may be present at the microphones in addition to the sound from a source. An extended model that accounts for multiple sources and additional noise is

$$\mathbf{p} = \mathbf{A}\mathbf{q} + \mathbf{n}, \tag{7}$$

where $\mathbf{n}$ represents the noise. With equal amplitude of the noise at each microphone, the cross spectral matrix used in (3) is then

$$\mathbf{G} = \mathbf{A}\mathbf{q}\mathbf{q}^H\mathbf{A}^H + n^2\mathbf{I}. \tag{8}$$

For $M$ sources this matrix has $M$ possibly different eigenvalues ($\mathbf{A}\mathbf{q}\mathbf{q}^H\mathbf{A}^H$ has rank $M$) and $N - M$ equal eigenvalues $n^2$. The latter are said to span the noise subspace of $\mathbf{G}$, while the first span the signal subspace. From the respective eigenvectors individual components of the cross spectral matrix may be resynthesised:

$$\mathbf{G} = \sum_{i=1}^{M}\mathbf{G}_i + \sum_{i=M+1}^{N}\mathbf{G}_i = \sum_{i=1}^{M}\mathbf{G}_i + \mathbf{G}_n. \tag{9}$$

The matrix $\mathbf{G}_n$, computed from the set of noise subspace eigenvectors, may be used in the multiple signal classification (MUSIC) estimator [3]

$$B(\mathbf{x}_t) = \frac{\mathbf{a}^H(\mathbf{x}_t)\mathbf{a}(\mathbf{x}_t)}{\mathbf{a}^H(\mathbf{x}_t)\mathbf{G}_n\mathbf{a}(\mathbf{x}_t)}.$$

This estimator exhibits sharp local maxima at the location of sources. Because it is constructed using only the noise subspace, no quantitative information is available.

## 2.4 Orthogonal Beamforming

Another possibility is to use the components from the signal subspace for component-based delay-and-sum beamforming [4], where $\mathbf{G}$ is replaced by $\mathbf{G}_i$ in Equation (5). The individual beamformer output powers $B_i$ map approximately to one source or source mechanism.

A further refinement [5] of this idea is to estimate the location of each of the $M$ sources from the maximum of the respective $B_i$ and to assess the strength of the sources from the corresponding eigenvalues. This allows to synthesise a spatial filter with high resolution and good quantitative estimation of source strengths. Because the orthogonality of eigenvectors is vital to the method, the name orthogonal beamforming (OB) was proposed.

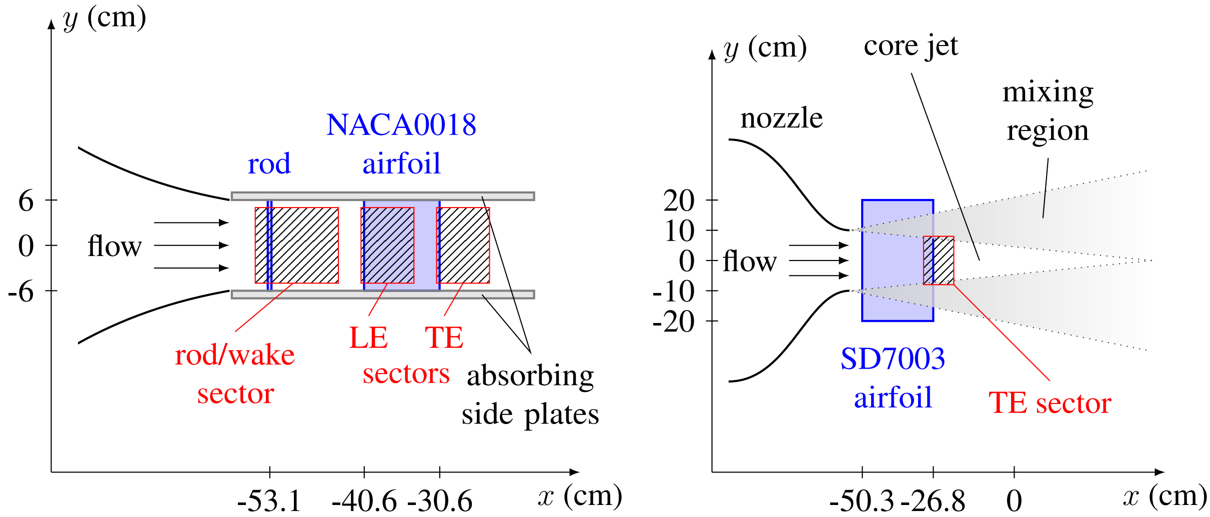


Figure 1: Schema of the set-up for rod-aerofoil noise test with sectors for integration of rod / wake noise, leading edge (LE) and trailing edge (TE) noise (*left*) and schema of the set-up for the measurement of aerofoil trailing edge noise (*right*)

### 2.5 DAMAS

The delay-and-sum beamformer output is usually plotted using a grid of locations $x_t$. The result is a map of apparent sound pressure contributions. Thus, DS beamforming may be interpreted as an imaging system that maps a source distribution $S(\mathbf{x})$ to an image $B(\mathbf{x})$. The properties of this map are characterised by the point spread function $P$. This function describes how the point sources at individual $\mathbf{x}_s$ are represented in the beamformer output:

$$B(\mathbf{x}_t) = \sum_{\mathbf{x}_s} P(\mathbf{x}_s, \mathbf{x}_t) S(\mathbf{x}_s) \qquad \text{for each} \quad \mathbf{x}_t. \tag{10}$$

As $P$ may be estimated theoretically and $B$ is available from the DS beamforming result, this system of equations may be solved for the unknown $S(\mathbf{x}_s)$ [6]. The solution requires a special variant of the Gauss-Seidel iterative solver that ensures the physically required positivity of $S$. As Equation (10) can be seen as a convolution operation, following [6] this approach is called "deconvolution approach for the mapping of acoustic sources" (DAMAS).

### 2.6 CLEAN-SC

Another approach for the deconvolution is the CLEAN algorithm chiefly used in radioastronomy. This algorithm defines the location of the highest peak of the DS beamformer output map $B(\mathbf{x}_t)$ as the location of a point source and subtracts the respective point spread function from the map. This is iteratively repeated until a stop criterion is reached. Among the many variants of this approach, one was developed with aeroacoustic application in mind. Unlike CLEAN, the CLEAN-SC approach [7] uses no theoretically computed point spread functions but those parts of the map that are spatially coherent to the highest peak. An effective computational algorithm avoids the explicit calculation of this coherence maps, making CLEAN-SC faster than CLEAN.

## 3 WIND TUNNEL SET-UPS

The results of the different methods are compared using tests on two set-ups in the aeroacoustic wind tunnel of the Brandenburg University of Technology at Cottbus (BTU). This tunnel is a small open jet facility with exchangeable nozzles and an open test section.

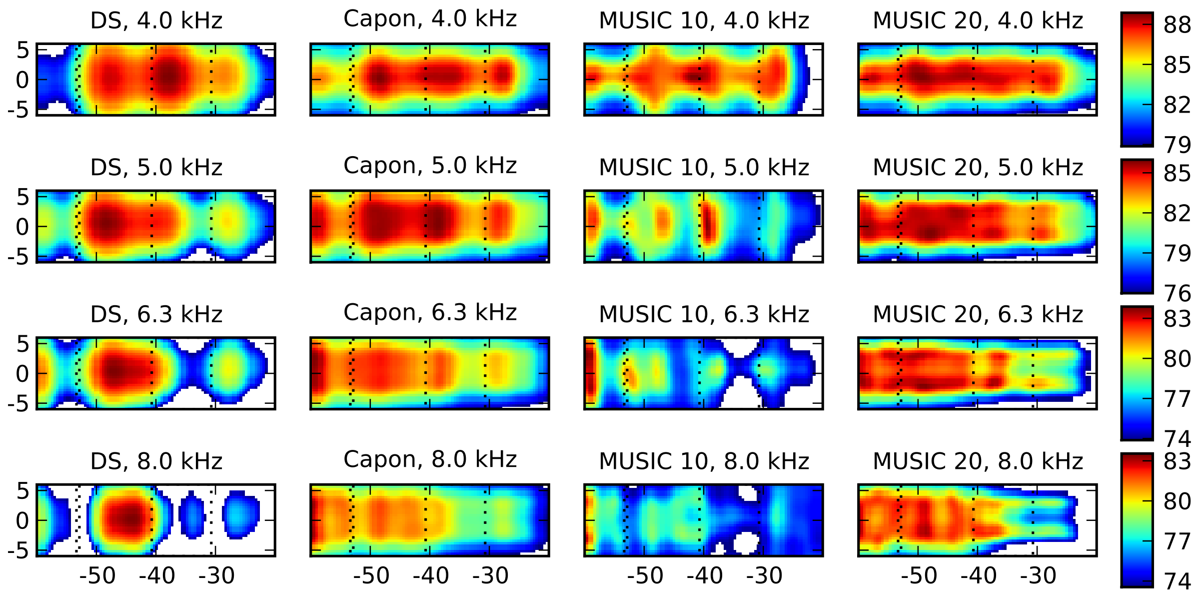


Figure 2: Maps of third-octave sound pressure contributions for the rod-aerofoil set-up: delay-and-sum (DS), Capon, MUSIC with $M = 10$ and $M = 20$ methods (MUSIC results are scaled with respect to the maximum value)

The first set-up focuses on the broadband noise generation at a NACA 0018 aerofoil in the wake of a small cylinder with circular cross-section. This rod-aerofoil configuration is placed between two sound-absorbing side-plates in front of a 120 mm by 147 mm nozzle. At a flow speed of 72 m/s, the vortex shedding frequency is approximately 2.9 kHz. Broadband noise is of interest in the frequency range above. Possible broadband noise sources are the rod and its wake and both the trailing edge (TE) and the leading edge (LE) of the aerofoil. A 38 microphone array with a 1.4 m by 0.4 m aperture was used for the measurement.

In the second set-up, the trailing edge (TE) noise at an SD7003 aerofoil was examined in an open jet of 200 mm diameter and a flow speed of 50 m/s. The aerofoil has a chord length of 235 mm and a span of 400 mm. It was placed in front of the nozzle with its mountings outside of the flow. Thus, possible sources present include the trailing edge (TE) and the leading edge (LE) of the aerofoil as well as the interaction of the jet shear layer with the aerofoil. A 56 microphone array was applied in this case.

## 4 RESULTS

The DS beamforming method is easily applied for source characterisation. In Fig. 2, results are shown for the frequency range of interest. While it is possible to identify the three main sources from the DS result, localisation is not very precise and it is not easy to guess the source strength, especially for lower frequencies. Capon results show generally better source localisation at lower frequencies, but the dynamic range of the map is even lower than for the DS results. Thus, the method is restricted to those few applications where this is not of concern. The MUSIC algorithm is capable of precise localisation of sources, if the the parameter $M$ (assumed number of sources) is chosen correctly ($M = 10$ gives good results in the example). With incorrect $M$, the results are meaningless as shown in Fig. 2 for $M = 20$. All methods compared so far exhibit a low dynamic

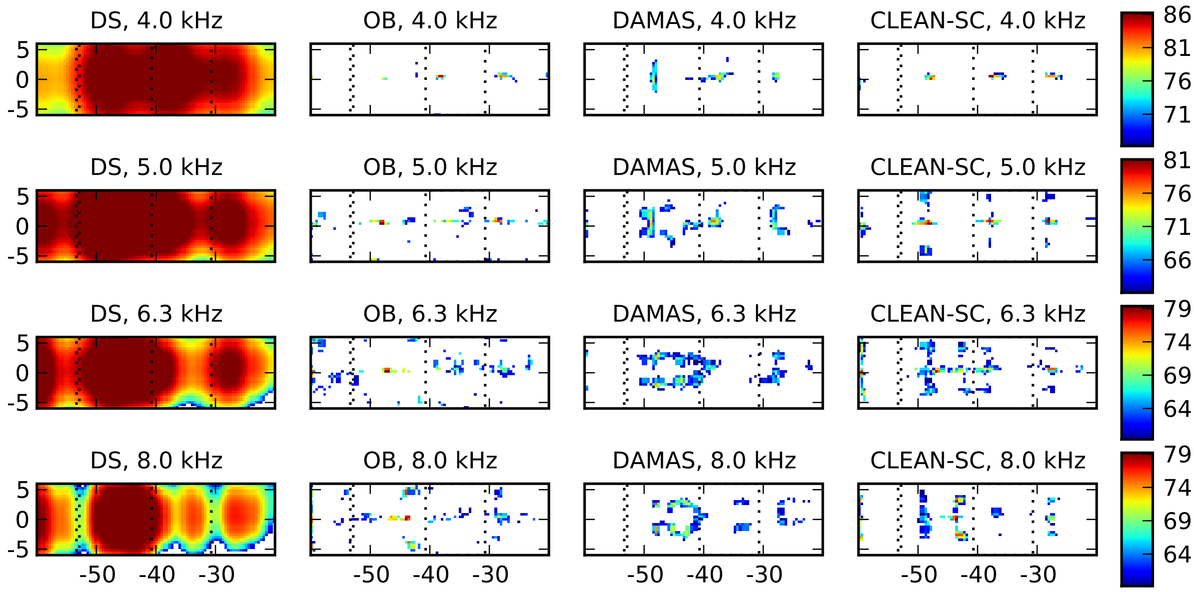


Figure 3: Maps of third-octave sound pressure contributions for the rod-aerofoil set-up: delay-and-sum (DS), orthogonal beamforming using 19 eigenvalues (OB), DAMAS with 5000 iterations and CLEAN-SC

range in the result maps. Moreover, the quantitative estimation of source strength and spectrum is difficult as apparent and real sources cannot always be distinguished.

The results of some deconvolution methods are compared in Fig. 3 to those of DS beamforming. While these methods are more complicated to implement, the quality of source characterisation is much improved. The maps have a higher dynamic range and the source locations are represented more clearly. Orthogonal beamforming is the fastest method with a computational effort comparable to DS beamforming. However, for low frequencies it suffers from a similar uncertainty in the localisation of sources. DAMAS results show a detailed map of sources, but the computational effort is extremely high. The CLEAN-SC method is a compromise with result maps of acceptable quality also for low frequencies and a very moderate computational effort. As shown in Fig. 4, all three methods have a comparable performance with regard to the quantitative estimation of source spectra. All methods reproduce the tonal noise at the rod vortex shedding frequency at 2.8 khz. At 4 kHz, OB fails to map the sources into the rod/wake sector, while both DAMAS and CLEAN-SC have some difficulties with the weak TE noise source at higher frequencies.

In the second set-up, the aerofoil TE noise is the only source of interest but is much weaker than the sources from the rod-aerofoil set-up. Because of the low turbulence in the tunnel flow, the aerofoil LE noise is absent except for the noise from the interaction of the wind tunnel shear

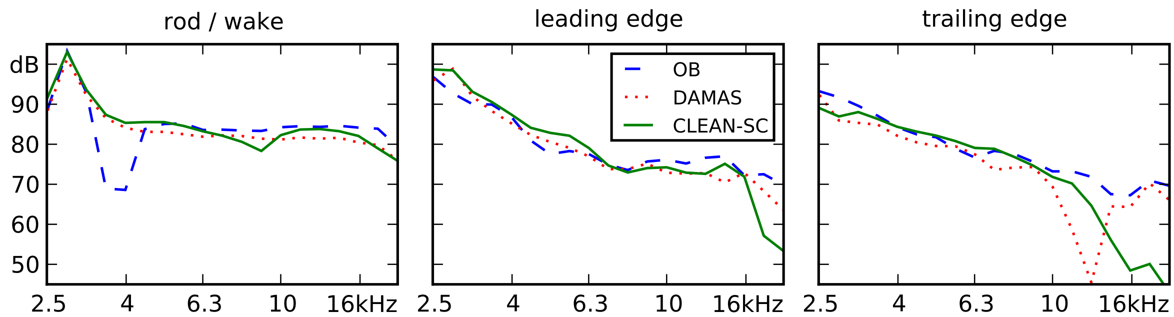


Figure 4: 1/6-octave spectra for the rod-aerofoil set-up: methods as in Fig. 3, spectra are estimated from an integration over sectors of the maps, respective sectors are shown in Fig. 1

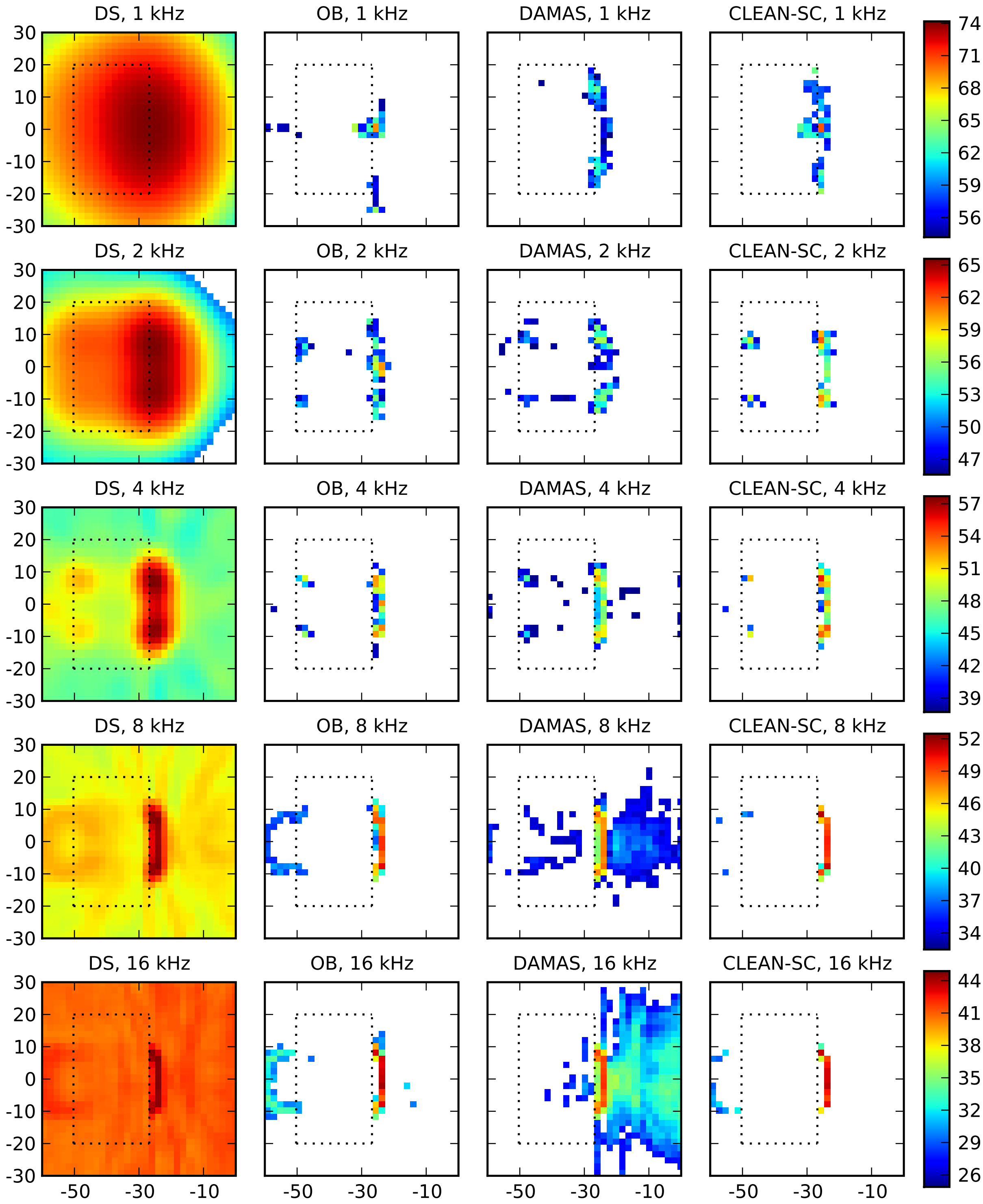


Figure 5: Maps of octave sound pressure contributions for the aerofoil set-up: delay-and-sum (DS), orthogonal beamforming using 16 eigenvalues (OB), DAMAS with 5000 iterations and CLEAN-SC

layer with the leading edge. Fig. 5 shows results computed with OB, DAMAS and CLEAN-SC compared to DS beamforming. Similar conclusions as in the case of the rod-aerofoil set-up may be drawn for these three methods. Additionally, it can be noted from the 8 and 16 kHz results that the performance of OB and CLEAN-SC is more robust with weak sources. The main reason is that DAMAS relies on the theoretical point spread functions and is not able to remove uncorrelated noise

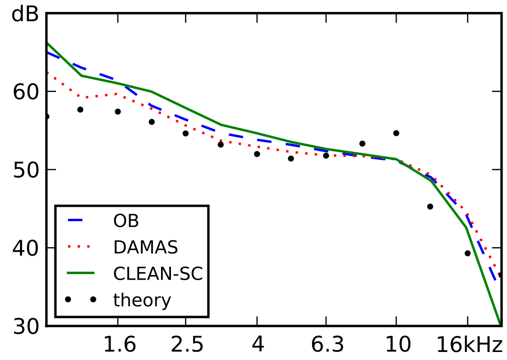


Figure 6. third-octave spectra of trailing edge noise: methods as in Fig. 5, theory after [8]

from the map. Finally, Fig. 6 shows TE spectra from the integration over the trailing edge sector. The results are of similar quality for all three methods and show good agreement with theoretical results in the considered frequency range.

## 5 CONCLUSION

A number of methods for the processing of microphone array measurements were compared with respect to their performance for the characterisation of aeroacoustic sound sources. Recent signal subspace and deconvolution methods allow generally an improved source characterisation compared to classic beamforming methods. Example results using different techniques show some discrepancies. Thus, not a single technique should be used for source characterisation, but the right one should be chosen depending on whether the location or the source strength is of primary interest.

## REFERENCES


[1] P. Stoica and R.L. Moses. *Introduction to spectral analysis*. Prentice Hall, Upper Saddle River, NY, 1997.

[2] J. Capon. High-resolution frequency-wavenumber spectrum analysis. *Proc. IEEE*, 57(8):1408–1418, 1969.

[3] R. Schmidt. Multiple emitter location and signal parameter estimation. *IEEE T. Antennas Propag.*, 34(3):276–280, 1986.

[4] E. Sarradj, C. Schulze, and A. Zeibig. Identification of noise source mechanisms using orthogonal beamforming. In *Noise and Vibration: Emerging Methods*, 2005.

[5] E. Sarradj. Quantitative source spectra from acoustic array measurements. In *Berlin Beamforming Conference*, 2008.

[6] T. F. Brooks and W. M. Humphreys. A deconvolution approach for the mapping of acoustic sources (DAMAS) determined from phased microphone arrays. AIAA paper 2004-2954, 2004.

[7] P. Sijtsma. CLEAN based on spatial source coherence. AIAA-paper 2007-3436, 2007.

[8] T. F. Brooks, D. S. Pope, and M. A. Marcolini. Airfoil self-noise and prediction. *NASA Reference Publication 1218*, 1989.